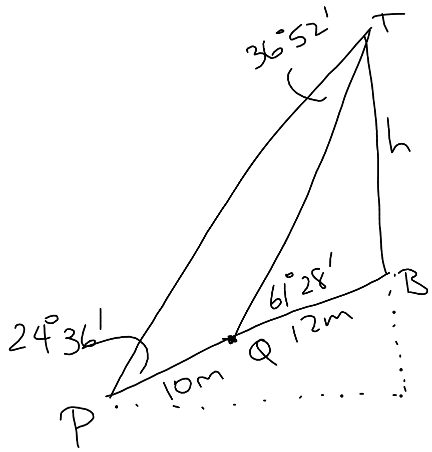
2b)

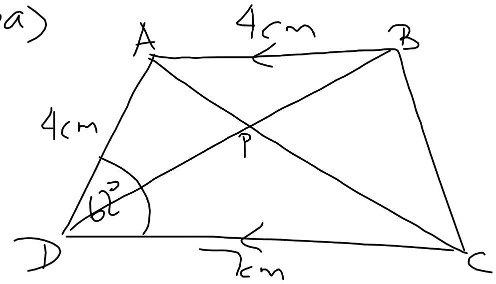


a) $TQ = \dfrac{10\sin 24^\circ 36'}{\sin 36^\circ 52'}$

b) $h^2 = TQ^2 + 12^2 - 2 \times TQ \times 12 \cos 61^\circ 28'$

$h = 10.61\,m$

6a)



$AC^2 = 4^2 + 7^2 - 2 \times 4 \times 7 \cos 62°$

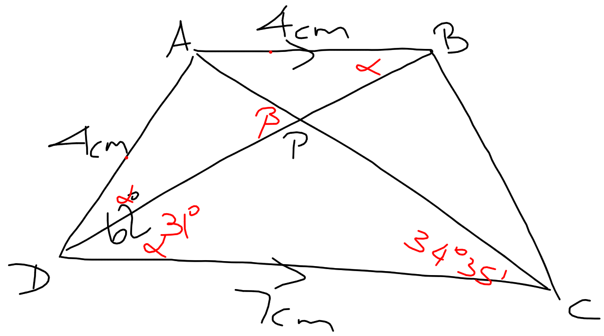
$AC = 6.2$ cm (to 1 dp)

$$\frac{\sin \angle ACD}{4} = \frac{\sin 62}{AC}$$

$$\sin \angle ACD = \frac{4 \sin 62}{AC}$$

$\angle ACD = 34° 35'$

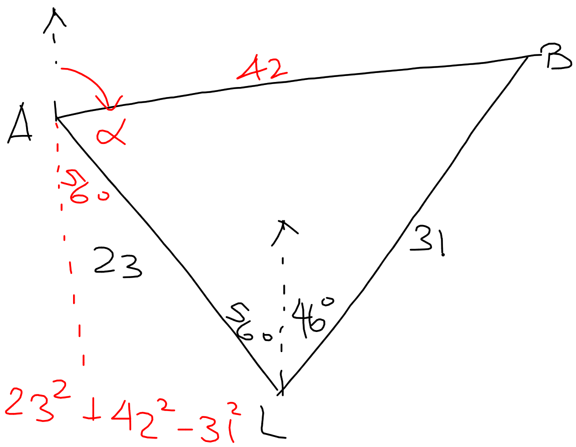
6 c)



$\beta = 31° + 34°35'$

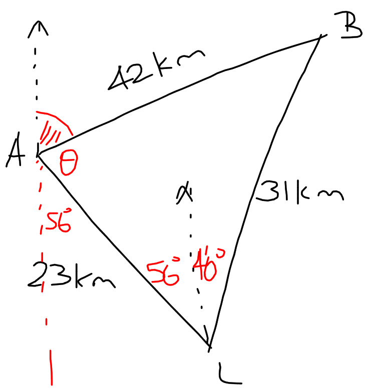
$= 65°35'$

10b)



$$\cos\alpha = \frac{23^2 + 42^2 - 31^2}{2 \times 23 \times 42}$$

$=$



$$\frac{\sin\theta}{31} = \frac{\sin 102}{42}$$

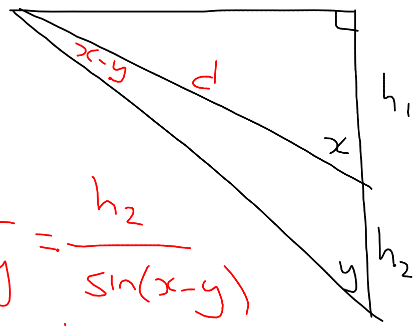
$$\sin\theta = \frac{31 \sin 102}{42}$$

$$\underline{\theta = 46°13'}$$

$$\text{bearing} = \underline{077°47'}$$

12

Prove: $h_1 = \dfrac{h_2 \cos x \sin y}{\sin(x-y)}$



$$\frac{d}{\sin y} = \frac{h_2}{\sin(x-y)}$$

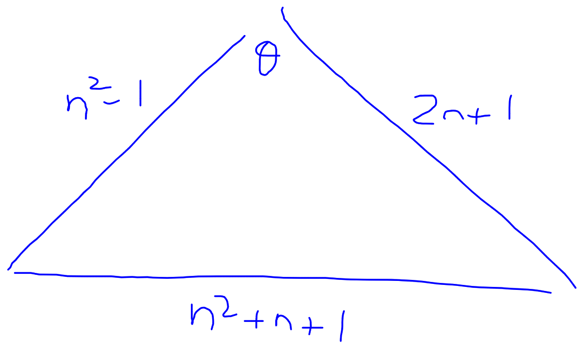
$$d = \frac{h_2 \sin y}{\sin(x-y)}$$

$$\cos x = \frac{h_1}{d}$$

$$h_1 = d \cos x$$

$$= \frac{h_2 \cos x \sin y}{\sin(x-y)}$$

16/



$$\cos\theta = \frac{(n^2-1)^2 + (2n+1)^2 - (n^2+n+1)^2}{2(n^2-1)(2n+1)}$$

$$= \frac{\cancel{n^4} - 2n^2 + 1 + 4n^2 + 4n + 1 \cancel{-n^4} - n^2 - 1 - 2n^3 - 2n^2 - 2n}{2(n^2-1)(2n+1)}$$

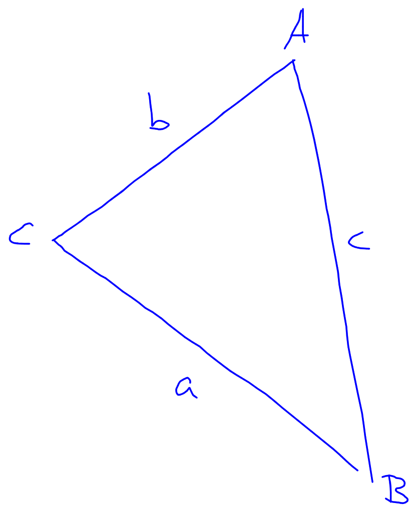
$$= \frac{-2n^3 - n^2 + 2n + 1}{2(n^2-1)(2n+1)}$$

$$= \frac{-(n^2-1)(2n+1)}{2(n^2-1)(2n+1)}$$

$$= -\frac{1}{2}$$

$$\underline{\theta = 120^\circ}$$

18



$$a\cos A = b\cos B$$

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\frac{ab^2+ac^2-a^3}{2bc} = \frac{a^2b+bc^2-b^3}{2ac}$$

$$a^2b^2+a^2c^2-a^4 = a^2b^2+b^2c^2-b^4$$

$$a^2c^2-b^2c^2-a^4+b^4=0$$

$$c^2(a^2-b^2)-(a^2+b^2)(a^2-b^2)=0$$

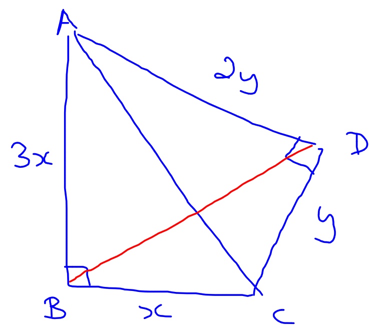
$$(a^2-b^2)(c^2-a^2-b^2)=0$$

$$a^2-b^2=0 \quad \text{or} \quad c^2-a^2-b^2=0$$

$$a^2=b^2 \qquad c^2=a^2+b^2$$

$$a=b$$

20/



equate AC^2.

$$9x^2 + x^2 = 4y^2 + y^2$$

$$10x^2 = 5y^2$$

$$y^2 = 2x^2$$

equate BD^2

$$9x^2 + 4y^2 - 12xy\cos\angle BAD = x^2 + y^2 - 2xy\cos\angle DCB$$

$$= x^2 + y^2 - 2xy\cos(180 - \angle BAD)$$

$$= x^2 + y^2 + 2xy\cos\angle BAD$$

$$14xy\cos\angle BAD = 8x^2 + 3y^2$$

$$\cos\angle BAD = \frac{8x^2 + 3y^2}{14xy}$$

$$\left(\begin{array}{l} y^2 = 2x^2 \\ y = \sqrt{2}x \end{array}\right)$$

$$= \frac{8x^2 + 6x^2}{14\sqrt{2}x^2}$$

$$= \frac{1}{\sqrt{2}}$$

$$\angle BAD = 45^\circ$$

22/ a) $4(ab+cd)^2 - (a^2+b^2-c^2-d^2)^2$

$$= \left[2(ab+cd) + (a^2+b^2-c^2-d^2)\right]\left[2(ab+cd) - (a^2+b^2-c^2-d^2)\right]$$

$$= \left[(a^2+2ab+b^2) - (c^2-2cd+d^2)\right]\left[(c^2+2cd+d^2) - (a^2-2ab+b^2)\right]$$

$$= \left[(a+b)^2 - (c-d)^2\right]\left[(c+d)^2 - (a-b)^2\right]$$

$$= \left[(a+b)+(c-d)\right]\left[(a+b)-(c-d)\right]\left[(c+d)+(a-b)\right]\left[(c+d)-(a-b)\right]$$

$$= (a+b+c-d)(a+b-c+d)(a-b+c+d)(-a+b+c+d)$$

b) $s = \frac{1}{2}(a+b+c+d)$

$$(a+b+c-d)(a+b-c+d)(a-b+c+d)(-a+b+c+d)$$

$$= 16\left(\frac{1}{2}(a+b+c-d)\frac{1}{2}(a+b-c+d)\frac{1}{2}(a-b+c+d)\frac{1}{2}(-a+b+c+d)\right)$$

$$= 16\left(\frac{1}{2}(a+b+c+d-2d)\frac{1}{2}(a+b+c+d-2c)\frac{1}{2}(a+b+c+d-2b)\frac{1}{2}(a+b+c+d-2a)\right)$$

$$= 16(s-d)(s-c)(s-b)(s-a)$$

c) equate SQ^2

$$a^2 + b^2 - 2ab\cos\theta = c^2 + d^2 - 2cd\cos(180-\theta)$$

$$a^2 + b^2 - c^2 - d^2 = 2ab\cos\theta + 2cd\cos\theta$$

$$\cos\theta = \frac{a^2+b^2-c^2-d^2}{2(ab+cd)}$$

$$\sin^2\theta = 1 - \frac{(a^2+b^2-c^2-d^2)^2}{4(ab+cd)^2}$$

$$= \frac{4a^2b^2 + 8abcd + 4c^2d^2 - a^4 - b^4 - c^4 - d^4 - 2a^2b^2 + 2a^2c^2 + 2a^2d^2 + 2b^2c^2 + 2b^2d^2 - 2c^2d^2}{4(ab+cd)^2}$$

$$= \frac{8abcd + 2a^2b^2 + 2c^2d^2 + 2a^2c^2 + 2a^2d^2 + 2b^2c^2 + 2b^2d^2 - a^4 - b^4 - c^4 - d^4}{4(ab+cd)^2}$$

$$= \frac{8abcd + 4a^2b^2 + 4c^2d^2 - (a^2+b^2-c^2-d^2)^2}{4(ab+cd)^2}$$

$$= \frac{4(ab+cd)^2 - (a^2+b^2-c^2-d^2)^2}{4(ab+cd)^2}$$

$$= \frac{4\,\cancel{16}\,(s-a)(s-b)(s-c)(s-d)}{\cancel{4}\,(ab+cd)^2}$$

$$\sin\theta = \frac{2\sqrt{(s-a)(s-b)(s-c)(s-d)}}{(ab+cd)}$$

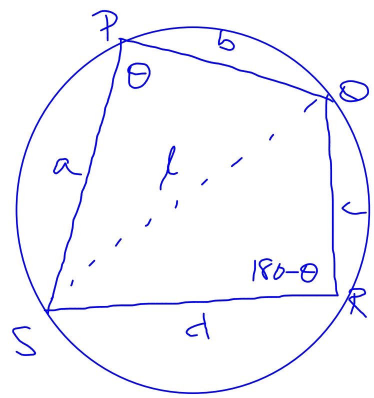
e)

$$A = \frac{1}{2}ab\sin\theta + \frac{1}{2}cd\sin(180-\theta)$$

$$= \frac{1}{2}(ab+cd)\sin\theta$$

$$= \frac{1}{2}(ab+cd) \times \frac{2\sqrt{(s-a)(s-b)(s-c)(s-d)}}{(ab+cd)}$$

$$= \sqrt{(s-a)(s-b)(s-c)(s-d)}$$



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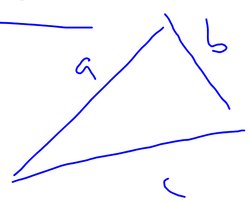

$$s = \frac{a+b+c}{2}$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$