

# GOSFORD HIGH SCHOOL



2016

## TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

### Mathematics Extension 1

- **General Instructions**
- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen  
Black pen is preferred
- Board-approved calculators may be used
- A Reference Sheet is provided at the back of this paper
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total Marks – 70

**Section I** Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

**Section II** Pages 6 – 12

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

**Section I**

**10 marks**

**Attempt Questions 1 – 10.**

**Allow about 15 minutes for this section.**

Use the multiple-choice answer sheet for Questions 1 – 10.

1. What is the value of  $\lim_{x \rightarrow 0} \frac{\sin 3x}{2x}$

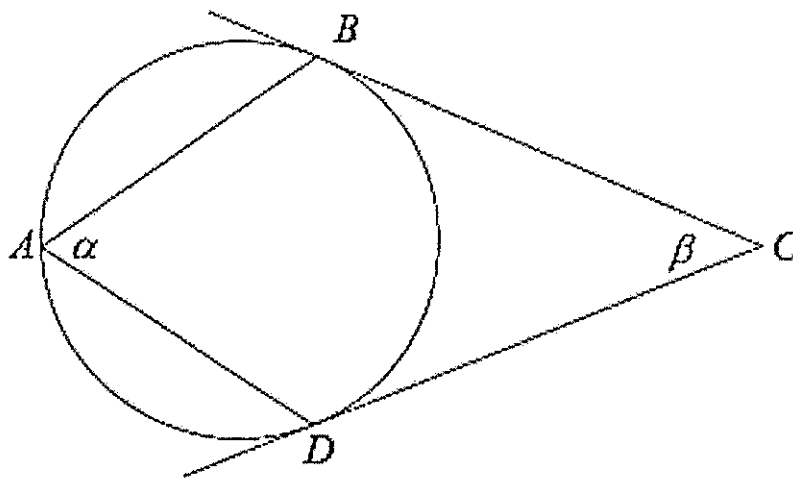
(A) 0

(B)  $\frac{2}{3}$

(C) 1

(D)  $\frac{3}{2}$

2. In the diagram below,  $BC$  and  $DC$  are tangents. Which statement is correct?



(A)  $\alpha + \beta = 180^\circ$

(B)  $2\alpha + \beta = 180^\circ$

(C)  $\alpha + 2\beta = 180^\circ$

(D)  $2\alpha - \beta = 180^\circ$

3. When the polynomial  $P(x) = x^4 + ax + 2$  is divided by  $x^2 + 1$  the remainder is  $2x + 3$

The value of  $a$  is?

- (A) 1
- (B) 2
- (C) 0
- (D) 3

4. Which of the following is equal to  $4^{\log_2 a}$ ?

- (A)  $2a$
- (B)  $a^2$
- (C)  $a$
- (D)  $4a$

5. Evaluate  $\frac{\sin A}{\sin A + \cos A} - \frac{\sin A}{\sin A - \cos A} =$

- (A)  $\cot 2A$
- (B)  $\operatorname{cosec} 2A$
- (C)  $\tan 2A$
- (D)  $\sec 2A$

6. Which of the following is an expression for  $\frac{d}{dx} \sin^{-1}(2x-1)$  ?

(A)  $\frac{-1}{\sqrt{x(1-x)}}$

(B)  $\frac{-1}{2\sqrt{x(1-x)}}$

(C)  $\frac{1}{2\sqrt{x(1-x)}}$

(D)  $\frac{1}{\sqrt{x(1-x)}}$

7. Find  $\int \frac{1}{9+25x^2} dx$ .

(A)  $\frac{1}{15} \tan^{-1} \frac{5x}{3} + C$

(B)  $\frac{1}{25} \tan^{-1} \frac{5x}{3} + C$

(C)  $\frac{1}{25} \tan^{-1} \frac{3x}{5} + C$

(D)  $\frac{1}{15} \tan^{-1} \frac{3x}{5} + C$

8. A particle is oscillating in Simple Harmonic Motion where its position  $x$  metres from a fixed point  $O$  on the same line as its motion after  $t$  seconds is given by  $x = 2\cos\left(3t + \frac{\pi}{6}\right)$ . What is the maximum speed of the particle?

- (A) 2 m/s
- (B) 6 m/s
- (C) 0 m/s
- (D)  $\frac{\pi}{9}$  m/s

9. The solution to  $|2x - 1| \leq |x - 2|$  is

- (A)  $x \leq 1$
- (B)  $x \geq 1$
- (C)  $-1 \leq x \leq 1$
- (D)  $x \leq -1$  or  $x \geq 1$

10. A metal disc of 5 cm radius expands when heated. If the radius is increasing at a rate of 0.02 cm/sec, the rate at which the area of one of the faces is increasing is given by:

- (A)  $\frac{\pi}{10}$  cm<sup>2</sup>/sec
- (B)  $\frac{\pi}{5}$  cm<sup>2</sup>/sec
- (C)  $\frac{2\pi}{5}$  cm<sup>2</sup>/sec
- (D)  $\frac{5\pi}{2}$  cm<sup>2</sup>/sec

## Section II

60 marks

Attempt Questions 11 – 14.

Allow about 1 hour and 45 minutes for this section.

Answer each question in a separate writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

**Question 11** (15 marks) Use a new writing booklet.

- (a) Solve the inequality  $\frac{3x-2}{x+1} > 2$  2
- (b)  $M(-1,7)$  and  $N(3,1)$  are two points. If point  $L(\overset{7}{-5}, \overset{-5}{2})$  divides the interval  $MN$  externally in the ratio  $k:1$ , find the value of  $k$ . 2
- (c) Find  $\int \frac{1+6x}{1+x^2} dx$  2
- (d) The two curves  $y = x^3$  and  $y = 2 - x^2$  intersect at  $(1,1)$ . 3  
Find the acute angle between the two curves at  $(1,1)$ .
- (e) Use Mathematical Induction to show that for all positive integers  $n$  3  
 $1^2 + 3^2 + 5^2 + 7^2 + \dots + (2n-1)^2 = \frac{n}{3}(2n-1)(2n+1)$
- (f) Use the substitution  $u = x^3 - 1$  to evaluate  $\int_0^2 \frac{x^2}{(x^3-1)^2} dx$ . 3

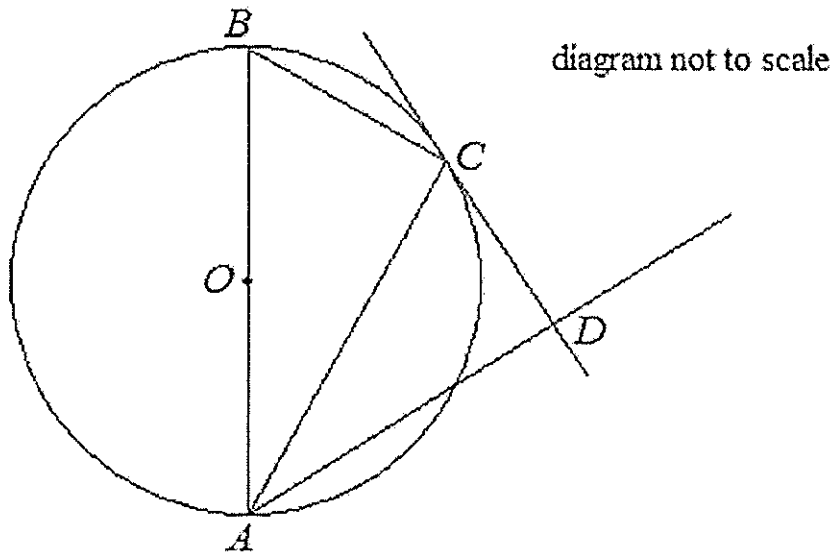
**End of Question 11.**

**Question 12** (15 marks) Use a new writing booklet.

- (a) The three numbers  $a, b, c$  are consecutive terms in an arithmetic progression. 2

Show that the three numbers  $e^a, e^b, e^c$  are consecutive terms in a geometric progression.

- (b) 3



In the diagram  $AOB$  is the diameter of a circle centre  $O$ , and  $C$  is the point of contact of the tangent  $DC$  such that  $AC$  bisects  $\angle DAB$ .

Copy this diagram into your booklet.

Prove that  $AD$  is perpendicular to  $DC$ .

- (c) The polynomials  $P(x)$  and  $Q(x)$  are such that  $P(x) = (x^2 - 1)Q(x) + ax + b$  for some constants  $a$  and  $b$ .  $(x + 1)$  is a factor of  $P(x)$  and when  $P(x)$  is divided by  $(x - 1)$  the remainder is 2. Find the remainder when  $P(x)$  is divided by  $(x^2 - 1)$ . 3

**Question 12 continues on page 8.**

**Question 12 continued**

- (d) Find the exact area between the curve  $y = \sin^{-1} x$ , the x-axis and the lines  $x = \frac{1}{2}$  and  $x = 1$ . **3**
- (e) Find the equation of the vertical and horizontal asymptotes of the curve  $y = \frac{2x^2 + 1}{x^2 - 4x}$  **2**
- (f) For what values of  $x$  will  $1 - \tan^2 x + \tan^4 x - \tan^6 x + \dots$  have a limiting sum for  $0 \leq x \leq 2\pi$ ? **2**

**End of Question 12.**

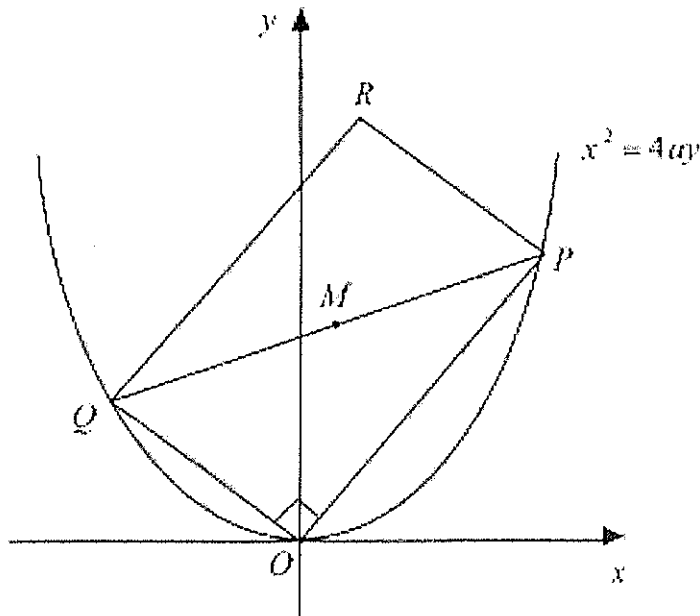


**Question 13** (15 marks) Use a new writing booklet.

(a) Show that  $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin^2 3x \, dx = \frac{1}{2} \left( \frac{\pi}{12} - \frac{1}{6} \right)$

3

(b)



$P(2ap, ap^2)$  and  $Q(2aq, aq^2)$  are two points which move on the parabola  $x^2 = 4ay$  such that  $\angle POQ = 90^\circ$ , where  $O$  is the origin.

$M = \left( a(p+q), \frac{1}{2}a(p^2+q^2) \right)$  is the midpoint of  $PQ$ .  $R$  is the point such that  $OPRQ$  is a rectangle.

(i) Show that  $pq = -4$

1

(ii) Show that  $R$  has coordinates  $(2a(p+q), a(p^2+q^2))$ .

1

(iii) Find the equation of the locus of  $R$ .

2

**Question 13 continues on page 10.**

**Question 13 continued**

- (c) The acceleration of a particle is given by  $\ddot{x} = 4(x + 1) \text{ ms}^{-2}$ . Initially, the particle is at the origin and velocity is  $2 \text{ ms}^{-1}$ .
- (i) Show that the velocity,  $v$ , at any position,  $x$ , is given by  $v = 2x + 2$ . 2
- (ii) Hence show that  $x = e^{2t} - 1$ . 2
- (d) (i) Show that  $e^x + x = 3$  has a root between  $x = 0$  and  $x = 1$ . 2
- (ii) By taking  $x = 0.8$  as an approximate solution, use one application of Newton's Method to find a better approximation, correct to 3 significant figures. 2

**End of Question 13.**

**Question 14** (15 marks) Use a new writing booklet

(a) The depth of water  $y$  metres in a tidal creek is given by  $y = 5 - 4 \cos \frac{t}{2}$ , for  $0 \leq t \leq 4\pi$ .

The time,  $t$ , being measured in hours.

(i) Draw a neat sketch of  $y = 5 - 4 \cos \frac{t}{2}$ , showing all important features 2

(ii) If the low tide one day is at 1.00 p.m., when is the earliest time that a ship requiring 3 m of water can enter the creek? Give your answer to the nearest minute. 2

(b) At time  $t$  years the number,  $N$  of individuals in a population is given by  $N = 500 - 400e^{-0.1t}$ .

(i) Show that  $\frac{dN}{dt} = -0.1(N - 500)$ . 1

(ii) Find the population size for which the rate of growth of the population is half the initial rate of growth. 2

(c) A group of 10 people, consisting of 6 girls and 4 boys, decided to go to the movies where they sit together in the same row.

(i) How many ways can the 10 people be seated in a row? 1

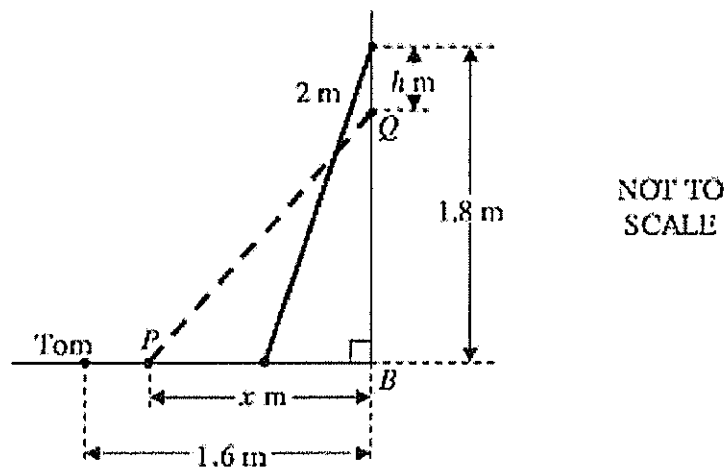
(ii) How many different arrangements of seating are possible where the 4 boys are all seated together? 2

(iii) How many ways can at least one of the boys be separated from the other boys? 1

**Question 14 continues on page 12.**

**Question 14 continued**

- (d) The diagram shows a ladder  $PQ$ , 2 metres in length, leaning against a wall such that the top of the ladder,  $Q$ , initially reaches 1.8 metres up the wall. The base of the ladder,  $P$ , is  $x$  metres from the base of the wall,  $B$ .



The ladder begins to slide down the wall at the rate of 0.5 metres per minute such that the top of the ladder is  $h$  metres below its original position after  $t$  minutes.

- (i) Show that  $t$  minutes after the ladder begins to slide down the wall, 1  

$$h = 1.8 - \sqrt{4 - x^2}.$$
- (ii) Tom is standing on the ground 1.6 metres from the base of the wall in a direct line with the ladder. 3

At what rate does the base of the ladder hit Tom?

**End of Exam.**

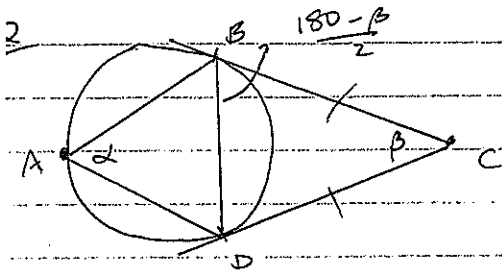
# Exam 1

$$1 \quad \lim_{x \rightarrow 0} \frac{\sin 3x}{2x}$$

$$= \lim_{x \rightarrow 0} \frac{\sin 3x}{3x \times \frac{2}{3}}$$

$$= \frac{3}{2} \lim_{x \rightarrow 0} \frac{\sin 3x}{3x}$$

$$= \frac{3}{2} \quad (D)$$



$$d = \frac{180 - \beta}{2} \quad (\angle \text{ in alternate segments})$$

$$\therefore 2d + \beta = 180$$

(B)

$$3 \quad P(x) = (x^2 + 1)Q(x) + (2x + 3)$$

$$+ Q(x) = (x^2 + bx + c)$$

$$c + 1 = 0 \quad \therefore c = -1$$

$$b + 2 = a \quad b = 0$$

$$\therefore a = 2 \quad (B)$$

$$4 \quad \log_2 a$$

$$= (2^2)^{\log_2 a}$$

$$= 2^{\log_2 a^2}$$

$$= a^2 \quad (B)$$

$$5 \quad \frac{\sin^2 A - \sin A \cos A - \sin^2 A - \sin A \cos A}{\sin^2 A - \cos^2 A}$$

$$\text{LHS} = \frac{-2 \sin A \cos A}{\sin^2 A - \cos^2 A}$$

$$= \frac{-2 \sin A \cos A}{-(\cos^2 A - \sin^2 A)}$$

$$= \frac{-(\sin 2A)}{-(\cos 2A)}$$

$$= \tan 2A \quad (C)$$

$$6 \quad \frac{d}{dx} \sin^{-1}(2x-1)$$

$$\text{let } u = 2x-1$$

$$\frac{du}{dx} = 2$$

$$\frac{dy}{du} \times \frac{du}{dx}$$

$$y = \sin^{-1} u$$

$$\frac{dy}{du} = \frac{1}{\sqrt{1-u^2}}$$

$$= \frac{2}{\sqrt{1-(2x-1)^2}}$$

$$= \frac{2}{\sqrt{1-4x^2+4x-1}}$$

$$= \frac{2}{\sqrt{4x(1-x)}}$$

$$= \frac{2}{2\sqrt{x(1-x)}}$$

$$= \frac{1}{\sqrt{x(1-x)}} \quad (D)$$

$$7 \quad \int \frac{1}{25(x^2 + \frac{9}{25})} dx$$

$$= \frac{1}{25} \times \frac{5}{3} \tan^{-1} \frac{5x}{3} + c$$

$$= \frac{1}{15} \tan^{-1} \frac{5x}{3} + c \quad (A)$$

$$8 \quad x = 2 \cos(3t + \frac{\pi}{6})$$

$$\text{max speed at } x=0$$

$$\therefore 3t + \frac{\pi}{6} = \frac{\pi}{2}$$

$$3t = \frac{\pi}{3}$$

$$t = \frac{\pi}{9} \quad (B)$$

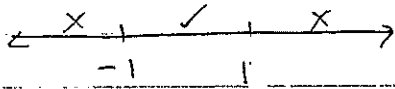
$$x = -6 \sin(3t + \frac{\pi}{6})$$

$$\text{speed} = |-6 \sin(\frac{\pi}{3} + \frac{\pi}{6})| = 6 \text{ m/s}$$

$$7 \quad |2x-1| \leq |x-2|$$

C.P

$x < \frac{1}{2}$	$\frac{1}{2} < x < 2$	$x > 2$
$2x+1 = -x+2$	$2x-1 = -x+2$	$2x-1 = x-2$
$-x = 1$	$3x = 3$	$x = -1$
$x = -1$	$x = 1$	X



$$x = -2 \quad 5 \leq 4 \quad \times$$

$$x = 0 \quad 1 \leq 2 \quad \checkmark$$

$$x = 2 \quad 3 \leq 0 \quad \times$$

$$-1 < x < 1 \quad \textcircled{C}$$

$$\frac{0}{\frac{dA}{dt}} = 0.02 \quad \frac{dA}{dt} = ?$$

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r \quad \frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

$$= 2\pi r \times 0.02$$

$$\text{at } r = 5$$

$$\frac{dA}{dt} = 2 \times \pi \times 5 \times 0.02$$

$$= \frac{\pi}{5} \text{ cm}^2/\text{sec}$$

$\textcircled{B}$

$$\frac{3x-2}{x+1} > 2$$

C.P at

$$x = -1$$

$$3x-2 = 2x+2$$

$$x = 4$$

x	3x-2	2x+2
-2	-8	-2
0	-2	2
5	13	12

$$x < -1, x > 4$$

(2)

$$M(-1, 7) \quad N(3, 1)$$

$$k: -1 \quad (-5, 2)$$

$$-5 = \frac{3k+1}{k-1}$$

$$-5k+5 = 3k+1$$

$$4 = 8k$$

$$k = \frac{1}{2}$$

(2)

$$\int \frac{1+6x}{1+x^2} dx$$

$$= \int \left( \frac{1}{1+x^2} + \frac{6x}{1+x^2} \right) dx$$

$$= \tan^{-1} x + 3 \ln(1+x^2) + c$$

(2)

$$y = x^3$$

$$y = 2-x^2$$

$$y' = 3x^2$$

$$y' = -2x$$

$$m_1 = 3$$

$$m_2 = -2$$

$$\tan \alpha = \left| \frac{3+2}{1-6} \right|$$

$$= |1|$$

$$\therefore \alpha = 45^\circ$$

(3)

e) Prove

$$1^2 + 3^2 + \dots + (2n-1)^2 = \frac{n}{3} (2n-1)(2n+1)$$

Prove n=1

$$LHS = 1^2$$

$$RHS = \frac{1}{3} (2-1)(2+1)$$

$$= 1$$

$$= 1$$

$$\therefore LHS = RHS$$

$$\therefore \text{true } n=1$$

(1)

Assume true n=k k is the integer

ie

$$1^2 + 3^2 + \dots + (2k-1)^2 = \frac{k}{3} (2k-1)(2k+1)$$

Prove true n=k+1

$$LHS = 1^2 + 3^2 + \dots + (2k-1)^2 + (2(k+1)-1)^2$$

$$= \frac{k}{3} (2k-1)(2k+1) + (2k+2-1)^2$$

$$= \frac{k}{3} (2k-1)(2k+1) + (2k+1)^2$$

$$= (2k+1) \left[ \frac{k}{3} (2k-1) + (2k+1) \right]$$

$$= (2k+1) \left( \frac{2k^2}{3} - \frac{k}{3} + 2k+1 \right)$$

$$= \frac{1}{3} (2k+1) (2k^2 - k + 6k + 3)$$

$$= \frac{1}{3} (2k+1) (2k^2 + 5k + 3)$$

$$= \frac{1}{3} (2k+1) (2k+3)(k+1)$$

$$= \frac{k+1}{3} (2(k+1)-1)(2(k+1)+1)$$

\therefore true n=k+1

(3)

$$f) \int_0^2 \frac{x^2}{(x^3-1)^2} dx$$

$$u = x^3 - 1$$

$$\frac{du}{dx} = 3x^2$$

$$\frac{1}{3} \int_{-1}^7 \frac{1}{u^2} du$$

$$x=2 \quad u=7$$

$$x=0 \quad u=-1$$

$$= \frac{1}{3} \int_{-1}^7 u^{-2} du$$

$$= -\frac{1}{3} \left[ u^{-1} \right]_{-1}^7$$

$$= -\frac{1}{3} \left( \frac{1}{7} + 1 \right)$$

$$= -\frac{8}{21}$$

Q12

$$a) \quad b-a = c-b = d \quad |$$

$$\frac{e^b}{e^a} = e^{b-a}$$

$$= e^{c-b} \quad (\text{from above})$$

$$= \frac{e^c}{e^b}$$

$$\therefore e^a, e^b, e^c \text{ are in G.P.} \quad (2)$$

$$b) \quad \angle ACB = 90^\circ \quad (\angle \text{ in semi-circle})$$

$$\angle BAC = \angle CAD$$

(given AC bisects  $\angle DAB$ )

$$\angle DCA = \angle CBA \quad (\angle \text{ in alternate segment})$$

$$\therefore \triangle ABC \cong \triangle ACD \quad (\text{equiangular})$$

$$\therefore \angle ADC = \angle ACB = 90^\circ$$

(corresponding  $\angle$ 's in similar  $\triangle$ 's)

(3)

$$c) \quad P(x) = (x^2-1)Q(x) + ax+b$$

$$P(-1) = 0$$

$$P(1) = 2$$

$$\therefore -a+b=0 \quad |$$

$$\underline{a+b=2} \quad |$$

$$2b=2$$

$$b=1$$

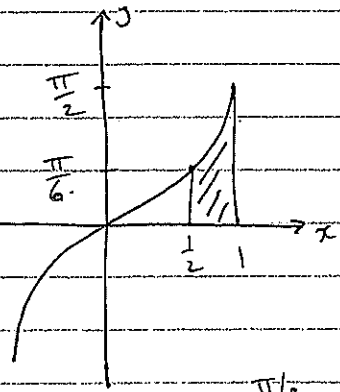
$$a=1$$

$$\therefore \text{Remainder} = x+1 \quad |$$

(3)



D)



$$\text{Area} = \frac{\pi}{2} - \frac{\pi}{6} \times \frac{1}{2} - \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin y \, dy$$

$$= \frac{\pi}{2} - \frac{\pi}{12} + \left[ \cos y \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} \leftarrow \frac{1}{2}$$

$$= \frac{5\pi}{12} + \left( 0 - \frac{\sqrt{3}}{2} \right) \frac{1}{2}$$

$$= \frac{5\pi - 6\sqrt{3}}{12} \quad \text{u2} \quad (3)$$

b)  $y = \frac{2x^2 + 1}{x^2 - 4x}$

vert. asympt at

$$x(x-4) = 0$$

$$x=0, x=4 \quad \frac{1}{2}$$

horiz asympt at

$$\lim_{x \rightarrow \infty} \frac{2x^2}{x^2} + \frac{1}{x^2} = \frac{x^2}{x^2} - \frac{4x}{x^2}$$

$$y = 2 \quad \frac{1}{2}$$

(2)

e)  $1 - \tan^2 x + \tan^4 x \dots$

limiting sum if  $|r| < 1$

$$\therefore |-\tan^2 x| < 1 \quad \frac{1}{2}$$

$$-1 < \tan x < 1$$

$$\begin{aligned} -\frac{\pi}{4} < x < \frac{\pi}{4} & \text{ but } 0 \leq x < 2\pi \\ 0 < x < \frac{\pi}{4} & \frac{3\pi}{4} < x < \frac{5\pi}{4} \quad \frac{7\pi}{4} < x < 2\pi \\ & \uparrow \text{ any } \frac{1}{2} \quad (2) \end{aligned}$$

13 a) LHS =  $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin^2 3x \, dx$

$$\cos 6x = 1 - 2\sin^2 3x$$

$$\sin^2 3x = \frac{1}{2} - \frac{1}{2} \cos 6x$$

$$= \frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (1 - \cos 6x) \, dx \quad \frac{1}{2}$$

$$= \frac{1}{2} \left[ x - \frac{\sin 6x}{6} \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} \quad \frac{1}{2}$$

$$= \frac{1}{2} \left( \frac{\pi}{3} - \frac{\sin 2\pi}{6} - \frac{\pi}{4} + \frac{\sin \frac{3\pi}{2}}{6} \right)$$

$$= \frac{1}{2} \left( \frac{\pi}{3} - \frac{\pi}{4} - \frac{1}{6} \right)$$

$$= \frac{1}{2} \left( \frac{\pi - 2}{12} \right) \quad \frac{1}{2} \quad (3)$$

b) OQ:  $m = \frac{aq^2}{2aq}$  OP:  $m_2 = \frac{p}{2}$

$$m_1 = \frac{q}{2}$$

$$m_1 m_2 = -1$$

∴ OQ ⊥ OP

$$\therefore \frac{q}{2} \times \frac{p}{2} = -1$$

$$\therefore pq = -4 \quad \frac{1}{2}$$

(1)

i) m is midpoint of OR

(diagonals of rectangle bisect each other)

$R(x, y)$

$$\therefore a(p+q) = \frac{x+0}{2}$$

(1)

$$\therefore x = 2a(p+q)$$

$$\frac{1}{2}a(p^2+q^2) = \frac{y+0}{2}$$

$$\therefore y = a(p^2+q^2)$$

$$\therefore R(2a(p+q), a(p^2+q^2))$$

i)  $p+q = \frac{x}{2a}$

$$+ y = a((p+q)^2 - 2pq) \quad (1)$$

$$= a\left(\frac{x^2}{4a^2} - 2(-4)\right)$$

$$= a\left(\frac{x^2}{4a^2} + 8\right) \quad \checkmark$$

$$= \frac{x^2}{4a} + 8a$$

$$x^2 = 4a(y - 8a) \quad (2)$$

c)  $\ddot{x} = 4(x+1)$

at  $t=0, x=0, v=2$

i)  $\frac{d}{ds}\left(\frac{1}{2}v^2\right) = 4x+4$

$$\frac{1}{2}v^2 = 2x^2 + 4x + c \quad \checkmark$$

at  $x=0, v=2 \quad \therefore c=2$

$$v^2 = 4x^2 + 8x + 4$$

$$v^2 = 4(x^2 + 2x + 1) \\ = 4(x+1)^2$$

$$\therefore v = \sqrt{4(x+1)^2} \quad \checkmark$$

$$v = 2(x+1) \quad (2)$$

ii)  $\frac{dx}{dt} = 2(x+1)$

$$\frac{dt}{ds} = \frac{1}{2(x+1)}$$

$$\therefore t = \frac{1}{2} \int \frac{1}{x+1} ds$$

$$t = \frac{1}{2} \ln(x+1) + c \quad \checkmark$$

at  $t=0, x=0$

$$\therefore 0 = \frac{1}{2} \ln 1 + c \quad \therefore c=0$$

$$\therefore t = \frac{1}{2} \ln(x+1)$$

$$2t = \ln(x+1)$$

$$e^{2t} = x+1$$

$$x = e^{2t} - 1 \quad \checkmark \quad (2)$$

d) i)  $f(x) = e^x + x - 3$

$$f(0) = e^0 + 0 - 3$$

$$= 1 - 3$$

$$= -2 < 0 \quad \checkmark$$

$$f(1) = e + 1 - 3$$

$$= e - 2 > 0 \quad \checkmark$$

$$\therefore f(0) < 0, f(1) > 0$$

$\therefore$  root lies between

$$x=0 \text{ and } x=1$$

(2)

1)  $a = 0.8$

$f(x) = e^x + x - 3$

$f'(x) = e^x + 1$

$f(0.8) = e^{0.8} - 2.2$

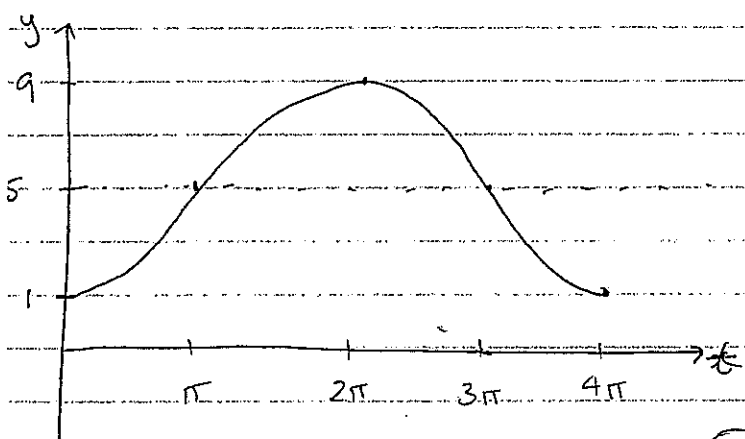
$f'(0.8) = e^{0.8} + 1$

$a = 0.8 - \frac{e^{0.8} - 2.2}{e^{0.8} + 1}$

$a = 0.792$  (3 s.f.)

(2)

4  
a) i)  $y = 5 - 4 \cos \frac{t}{2}$



(2)

1)  $3 = 5 - 4 \cos \frac{t}{2}$

$\cos \frac{t}{2} = \frac{1}{2}$

$\therefore \frac{t}{2} = \frac{\pi}{3}$

$t = \frac{2\pi}{3} h$

$\approx 2.09 h$

earliest time at 3:06 pm  
(nearest minute)

(2)

b)  $N = 500 - 400e^{-0.1t}$

i)  $\frac{dN}{dt} = -400 \times 0.1 e^{-0.1t}$   
 $= -0.1(-400e^{-0.1t})$   
 $= -0.1(500 - 400e^{-0.1t} - 500)$   
 $= -0.1(N - 500)$

(1)

ii) at  $t = 0$

$\frac{dN}{dt} = 40$

$\therefore \frac{dN}{dt} = 20$

$20 = -0.1(N - 500)$

$N = 300$

(2)

c) i)  $10! = 3628800$

(1)

ii)  $7! \times 4! = 120960$

(2)

iii)  $3628800 - 120960 = 3507840$

(1)

d) i)  $(1.8 - h)^2 + x^2 = 2^2$

$(1.8 - h)^2 = 4 - x^2$

$1.8 - h = \sqrt{4 - x^2}$

$h = 1.8 - \sqrt{4 - x^2}$

(1)

ii) at  $x = 1.6 m$

$\frac{dh}{dx} = -\frac{1}{2}(4 - x^2)^{-1/2} \times -2x$

$= \frac{x}{\sqrt{4 - x^2}}$

1

$$4 \frac{dh}{dt} = 0.5$$

$$\therefore \frac{dx}{dt} = \frac{dx}{dh} \times \frac{dh}{dt} \quad \text{at } x = 1.6$$

$$\frac{dx}{dt} = \frac{\sqrt{4-x^2}}{x} \times 0.5 \quad \checkmark$$

$$= \frac{\sqrt{4-1.6^2}}{1.6} \times 0.5$$

$$= 0.375 \text{ m/s} \quad \checkmark$$

(3)