4d)

$$y = (2-3x^2)^4 (2+3x^2)^5$$

$$\frac{dy}{dx} = (2-3x^2)^4 \left[5(2+3x^2)^4(6x)\right] + (2+3x^2)^5\left[4(2-3x^2)^3(-6x)\right]$$

$$= 30x(2-3x^2)^{\cancel{4}}(2+3x^2)^4 - 24x(2+3x^2)^5(2-3x^2)^3$$

$$= 6x(2-3x^2)^3(2+3x^2)^4\left[5(2-3x^2) - 4(2+3x^2)\right]$$

$$= 6x(2-3x^2)^3(2+3x^2)^4(2-27x^2)$$

5/

$$y = (x^2-10)^3 x^4$$

$$\frac{dy}{dx} = (x^2-10)^3(4x^3) + (x^4)\left[3(x^2-10)^2(2x)\right]$$

$$= 4x^3(x^2-10)^3 + 6x^5(x^2-10)^2$$

$$= 2x^3(x^2-10)^2\left[2(x^2-10)+3x^2\right]$$

$$= 2x^3(x^2-10)^2(5x^2-20)$$

$$= 10x^3(x^2-10)^2(x^2-4)$$

6c)

$$y = 10x^2\sqrt{2x-1}$$

$$\frac{dy}{dx} = (10x^2)\left[\frac{1}{2}(2x-1)^{-\frac{1}{2}}(2)\right] + (2x-1)^{\frac{1}{2}}(20x)$$

$$= 10x^2(2x-1)^{-\frac{1}{2}} + 20x(2x-1)^{\frac{1}{2}}$$

$$= 10x(2x-1)^{-\frac{1}{2}}\left[x + 2(2x-1)\right]$$

$$= 10x(2x-1)^{-\frac{1}{2}}(5x-2)$$

$$= \frac{10x(5x-2)}{\sqrt{2x-1}}$$

7 a) $y = x\sqrt{1-x^2}$

$1-x^2 \geq 0$

$1 \geq x^2$

$x^2 \leq 1$

$\underline{D}: \underline{-1 \leq x \leq 1}$

b) $y = x(1-x^2)^{\frac{1}{2}}$

$\frac{dy}{dx} = (x)\left[\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)\right] + (1-x^2)^{\frac{1}{2}}(1)$

$= -x^2(1-x^2)^{-\frac{1}{2}} + (1-x^2)^{\frac{1}{2}}$

$= (1-x^2)^{-\frac{1}{2}}\left[-x^2 + (1-x^2)\right]$

$= (1-x^2)^{-\frac{1}{2}}(1-2x^2)$

$= \underline{\underline{\frac{1-2x^2}{\sqrt{1-x^2}}}}$

c) tangent is horizontal when $\frac{dy}{dx} = 0$

ie $1 - 2x^2 = 0$

$x^2 = \frac{1}{2}$

$x = \pm \frac{1}{\sqrt{2}}$

tangent is horizontal at $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{2}\right)$ and $\left(\frac{1}{\sqrt{2}}, \frac{1}{2}\right)$

$y = x\sqrt{1-x^2}$

$x = \frac{1}{\sqrt{2}}$, $y = \frac{1}{\sqrt{2}} \times \sqrt{1 - \frac{1}{2}}$

$= \frac{1}{2}$.

d) when $x = 0$, $\frac{dy}{dx} = \frac{1-2(0)^2}{\sqrt{1-(0)^2}}$

$= 1$

T: $y = x$

N: $y = -x$.

8a) $y = a(x-\alpha)(x-\beta)$

$\frac{dy}{dx} = a(x-\alpha)(1) + (x-\beta)(a)$

$= a(2x-\alpha-\beta)$

b) $x=\alpha$, $\frac{dy}{dx} = a(\alpha - \beta)$

$x=\beta$, $\frac{dy}{dx} = a(\beta-\alpha)$

$M = \left(\frac{\alpha+\beta}{2} , \quad \right)$

c) tangent is horizontal when $\frac{dy}{dx} = 0$

$$2x - \alpha - \beta = 0$$

$$x = \frac{\alpha + \beta}{2}$$

$\therefore$ V is directly above or below

as $x_V = x_M$

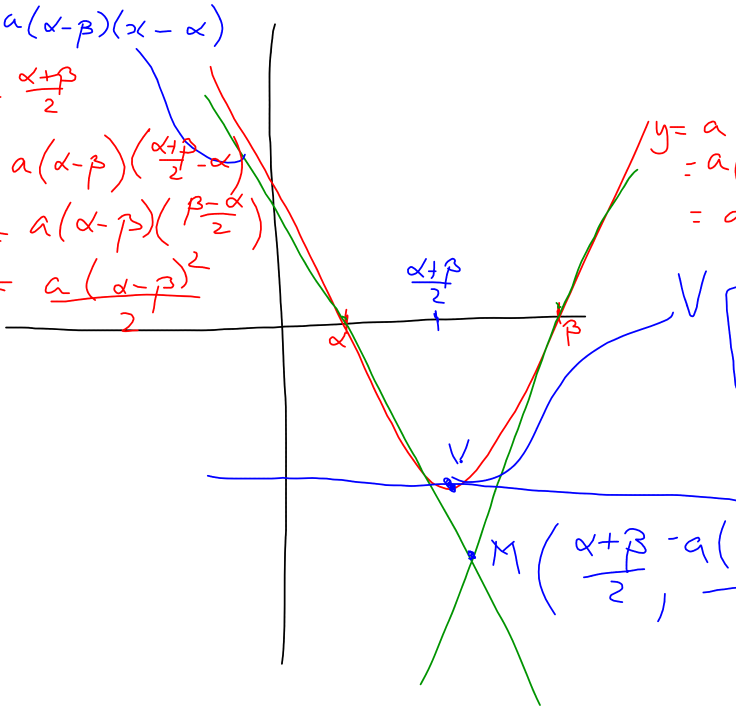
$y = a(\alpha-\beta)(x-\alpha)$

$x = \frac{\alpha+\beta}{2}$

$y = a(\alpha-\beta)\left(\frac{\alpha+\beta}{2}-\alpha\right)$

$= a(\alpha-\beta)\left(\frac{\beta-\alpha}{2}\right)$

$= \frac{a(\alpha-\beta)^2}{2}$



$y = a(x-\alpha)(x-\beta)$

$= a\left(\frac{\alpha+\beta}{2}-\alpha\right)\left(\frac{\alpha+\beta}{2}-\beta\right)$

$= a\left(\frac{\beta-\alpha}{2}\right)\left(\frac{\alpha-\beta}{2}\right)$

$V\left[\frac{\alpha+\beta}{2}, \frac{-a(\alpha-\beta)^2}{4}\right]$

$M\left(\frac{\alpha+\beta}{2}, \frac{-a(\alpha-\beta)^2}{2}\right)$

9

$$f(x) = (x-a)^n q(x)$$

$$f'(x) = (x-a)^n q'(x) + q(x)\left[n(x-a)^{n-1}(1)\right]$$

$$= (x-a)^n q'(x) + n(x-a)^{n-1} q(x)$$

$$= (x-a)^{n-1}\left[(x-a)q'(x) + nq(x)\right]$$

$$\lim_{x \to a} f'(x) = 0$$

$\therefore$ at $x = a$ there is a horizontal tangent.

13a)

$$f(x) = x^5(x-1)^4(x-2)^3$$

$$f'(x) = x^5(x-1)^4\left[3(x-2)^2(1)\right] + x^5(x-2)^3\left[4(x-1)^3(1)\right] + (x-1)^4(x-2)^3(5x^4)$$

$$= 3x^5(x-1)^4(x-2)^2 + 4x^5(x-2)^3(x-1)^3 + 5x^4(x-1)^4(x-2)^3$$

$$= x^4(x-1)^3(x-2)^2\left[3x(x-1) + 4x(x-2) + 5(x-1)(x-2)\right]$$

$$= x^4(x-1)^3(x-2)^2(12x^2-26x+10)$$

$$= 2x^4(x-1)^3(x-2)^2(6x^2-13x+5)$$

$$= 2x^4(x-1)^3(x-2)^2(3x-5)(2x-1)$$