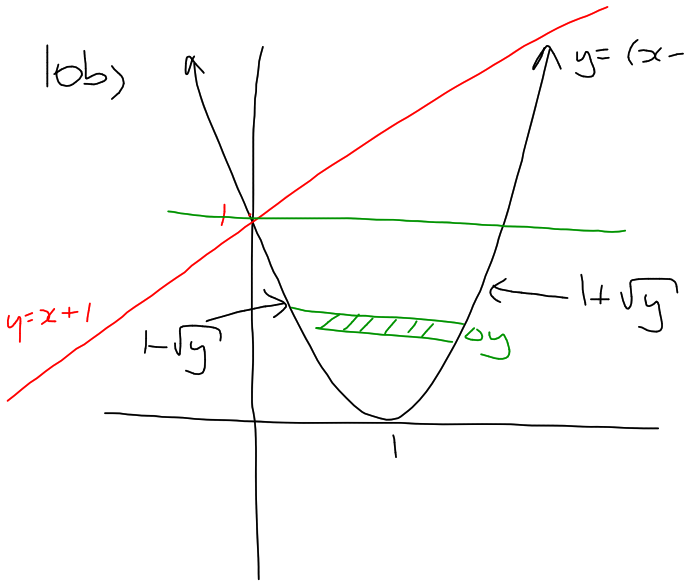
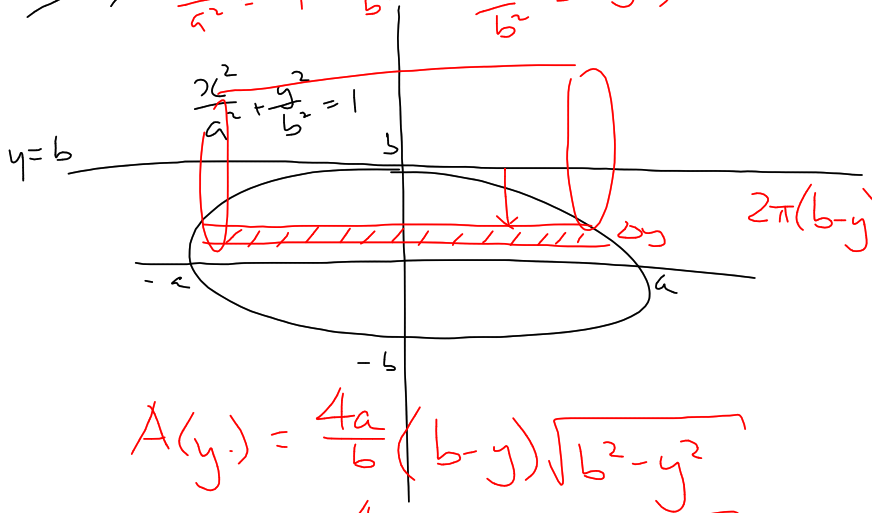
(b)

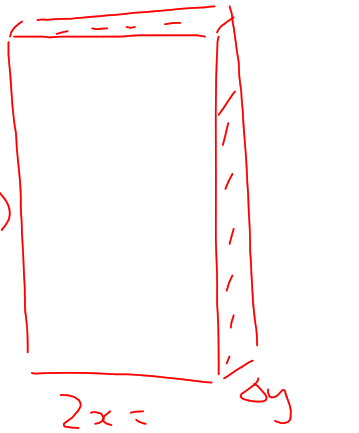


$y = (x-1)^2 \Rightarrow (x-1) = \pm\sqrt{y}$

$x = 1 \pm \sqrt{y}$

15 b) $\frac{x^2}{a^2} = 1 - \frac{y^2}{b^2} = \frac{a^2}{b^2}(b^2 - y^2)$





$2x = \frac{2a}{b}\sqrt{(b^2 - y^2)}$

$A(y) = \frac{4a}{b}(b-y)\sqrt{b^2-y^2}$

$\Delta V = \frac{4a}{b}(b-y)\sqrt{b^2-y^2}\,\Delta y$

$V = \lim_{\Delta y \to 0} \sum_{y=-b}^{b} \frac{4a}{b}(b-y)\sqrt{b^2-y^2}\,\Delta y$

$= \frac{4a}{b}\pi\int_{-b}^{b}(b-y)\sqrt{b^2-y^2}\,dy$

$= 4a\pi\int_{-b}^{b}\sqrt{b^2-y^2}\,dy - \frac{4a}{b}\pi\int_{-b}^{b} y\sqrt{b^2-y^2}\,dy$

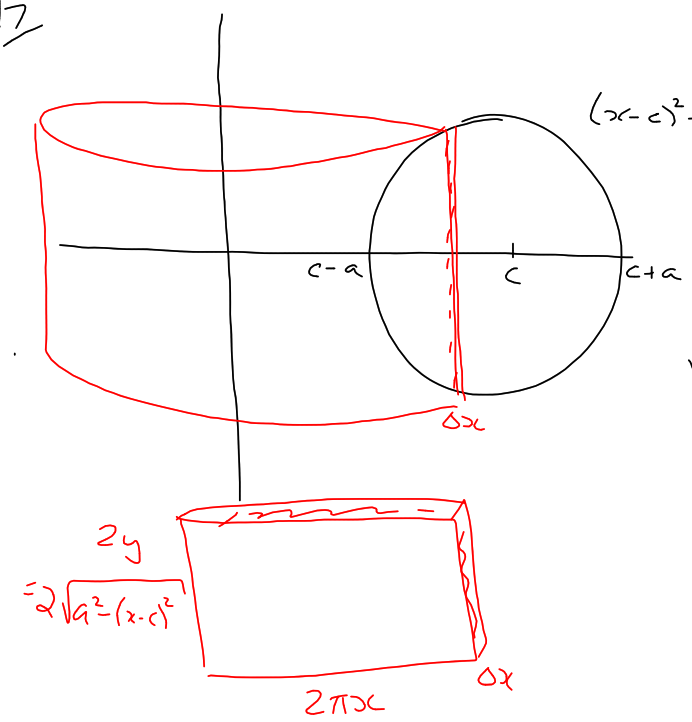
$= 4a\pi \times \frac{1}{2}\pi b^2 - 0$

$= 2ab^2\pi^2$

c) $V = \frac{4ac}{b} \times \frac{1}{2}\pi b^2$

$= \underline{2abc\pi^2}$.

17



$(x-c)^2+y^2=a^2 \Rightarrow y=\sqrt{a^2-(x-c)^2}$

$A(x)=4\pi x\sqrt{a^2-(x-c)^2}$

$\Delta V=4\pi x\sqrt{a^2-(x-c)^2}\,\Delta x$

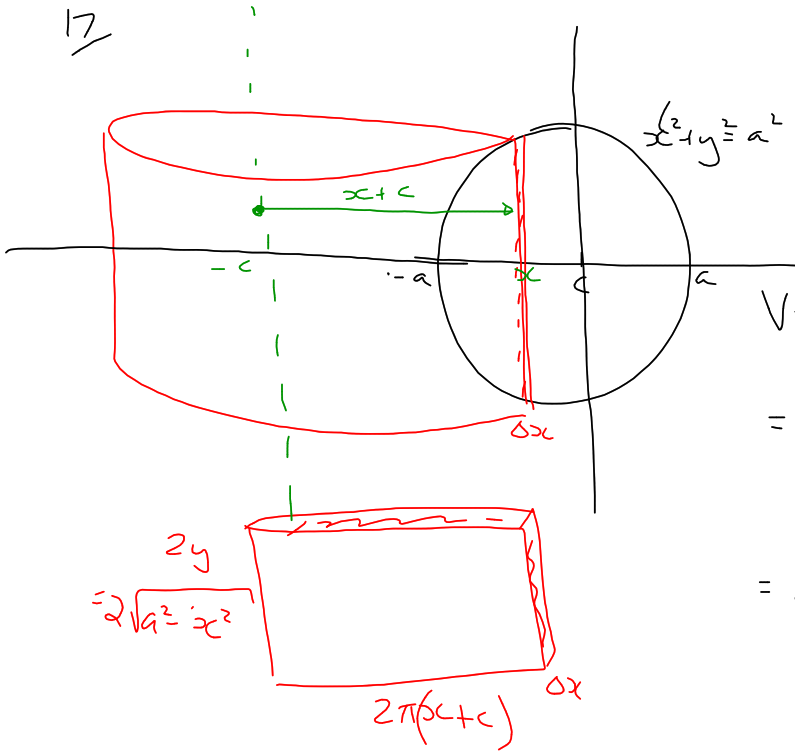
$$V=4\pi\int_{c-a}^{c+a} x\sqrt{a^2-(x-c)^2}\,dx$$

$x=u+c$

$dx=du$

$$V=4\pi\int_{-a}^{a}(u+c)\sqrt{a^2-u^2}\,du$$

17



$$A(x) = 4\pi(x+c)\sqrt{a^2-x^2}$$

$$\Delta V = 4\pi(x+c)\sqrt{a^2-x^2}\,\delta x$$

$$V = \lim_{\delta x \to 0} \sum_{x=-a}^{a} 4\pi(x+c)\sqrt{a^2-x^2}\,\delta x$$

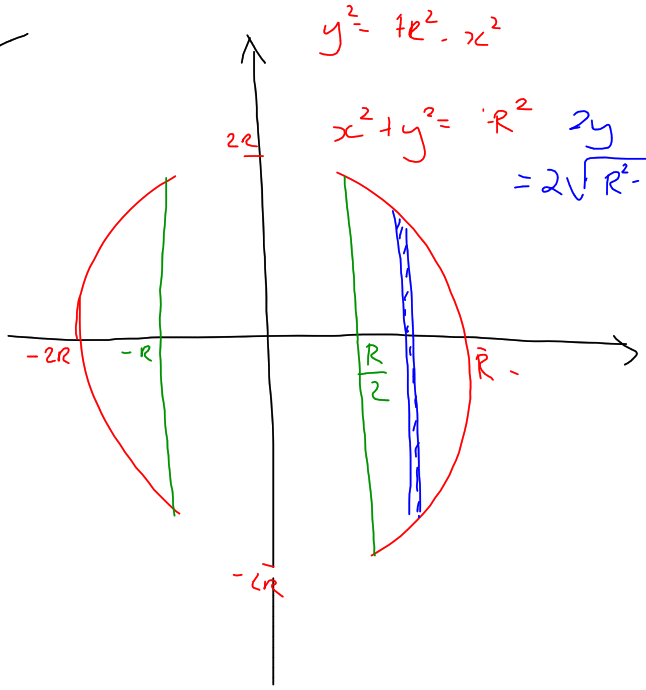
$$= 4\pi \int_{-a}^{a} (x+c)\sqrt{a^2-x^2}\,dx$$

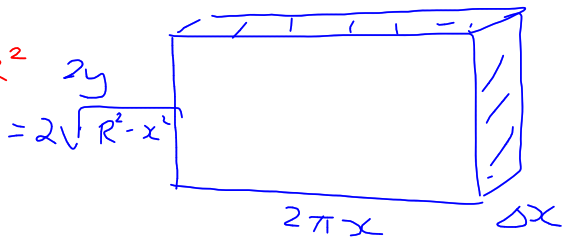
$$= 4\pi \int_{-a}^{a} x\sqrt{a^2-x^2}\,dx + 4\pi c \int_{-a}^{a} \sqrt{a^2-x^2}\,dx$$

$$= 0 + 4\pi c \times \frac{1}{2}\pi a^2$$

$$= 2\pi^2 a^2 c \text{ units}^3$$

18





$$A(x) = 4\pi x\sqrt{R^2 - x^2}$$

$$\Delta V = 4\pi x\sqrt{R^2 - x^2}\,\Delta x$$

$$V = \lim_{\Delta x \to 0} \sum_{x=\frac{R}{2}}^{R} 4\pi x\sqrt{R^2 - x^2}\,\Delta x$$

$$V = 4\pi \int_{\frac{R}{2}}^{R} x\sqrt{R^2 - x^2}\,dx$$

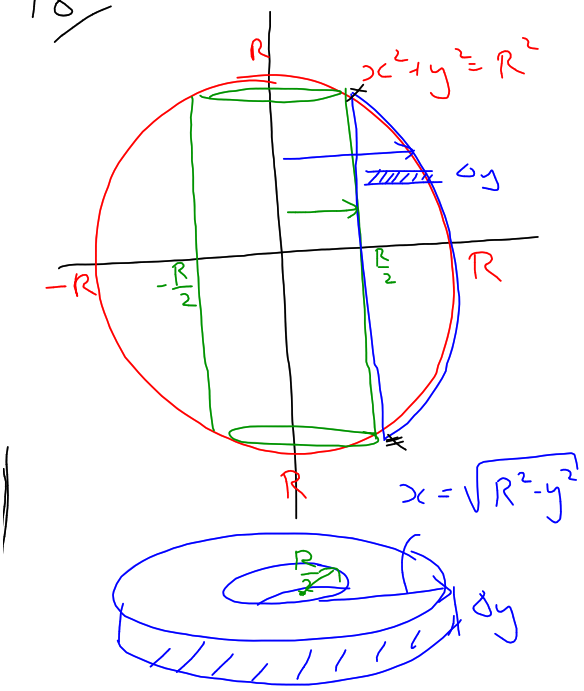
$$= -2\pi\left[\frac{2}{3}\left(R^2 - x^2\right)^{\frac{3}{2}}\right]_{\frac{R}{2}}^{R}$$

$$= -\frac{4\pi}{3}\left(0 - \frac{3R^2}{4}\right)^{\frac{3}{2}}$$

$$= \frac{4\pi}{3}\cdot\frac{3\sqrt{3}R^3}{8}$$

$$= \frac{\pi\sqrt{3}R^3}{2}$$

18/



when $x=\frac{R}{2}$,

$$y=\sqrt{R^2-\frac{R^2}{4}}$$

$$=\frac{\sqrt{3}R}{2}$$

$$A(y)=\pi\left[\left(\sqrt{R^2-y^2}\right)^2-\left(\frac{R}{2}\right)^2\right]$$

$$\Delta V=\pi\left(\frac{3R^2}{4}-y^2\right)\Delta y$$

$$V=\lim_{\Delta y\to 0}\sum_{y=-\frac{\sqrt{3}R}{2}}^{\frac{\sqrt{3}R}{2}}\pi\left(\frac{3R^2}{4}-y^2\right)\Delta y$$

$$=2\pi\int_0^{\frac{\sqrt{3}R}{2}}\left(\frac{3R^2}{4}-y^2\right)dy$$

$$=2\pi\left[\frac{3R^2}{4}y-\frac{1}{3}y^3\right]_0^{\frac{\sqrt{3}R}{2}}$$

$$=2\pi\left(\frac{3\sqrt{3}R^3}{8}-\frac{3\sqrt{3}R^3}{24}\right)$$

$$=\frac{12\pi\sqrt{3}R^3}{24}$$

$$=\frac{\pi\sqrt{3}R^3}{2}$$