2c) a, b, ae form a GP

$$b = \sqrt{a^2 e}$$

$$b^2 = a^2 e$$

$$e = \frac{b^2}{a^2}$$

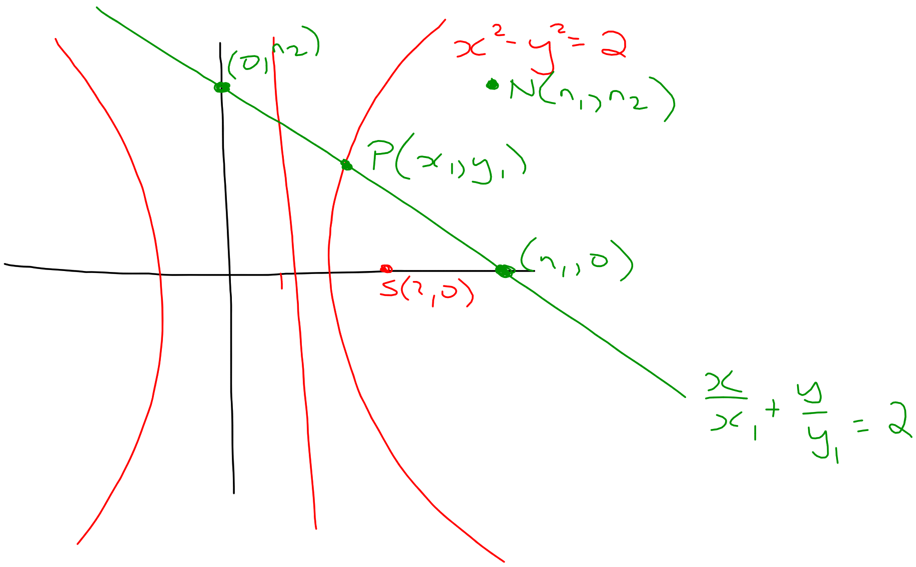
$$= \frac{a^2(e^2-1)}{a^2}$$

$$e = e^2 - 1$$

$$e^2 - e - 1 = 0$$

$$e = \frac{1+\sqrt{5}}{2} \quad (e>1)$$

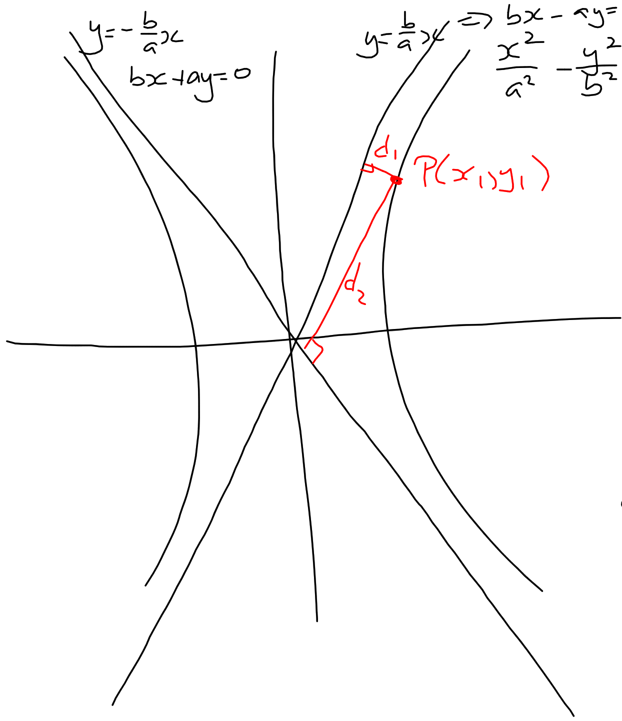
6c)



$N(2x_1, 2y_1)$

$$x^2 - y^2 = 4x_1^2 - 4y_1^2$$
$$= 4(x_1^2 - y_1^2)$$
$$= 4(2)$$
$$= \underline{\underline{8}}$$

10/



$y=\frac{b}{a}x \Rightarrow bx-ay=0$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow b^2x^2-a^2y^2=a^2b^2$$

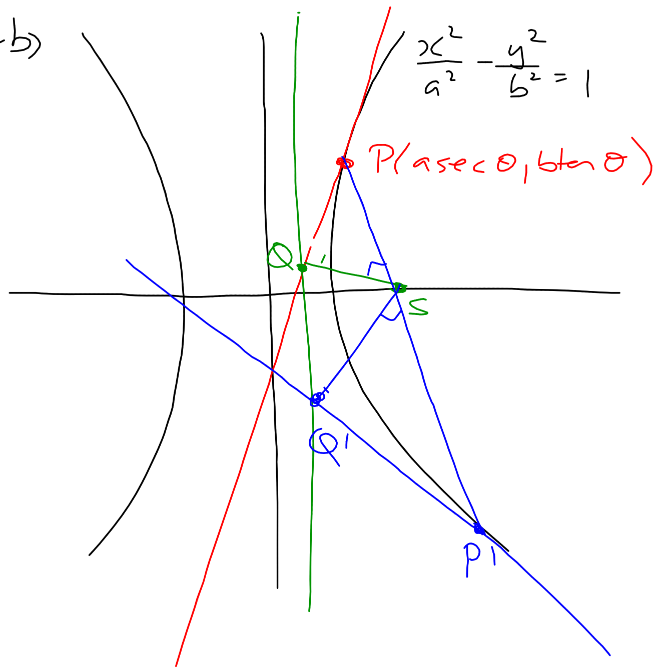
$$d_1=\frac{|bx_1-ay_1|}{\sqrt{b^2+a^2}}$$

$$d_2=\frac{|bx_1+ay_1|}{\sqrt{b^2+a^2}}$$

$$d_1d_2=\frac{|b^2x_1^2-a^2y_1^2|}{a^2+b^2}$$

$$=\underline{\underline{\frac{a^2b^2}{a^2+b^2}}}$$

12 b)



$\angle PSQ = 90$

$\angle P'SQ' = 90$

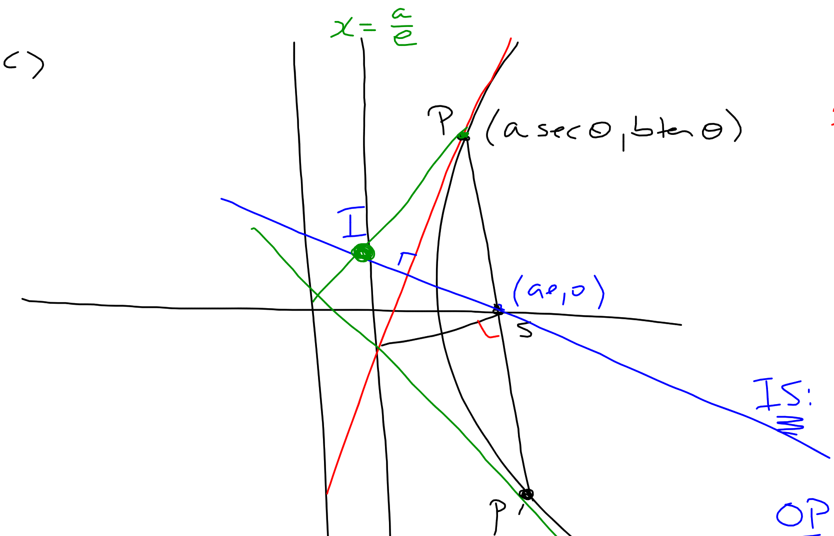
$\angle PSQ + \angle QSQ' + \angle P'SQ' = 180$

$\therefore \angle QSQ' = \underline{0^\circ}$

ie Q coincides with Q'

thus tangents intersect at Q which is on the directrix

12 c)



$$\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$$

$$m = \frac{b\sec\theta}{a\tan\theta}$$

$$\therefore m_{IS} = -\frac{a\tan\theta}{b\sec\theta}$$

IS: $y = -\frac{a\tan\theta}{b\sec\theta}(x-ae)$

OP: $y = \frac{b\tan\theta}{a\sec\theta}x$

pt of intersection (I)

$$\frac{b\tan\theta}{a\sec\theta}x = -\frac{a\tan\theta}{b\sec\theta}(x-ae)$$

$$\frac{bx}{a} = -\frac{a}{b}(x-ae)$$

$$b^2x = -a^2x + a^3e$$

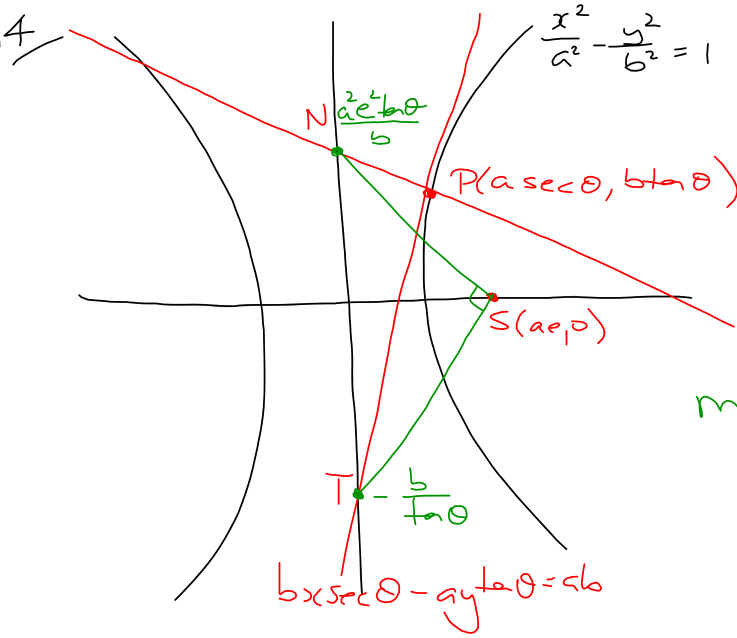
$$(a^2+b^2)x = a^3e$$

$$a^2e^2x = a^3e$$

$$x = \frac{a}{e}$$

$\therefore$ I lies on directrix

14



$by\sec\theta + ax\tan\theta = a^2e^2\sec\theta\tan\theta$

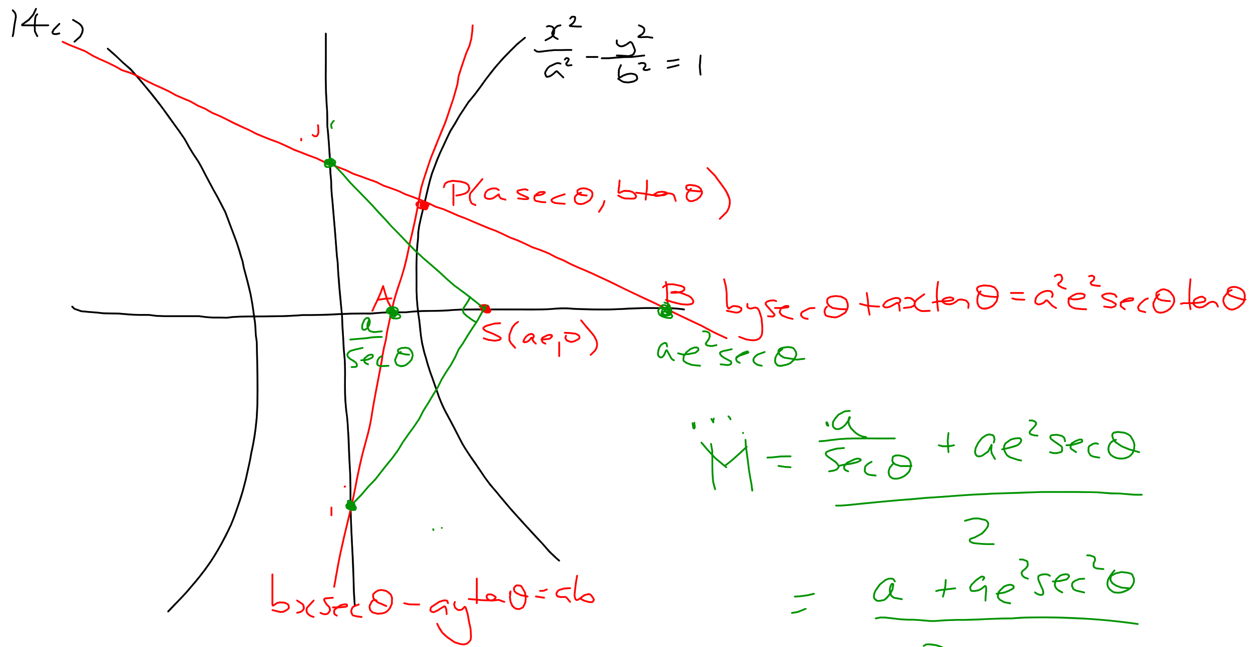
$$m_{NS} \times m_{ST} = \frac{-a^2e^2\cancel{\tan}\theta}{a\cancel{b}e} \times \frac{\cancel{b}}{ae\cancel{\tan}\theta}$$

$$= -1$$

$\therefore \triangle NTS$ is right angled

$\therefore$ points are concyclic
with NT diameter.

14 c)



$$\therefore M = \frac{\frac{a}{\sec\theta} + ae^2\sec\theta}{2}$$

$$= \frac{a + ae^2\sec^2\theta}{2\sec\theta}$$

$$= a\left[\frac{1+e^2\sec^2\theta}{2\sec\theta}\right]$$

$$\frac{1+e^2\sec^2\theta}{2\sec\theta} = e$$

$$e^2\sec^2\theta - 2e\sec\theta + 1 = 0$$

$$(e\sec\theta - 1)^2 = 0$$

$$e = \frac{1}{\sec\theta} \leq 1 \text{ for all } \theta$$

however $e > 1$ as it is a hyperbola

$$\therefore M \neq ae$$

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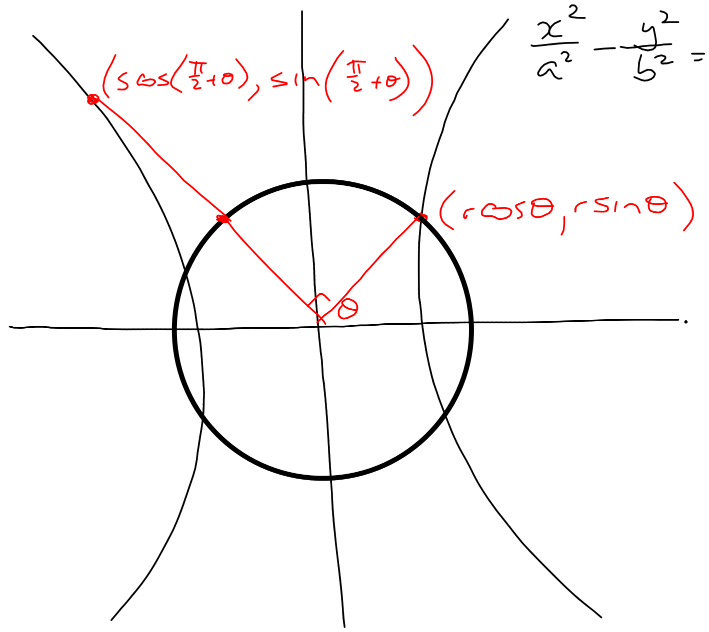


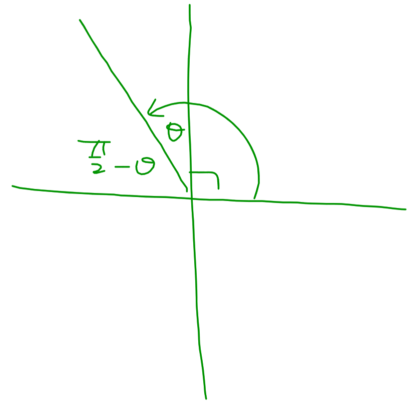
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{r^2\cos^2\theta}{a^2} - \frac{r^2\sin^2\theta}{b^2} = 1$$

$$r^2\left(\frac{b^2\cos^2\theta - a^2\sin^2\theta}{a^2b^2}\right) = 1$$

$$\frac{1}{r^2} = \frac{b^2\cos^2\theta - a^2\sin^2\theta}{a^2b^2}$$

$$\frac{s^2\cos^2\left(\frac{\pi}{2}+\theta\right)}{a^2} - \frac{s^2\sin^2\left(\frac{\pi}{2}+\theta\right)}{b^2} = 1$$



$$s^2\left[\frac{b^2\sin^2\theta - a^2\cos^2\theta}{a^2b^2}\right] = 1$$

$$\frac{1}{s^2} = \frac{b^2\sin^2\theta - a^2\cos^2\theta}{a^2b^2}$$

$$\frac{1}{r^2} + \frac{1}{s^2} = \frac{b^2\cos^2\theta - a^2\sin^2\theta}{a^2b^2} + \frac{b^2\sin^2\theta - a^2\cos^2\theta}{a^2b^2}$$

$$= \frac{b^2-a^2}{a^2b^2}$$

$$= \frac{1}{a^2} - \frac{1}{b^2}$$

when $a=b$, $e=\sqrt{2}$

$a>b$, $1<e<\sqrt{2}$

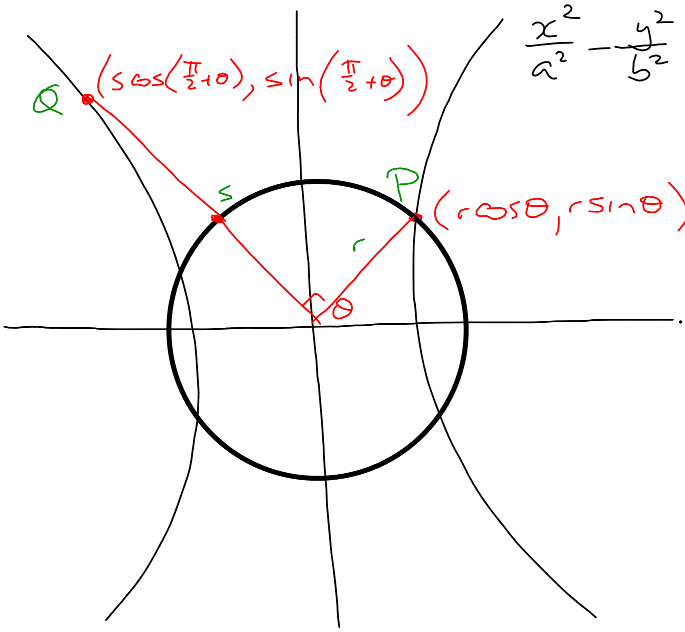
$b>a$, $e>\sqrt{2}$

$$\frac{1}{a^2} - \frac{1}{b^2} > 0$$

$$\frac{1}{a^2} > \frac{1}{b^2}$$

$$a^2 < b^2$$

$$\therefore \underline{\underline{e>\sqrt{2}}}$$



$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\frac{1}{OP^2}+\frac{1}{OQ^2}$$

$$=\frac{1}{r^2}+\frac{1}{s^2}$$

$$=\frac{1}{a^2}-\frac{1}{b^2}$$

$$= \underline{\underline{\text{constant}}}$$

$m_{OP} \times m_{OQ} = -1$

$\frac{r\sin\theta}{r\cos\theta} \times m_{OQ} = -1$

$m_{OQ} = \frac{-\cos\theta}{\sin\theta}$

$m_{OS} = \frac{s\cos\theta}{-s\sin\theta}$

$= \frac{-\cos\theta}{\sin\theta}$

$= m_{OQ}$

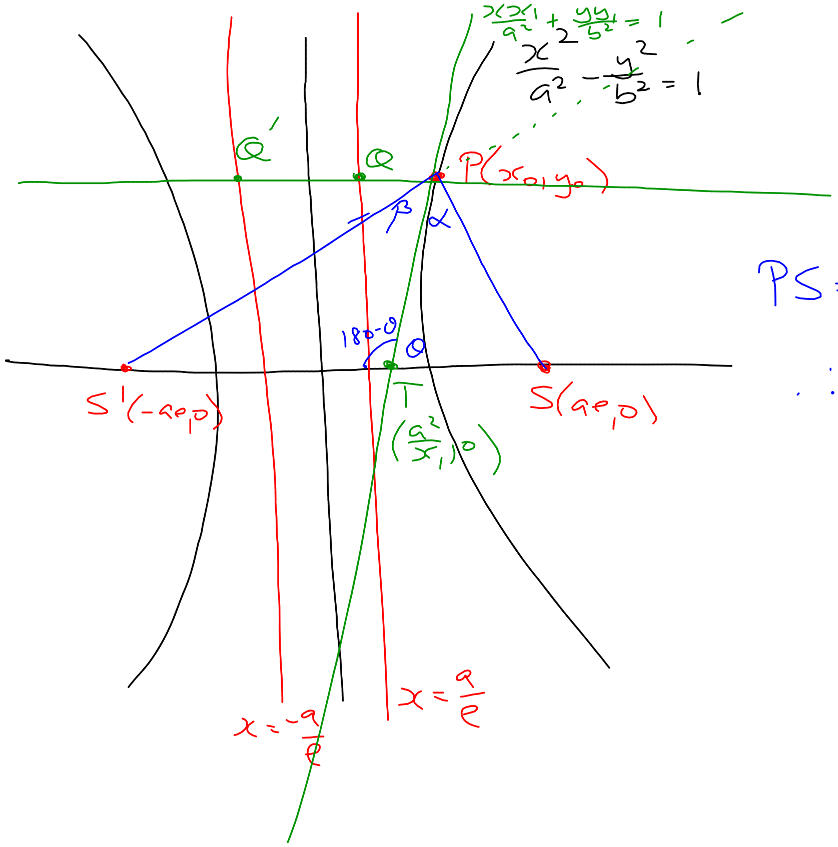
$\therefore$ $\underline{Q \text{ is } S}$

$$\frac{1}{OP^2} + \frac{1}{OQ^2} = \frac{1}{r^2\sin^2\theta + r^2\cos^2\theta} + \frac{1}{s^2\cos^2\theta + s^2\sin^2\theta}$$

$$= \frac{1}{r^2} + \frac{1}{s^2}$$

$$= \frac{1}{a^2} - \frac{1}{b^2}$$

18



$PS = ePQ \quad PS' = ePQ'$

$$\therefore \frac{PS}{PQ} = \frac{PS'}{PQ'} = e$$

$$\frac{PS}{PS'} = \frac{PQ}{PQ'}$$

$$\frac{PQ}{PQ'} = \frac{x_0 - \frac{a}{e}}{x_0 + \frac{a}{e}}$$

$$= \frac{ex_0 - a}{ex_0 + a}$$

$$\frac{ST}{S'T} = \frac{ae - \frac{a^2}{x_0}}{ae + \frac{a^2}{x_0}}$$

$$= \frac{x_0 e - a}{x_0 e + a}$$

$$= \frac{PQ}{PQ'}$$

$$= \frac{PS}{PS'}$$

c) In $\triangle PST$

$$\frac{PS}{\sin\theta} = \frac{ST}{\sin\alpha}$$

$$\frac{PS}{ST} = \frac{\sin\theta}{\sin\alpha}$$

$$\sin\alpha = \frac{ST\sin\theta}{PS}$$

In $\triangle PS'T$

$$\frac{PS'}{\sin(180-\theta)} = \frac{S'T}{\sin\beta}$$

$$\frac{PS'}{S'T} = \frac{\sin\theta}{\sin\beta}$$

$$\sin\beta = \frac{S'T\sin\theta}{PS'}$$

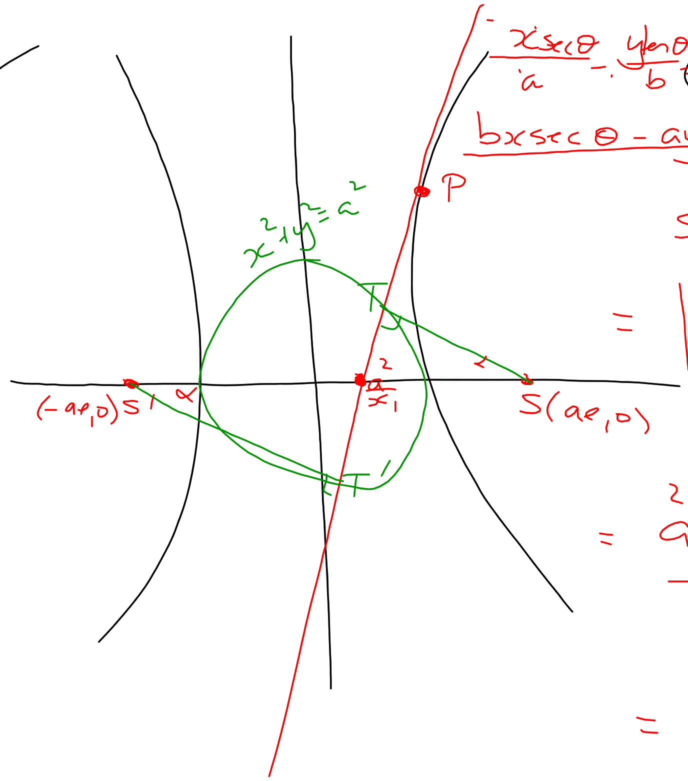
But $\frac{ST}{S'T} = \frac{PS}{PS'}$

$$\frac{ST}{PS} = \frac{S'T}{PS'}$$

$$\therefore \sin\alpha = \sin\beta$$

$$\underline{\alpha = \beta} \qquad (\text{as } \alpha, \beta \text{ are acute})$$

20/



$$\frac{x\sec\theta}{a}-\frac{y\tan\theta}{b}=1 \qquad S'T' \times ST = b^2$$

$$bx\sec\theta - ay\tan\theta - ab = 0$$

$$S'T' \times ST$$

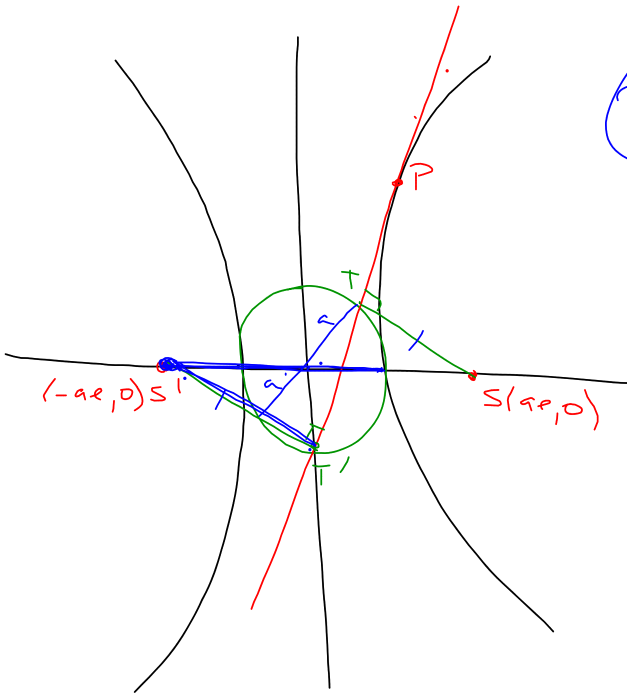
$$= \frac{|-abe\sec\theta - ab|}{\sqrt{b^2\sec^2\theta + a^2\tan^2\theta}} \times \frac{|abe\sec\theta - ab|}{\sqrt{b^2\sec^2\theta + a^2\tan^2\theta}}$$

$$= \frac{a^2b^2|e^2\sec^2\theta - 1|}{b^2\sec^2\theta + a^2\tan^2\theta}$$

$$= \frac{a^2b^2|(a^2+b^2)\sec^2\theta - a^2|}{a^2(b^2\sec^2\theta + a^2\tan^2\theta)}$$

$$= b^2 \times \frac{|a^2\tan^2\theta + b^2\sec^2\theta|}{(b^2\sec^2\theta + a^2\tan^2\theta)}$$

$$= \underline{\underline{b^2}}$$



$b^2 = a^2(e^2-1)$

(Product of intercepts of intersecting secants)

$$S'T' \times ST = (ae+a)(ae-a)$$

$$= a^2e^2 - a^2$$

$$= a^2(e^2-1)$$

$$= \underline{\underline{b^2}}$$