Rules For Differentiation

(1) $y = c$

$$f(x) = c$$

$$f(x+h) = c$$

$$f'(x) = \lim_{h\to 0} \frac{c-c}{h}$$

$$= \lim_{h\to 0} \frac{0}{h}$$

$$= \lim_{h\to 0} 0$$

$$= 0$$

(2) $y = kx$

$$f(x) = kx$$

$$f(x+h) = k(x+h)$$

$$= kx + kh$$

$$f'(x) = \lim_{h\to 0} \frac{kx+kh-kx}{h}$$

$$= \lim_{h\to 0} \frac{kh}{h}$$

$$= \lim_{h\to 0} k$$

$$= k$$

(3) $y = x^n$

$$a^2 - b^2 = (a-b)(a+b)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^4 - b^4 = (a-b)(a^3 + a^2b + ab^2 + b^3)$$

$$\vdots$$

$$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + a^2b^{n-3} + ab^{n-2} + b^{n-1})$$

$$f(x) = x^n \qquad f(x+h) = (x+h)^n$$

$$f'(x) = \lim_{h\to0}\frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h\to0}\frac{(x+h-x)\{(x+h)^{n-1} + (x+h)^{n-2}x + \ldots + (x+h)x^{n-2} + x^{n-1}\}}{h}$$

$$= \lim_{h\to0}\frac{h\{(x+h)^{n-1} + (x+h)^{n-2}x + \ldots + (x+h)x^{n-2} + x^{n-1}\}}{h}$$

$$= \lim_{h\to0}(x+h)^{n-1} + (x+h)^{n-2}x + \ldots + (x+h)x^{n-2} + x^{n-1}$$

$$f'(x) = \lim_{h\to 0}(x+h)^{n-1} + (x+h)^{n-2}x + \ldots + (x+h)x^{n-2} + x^{n-1}$$

$$= x^{n-1} + x^{n-1} + \ldots + x^{n-1} + x^{n-1}$$

$$= \underline{nx^{n-1}}$$

(4) $\underline{y = \frac{1}{x}}$

$$f(x) = \frac{1}{x}$$

$$f(x+h) = \frac{1}{x+h}$$

$$f'(x) = \lim_{h\to 0}\frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h\to 0}\frac{x-x-h}{hx(x+h)}$$

$$= \lim_{h\to 0}\frac{-h}{hx(x+h)}$$

$$= \lim_{h\to 0}\frac{-1}{x(x+h)}$$

$$= \underline{\frac{-1}{x^2}}$$

Note:

$$f(x) = x^{-1}$$

$$f'(x) = -x^{-2}$$

$$= \frac{-1}{x^2}$$

(5) $y=\sqrt{x}$

$$f(x)=\sqrt{x}$$

$$f(x+h)=\sqrt{x+h}$$

$$f'(x)=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\times\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$=\lim_{h\to 0}\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=\lim_{h\to 0}\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=\lim_{h\to 0}\frac{1}{\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=\frac{1}{\sqrt{x}+\sqrt{x}}$$

$$=\frac{1}{2\sqrt{x}}$$

Note:

$$f(x)=x^{\frac{1}{2}}$$

$$f'(x)=\frac{1}{2}x^{-\frac{1}{2}}$$

$$=\frac{1}{2\sqrt{x}}$$

e.g. $(i)\ y = 7$

$$\frac{dy}{dx} = 0$$

$(ii)\ y = 37x$

$$\frac{dy}{dx} = 37$$

$(iii)\ y = x^{10}$

$$\frac{dy}{dx} = 10x^9$$

$(iv)\ y = 3x^2 + 6x + 2$

$$\frac{dy}{dx} = 6x + 6$$

$(v)\ y = (2x+1)^2$

$$= 4x^2 + 4x + 1$$

$$\frac{dy}{dx} = 8x + 4$$

$(vi)\ y = 3x + \frac{1}{x^2}$

$$= 3x + x^{-2}$$

$$\frac{dy}{dx} = 3 - 2x^{-3}$$

$$= 3 - \frac{2}{x^3}$$

$(vii)\ y = x^2\sqrt{x}$

$$= x^{\frac{5}{2}}$$

$$\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}}$$

$$= \frac{5}{2}x\sqrt{x}$$

$(viii)$ If $f(x) = x^3 - 3$, find $f'(2)$

$$f(x) = x^3 - 3$$

$$f'(x) = 3x^2$$

$$f'(2) = 3(2)^2$$

$$= 12$$

(xix) Find the equation of the tangent to the curve $y = 5x^3 - 6x^2 + 2$ at the point $(1,1)$

$$y = 5x^3 - 6x^2 + 2$$

$$\frac{dy}{dx} = 15x^2 - 12x$$

$$\text{when } x = 1, \frac{dy}{dx} = 15(1)^2 - 12(1)$$

$$= 3$$

$\therefore$ required slope $= 3$

$$y - 1 = 3(x - 1)$$

$$y - 1 = 3x - 3$$

$$\underline{3x - y - 2 = 0}$$

(x) Find the points on the curve $y = x^3 - 12x$ where the tangents are horizontal

$$y = x^3 - 12x$$

$$\frac{dy}{dx} = 3x^2 - 12$$

tangents are horizontal when $\frac{dy}{dx} = 0$

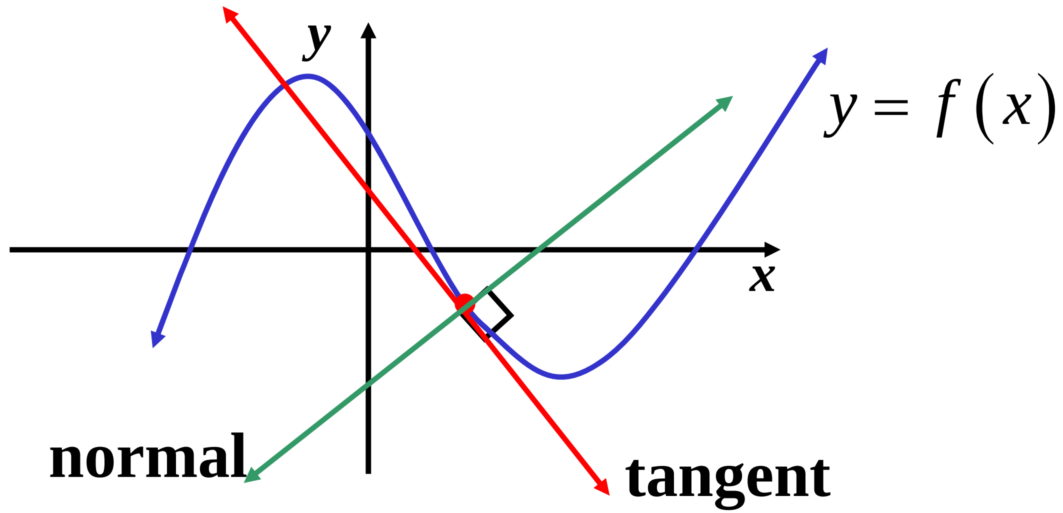
i.e. $3x^2 - 12 = 0$

$$x^2 = 4$$

$$x = \pm 2$$

$\therefore$ tangents are horizontal at $(-2, 16)$ and $(2, -16)$

A **normal** is a line perpendicular to the tangent at the point of contact



(xi) Find the equation of the normal to the curve $y = 4x^2 - 3x + 2$ at the point $(3, 29)$

$$y = 4x^2 - 3x + 2$$

$$\frac{dy}{dx} = 8x - 3$$

when $x = 3, \frac{dy}{dx} = 8(3) - 3$

$$= 21$$

$\therefore$ required slope $= -\frac{1}{21}$

$$y - 29 = -\frac{1}{21}(x - 3)$$

$$21y - 609 = -x + 3$$

$$x + 21y - 612 = 0$$

A note on setting out

When differentiating, you are actually solving an equation i.e. "**what you do to one side of the equation you do to the other**"

$$y = x^2 + 2x$$

$$\frac{d(y)}{dx} = \frac{d\left(x^2 + 2x\right)}{dx}$$

$$\frac{dy}{dx} = 2x + 2$$

"differentiate both sides with respect to *x*"

Exercise 7C; 1ace etc, 2ace etc, 3ace etc, 4bd, 5bdfh, 8bd, 9bd, 10ac, 12, 13b, 16, 21

Exercise 7D; 2ac, 3bd, 4ace, 6c, 7b, 11a, 13aei, 18, 22

Match the notation being used

(1) $y = x^2 + 2x$ ***OR*** $y = x^2 + 2x$

$\frac{dy}{dx} = 2x + 2$ $\quad y' = 2x + 2$

(2) $f(x) = x^2 + 2x$

$f'(x) = 2x + 2$

(3) $\frac{d\left(x^2 + 2x\right)}{dx} = 2x + 2$