Ext 1

NORTH SYDNEY BOYS HIGH SCHOOL

2016 HSC ASSESSMENT TASK 3 (TRIAL HSC)

Mathematics

Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher:
(Please tick or highlight)

- ○ Mr Berry
- ○ Mr Ireland
- ○ Dr Jomaa
- ○ Ms Lee
- ○ Mr Lin
- ○ Ms Ziaziaris

Student Number:

(To be used by the exam markers only.)

Question No	1-10	11	12	13	14	Total	Total
Mark	/10	/15	/15	/15	/15	/70	/100

Section I

10 Marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 - 10

---

1. What is the natural domain of the function $f(x) = \sqrt{x+1} - \sqrt{x+2}$

(A) $x \leq -2$

(B) $x \geq -1$

(C) $-1 \leq x \leq -2$

(D) $x \leq -2$ or $x \geq -1$

2. Given that $t = \tan\theta$, which of the following is the equivalent of $\sec 2\theta$?

(A) $\frac{1+t^2}{1-t^2}$

(B) $\frac{1+4t^2}{1-4t^2}$

(C) $2(\frac{1+t^2}{1-t^2})$

(D) $\frac{1+(\frac{t}{2})^2}{1-(\frac{t}{2})^2}$

3. Which of the following is the solution to $\frac{x+1}{x-2} \geq 1$?

(A) $x > 2$

(B) $-1 \leq x < 2$

(C) $x \leq -1$

(D) All real *x* but $x \neq 2$

4. Which of the following is equivalent to $\sqrt{3}\cos x - \sin x$?

(A) $2\cos\ (x + \frac{\pi}{3})$

(B) $2\cos(x - \frac{\pi}{3})$

(C) $2\cos\ (x + \frac{\pi}{6})$

(D) $2\cos\ (x - \frac{\pi}{6})$

5. What is the gradient of the tangent to the function $y = \sin^{-1} 2x$ at the point $\left(\frac{1}{4}, \frac{\pi}{6}\right)$?

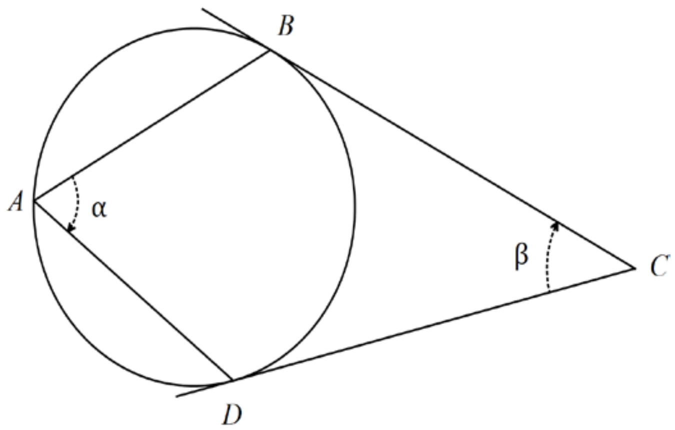
(A) $\frac{\pi}{6}$

(B) $\frac{1}{\sqrt{3}}$

(C) 1

(D) $\frac{4}{\sqrt{3}}$

6. In the diagram below, *BC* and *DC* are tangents, then:



(A) $2\alpha + \beta = 180°$

(B) $\alpha + 2\beta = 180°$

(C) $2\alpha - \beta = 90°$

(D)$2\beta - \alpha = 90°$

7. Let α, β and γ be the roots of $P(x) = x^3 + 2x^2 + 3x + 4$.

What is the value of $\frac{2}{\alpha} + \frac{2}{\beta} + \frac{2}{\gamma}$

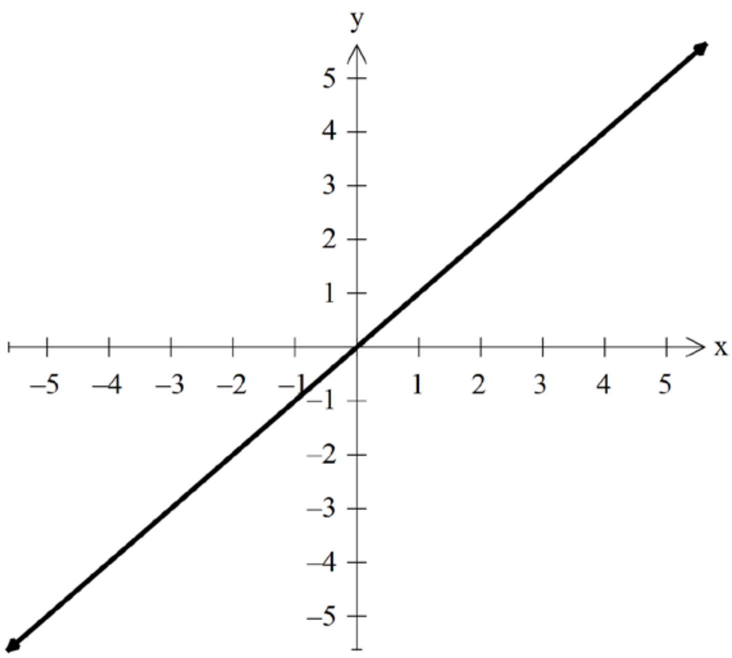
(A) $-\frac{3}{2}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $\frac{3}{2}$

8.



Which of the following could be used to describe the above graph:

(A) $\sin(\sin^{-1} x)$

(B) $\cos^{-1}(\cos x)$

(C) $\tan(\tan^{-1} x)$

(D) None of the above

9. What is the equation of the chord of contact to the parabola $x^2 = 8y$ from the point $(3, -2)$?

(A) $x - 8y = 0$

(B) $2x - 3y + 6 = 0$

(C) $x + 2y - 5 = 0$

(D) $3x - 4y + 8 = 0$

10. What is $\int \frac{dx}{\sqrt{x}+x}$?

(A) $\ln(x + \sqrt{x^2 - 1}) + C$

(B) $2\ln(1 + \sqrt{x}) + C$

(C) $\ln(x + \sqrt{x^2 + 1}) + C$

(D) $2\tan^{-1}(\sqrt{x}) + C$

Section II

60 Marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question on a NEW page. Extra writing booklets are available.

In Questions 11 -14, your responses should include all relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks) Start a NEW page.

(a) Given $A(2,1)$ and $B(7,3)$, find the coordinates of point *C* which divides the interval *AB* externally in the ratio $2:3$. **2**

(b) Find the value(s) of *k* such that $(x-2)$ is a factor of the polynomial:

$$P(x) = x^3 - 2x^2 + kx + k^2$$ **2**

(c) The acute angle between the lines $y = 2x + 2$ and $y = mx + 1$ is 45°.
Find the value(s) of *m*. **2**

(d) Use the substitution $u = e^x$ to find:

$$\int \frac{dx}{e^x + 9e^{-x}}$$ **3**

(e) Evaluate, expressing your answer in simplest form:

$$\int_0^{\frac{\pi}{6}} \sin(x)\cos^2(x)\,dx$$ **3**

(f) Find, showing full working:

$$\lim_{x\to 0} \frac{x^2}{2 - 2\cos 2x}$$ **3**

Question 12 (15 Marks) Start a NEW page.

(a) The polynomial $P(x) = 4x^3 + 2x^2 + 1$ has one real root in the interval $-1 < x < 1$.

(i) Find any stationary points and determine their nature. **2**

(ii) Find any points of inflexion **2**

(iii) Hence sketch $y = P(x)$ for $-1 < x < 1$ **1**

(iv) Let $x = -\frac{1}{4}$ be a first approximation to the root. Apply Newton's method once to obtain another approximation for the root. **2**

(v) Explain why the application of Newton's method in part (ii) was NOT effective in improving the approximation of the root. **1**

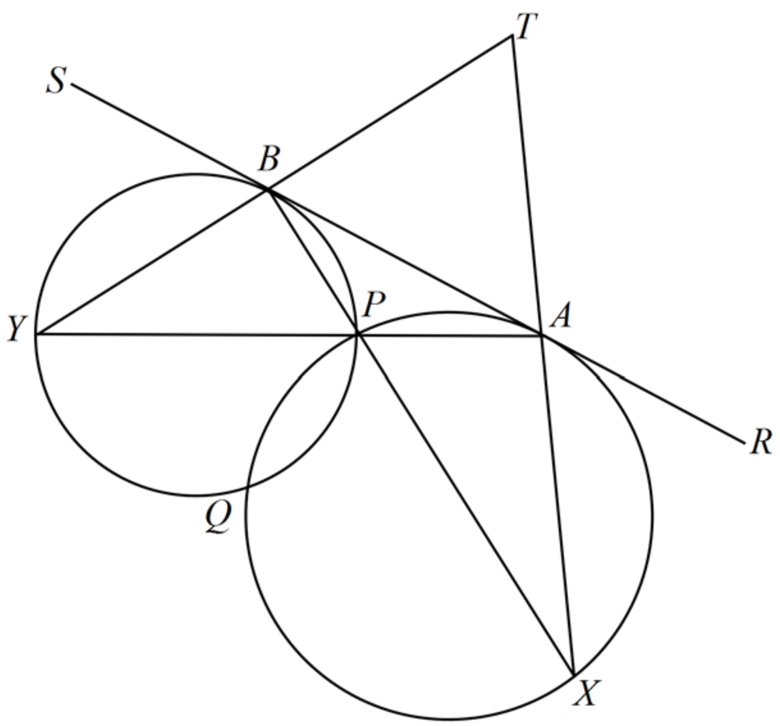
(b) Prove by mathematical induction that for $n \geq 1$:

$$\sum_{j=1}^{n} j \times 2^{j-1} = 1 + (n-1)2^n$$ **3**

(c)

AB is a common tangent in two circles which intersect at *P* and *Q* as illustrated in the diagram below.

XPB and *YPA* are straight lines. *XA* and *YB* intersect at *T*.



(i) Copy or trace this diagram into your writing book.

(ii) Explain why $\angle SBY = \angle BPY$ **1**

(iii) Prove that $AT = TB$ **3**

Question 13 (15 Marks) Start a NEW page.

(a) A young gentleman wishes to eat a meal of chicken tenders. His mother puts his chicken tenders in the oven and heats them to a temperature of $70°\text{C}$. They cool to $60°\text{C}$ in 5 minutes.

If the surrounding air temperature $S°\text{C}$ is $23°\text{C}$ and assuming Newton's Law of Cooling:

$$\frac{dT}{dt} = -k(T - S)$$

(i) Show that $T = 23 + Ae^{-kt}$ is a solution to the differential equation, where *A* is a constant. **1**

(ii) Find the temperature of the chicken tenders 15 minutes from the time they were taken out of the oven. **3**

(b) A polynomial *P(x)* has a remainder *p* when divided by $(x - p)$ and has a remainder *q* when divided by $(x - q)$, where *p* and *q* are real constants such that $p \neq q$.

Show that when *P(x)* is divided by $(x - p)(x - q)$, the remainder is *x*. **2**

(c) A particle is moving in a straight line. At time *t* seconds it has displacement *x* metres from a fixed point *O* on the line. Its velocity v m/s is given by $v = -\frac{1}{8}x^3$. The particle is initially 2 metres to the right of *O*.

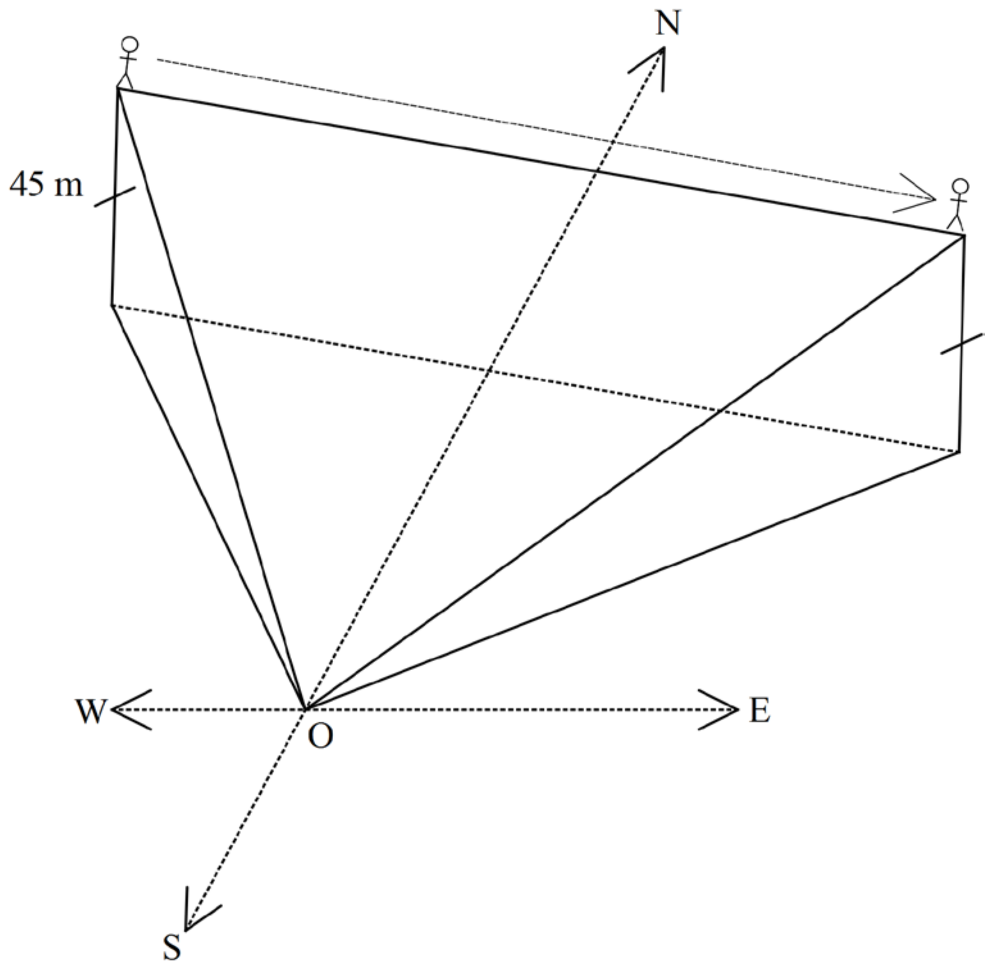
(i) Show that the acceleration a, is given by: $a = \frac{3}{64}x^5$ **2**

(ii) Find an expression for *x* in terms of *t*. **3**

(iii) Describe the limiting motion of the particle. **1**

Question 13 continues over page

(d)



An acrobat is walking at a constant speed along the top of a 45 m high tightrope. An observer at point *O* sees her on a bearing of $335°T$ from *O* with an angle of elevation of $28°$.

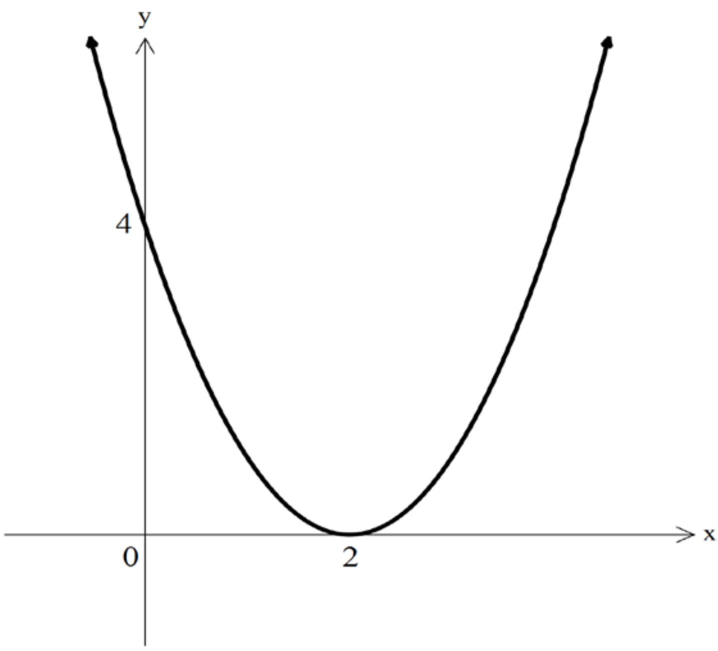
After 1 minute the acrobat has a bearing of $025°T$ from *O* and a new angle of elevation of $53°$.

Find the distance the acrobat has travelled in that minute. **3**

End of Question 13

Question 14 (15 Marks) Start a NEW page.

(a)



The sketch above shows the parabola $f(x) = (x - 2)^2$

(i) Explain why an inverse function, $f^{-1}(x)$ exists for $x \geq 2$. **1**

(ii) State the domain and range of this inverse function. **1**

(iii) For what value of *x* does $f(x) = f^{-1}(x)$? **1**

(iv) If $k < 2$, find, in simplest form, $f^{-1}(f(k))$ **1**

(b) From a point $A(p, q)$ perpendicular lines *AP* and *AQ* are drawn to meet the *x* and *y* axes at $P(p, 0)$ and $Q(0, q)$ respectively.

(i) Find the equation of *PQ* **1**

(ii) Show that the condition for the line *PQ* to be a tangent to the parabola

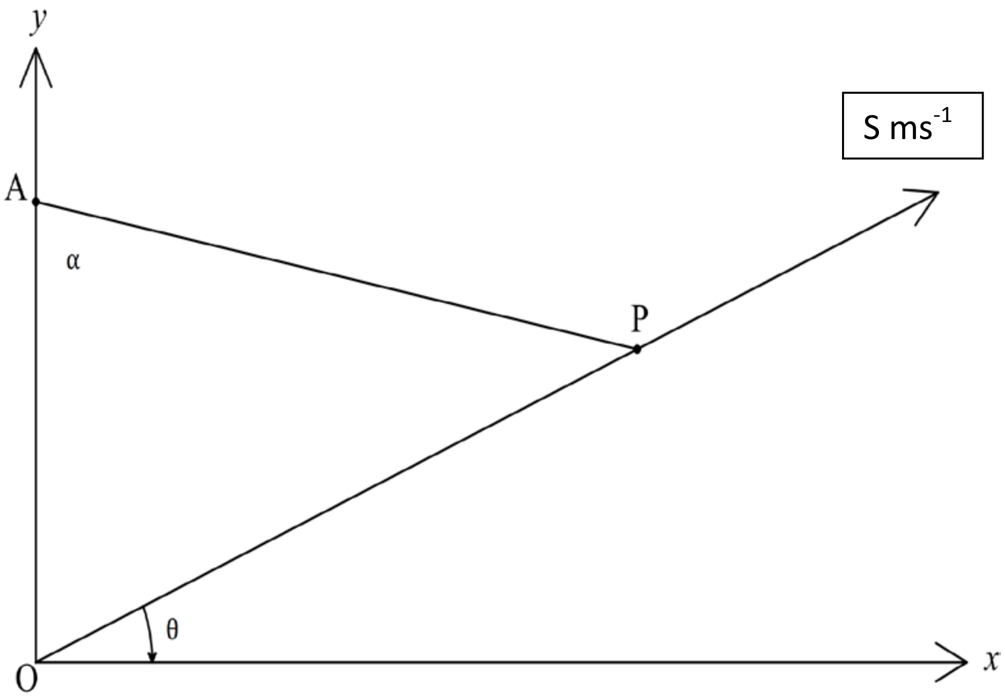
$x^2 = 4ay$ is $aq + p^2 = 0$. **3**

(iii) If the points $P(p, 0)$ and $Q(0, q)$ move on the *x* and *y* axes respectively such that *PQ* is a tangent to the parabola $x^2 = 4ay$ then the point $A(p, q)$ traces out a curve as *P* and *Q* move:

Find the equation of the locus of *A* and describe the path that it traces out, providing all relevant details. **2**

Question 14 continues over page

(c) A point A lies on the y-axis, 1 unit away from the origin in the positive direction. Another point P lies on the line $y = x\tan\theta$ where θ is a constant such that $0° < \theta < 90°$. P travels along the line with some speed S ms^{-1}. Let k be the distance from P to the origin, and let $\angle OAP$ be α .



(i) Prove that $k = \dfrac{\sin\alpha}{\cos(\alpha-\theta)}$ **1**

(ii) Let $\dot{\alpha} = \dfrac{d\alpha}{dt}$ and $\ddot{\alpha} = \dfrac{d^2\alpha}{dt^2}$. Show that when $\alpha = 2\theta$,

(1) $\dot{\alpha} = S\cos\theta$ **2**

(2) $\ddot{\alpha} = -\dot{\alpha} \times S \times k$ **2**

End of Examination.

MULTIPLE CHOICE ANSWERS

1. B

2. A

3. A

4. C

5. D

6. A

7. A

8. C

9. D

10. B

Question 11		
(a)	$C = \left(\dfrac{(3)\times(2)+(-2)\times(7)}{(3)+(-2)}, \dfrac{(3)\times(1)+(-2)\times 3}{3+(-2)}\right)$ $= (-8, -3)$	✓ x coordinate ✓ y coordinate
(b)	$2^3 - 2\times 2^2 + 2k + k^2 = 0$ $k^2 + 2k = 0$ $k(k+2) = 0$ $k = 0 \text{ or } k = -2$	✓ creating quadratic ✓ solving for k
(c)	$\tan 45° = \left\lvert\dfrac{m-2}{1+2m}\right\rvert$ case 1 : $\dfrac{m-2}{1+2m} = 1$ $m - 2 = 1 + 2m$ $m = -3$ case 2 : $\dfrac{m-2}{1+2m} = -1$ $m - 2 = -1 - 2m$ $3m = 1$ $m = \dfrac{1}{3}$	✓ substituting into formula ✓ solving for m
(d)	$u = e^x$ $\dfrac{du}{dx} = e^x$ $du = e^x\,dx$ $\displaystyle\int \frac{dx}{e^x + 9e^{-x}}$ $\displaystyle= \int \frac{dx}{e^x + \frac{9}{e^x}}$ $\displaystyle= \int \frac{e^x\,dx}{e^{2x} + 9}$ $\displaystyle= \int \frac{du}{u^2 + 9}$ $= \dfrac{1}{3}\tan^{-1}\dfrac{u}{3} + C$ $= \dfrac{1}{3}\tan^{-1}\dfrac{e^x}{3} + C$	✓ expression for *du* ✓ integral in terms of u ✓ final answer in terms of x

(e)	$\int_0^{\frac{\pi}{6}} \sin(x)\cos^2(x)\,dx = \left[-\frac{1}{3}\cos^3 x\right]_0^{\frac{\pi}{6}}$ $= -\frac{1}{3}\left(\left(\frac{\sqrt{3}}{2}\right)^3 - (1)^3\right)$ $= -\frac{3\sqrt{3}}{24} + \frac{1}{3}$ $= \frac{1}{3} - \frac{\sqrt{3}}{8}$	✓ applying reverse chain rule ✓ substituting into primitive ✓ final answer
(f)	$\lim_{x\to 0}\frac{x^2}{2-2\cos 2x} = \lim_{x\to 0}\frac{x^2}{2-2(1-2\sin^2 x)}$ $= \lim_{x\to 0}\frac{x^2}{4\sin^2 x}$ $= \frac{1}{4}\lim_{x\to 0}\frac{x^2}{\sin^2 x}$ $= \frac{1}{4}\times 1^2$ $= \frac{1}{4}$	✓ cos 2x in terms of sin x ✓ using x/sinx=1 ✓ final answer

Question 12

(a) (i)

$$P'(x) = 12x^2 + 4x$$

when $P'(x) = 0$

$12x^2 + 4x = 0$

$4x(3x+1) = 0$

$$x = 0 \text{ or } x = -\frac{1}{3}$$

Stationary points at $(0,1)$ and $\left(-\frac{1}{3}, \frac{29}{27}\right)$

✓ finding stationary points

$$P''(x) = 24x + 4$$

$$P''\left(-\frac{1}{3}\right) = -4$$
$$< 0$$

$\therefore \left(-\frac{1}{3}, \frac{29}{27}\right)$ is a maximum

✓ testing stationary points

$$P''(0) = 4$$
$$> 0$$

$\therefore (0, 1)$ is a minimum

(ii)

$P''(x) = 0$

$24x + 4 = 0$

$24x = -4$

$$x = -\frac{1}{6}$$

$$y = \frac{28}{27}$$

✓ finding point of inflexion

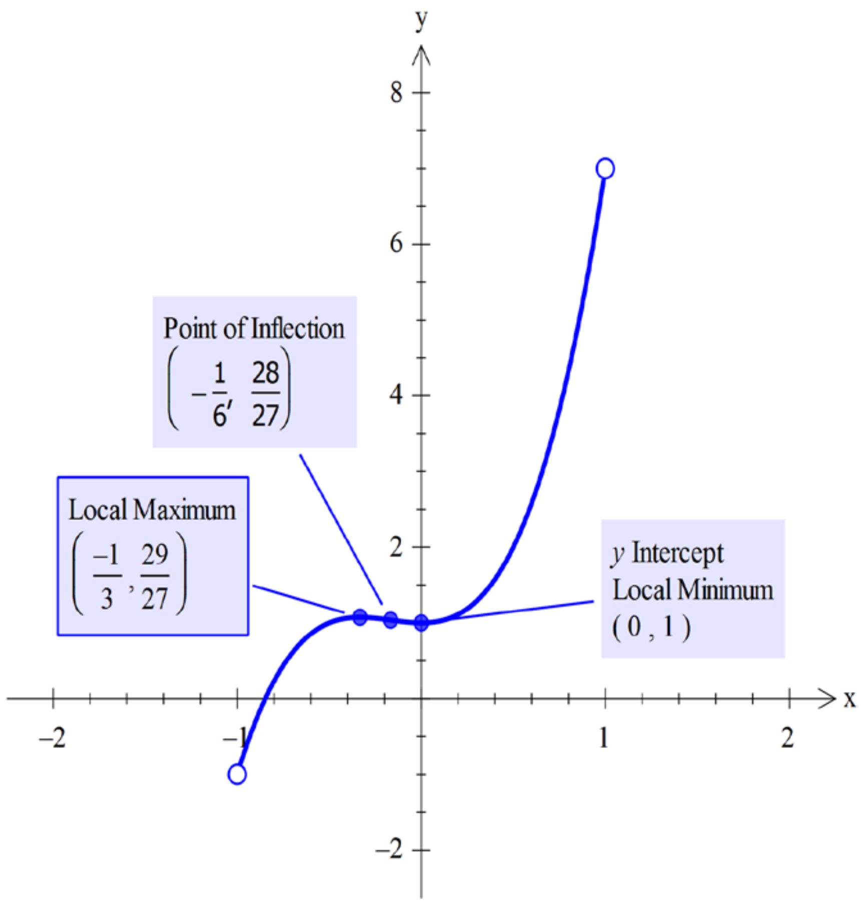
$\therefore$ possible point of inflexion at $\left(-\frac{1}{6}, \frac{28}{27}\right)$

x	-1	$-\frac{1}{6}$	0
$P''(x)$	-20	0	4

✓ showing change in concavity

$\therefore$ change in concavity so $\left(-\frac{1}{6}, \frac{28}{27}\right)$ is a point of inflexion

(iii)



✓ correct graph

(iv)

$$x_1 = x_0 - \frac{P(x_\circ)}{P'(x_0)}$$

$$= -\frac{1}{4} - \frac{4\left(-\frac{1}{4}\right)^3 + 2\left(-\frac{1}{4}\right)^2 + 1}{12\left(-\frac{1}{4}\right)^2 + 4\left(-\frac{1}{4}\right)}$$

$$= 4$$

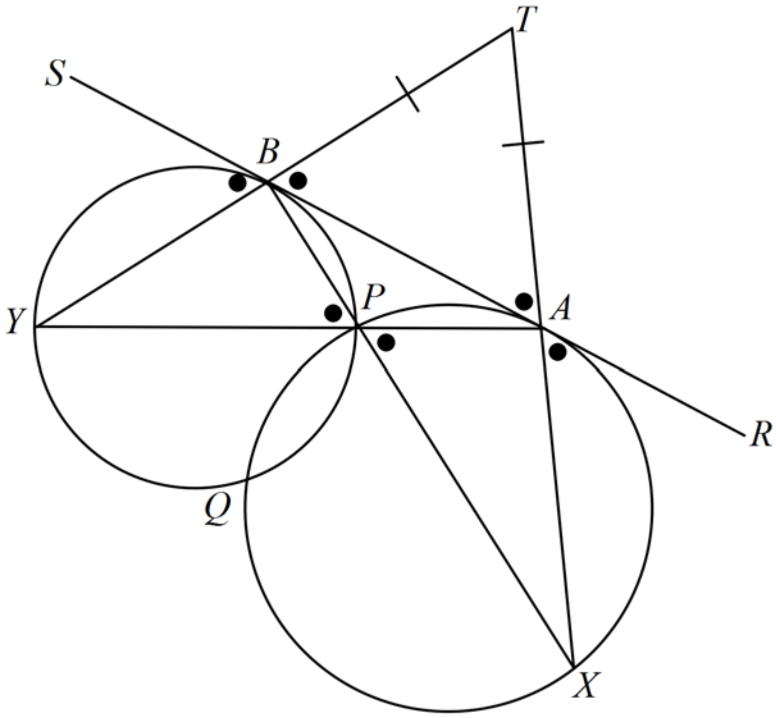
✓ correct formula and substitution for Newton's method

✓ correct approximation

(v) Because the initial approximation was on a different side of a turning point to the root.

✓ valid reason

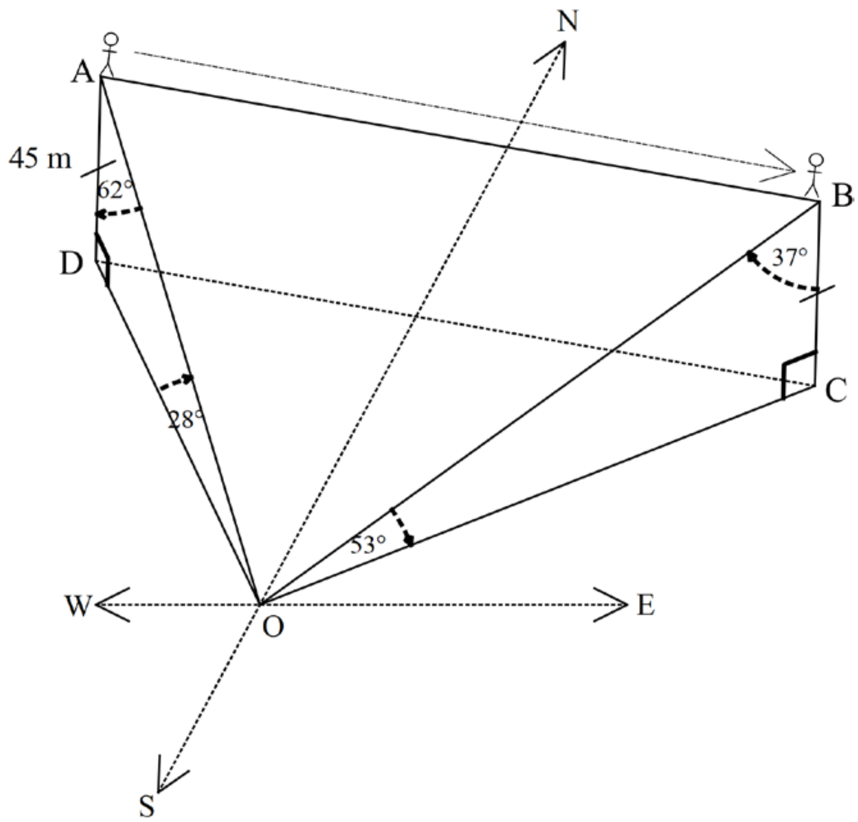
(b)	<u>Base case $n = 1$.</u> $\text{LHS} = 1 \times 2^{1-1}$ $= 1$ $\text{RHS} = 1 + (1-1) \times 2^{1}$ $= 1$ $\text{LHS} = \text{RHS}$ $\therefore$ true for $n = 1$	✓ testing base case
	<u>Assume true for *n*=*k*</u> assume $\sum_{j=1}^{k} j \times 2^{j-1} = 1 + (k-1)2^{k}$	
	<u>Prove true for *n* = *k* +1</u> Prove that: $\sum_{j=1}^{k+1} j \times 2^{j-1} = 1 + ((k+1)-1)2^{k+1}$ - $\text{LHS} = \sum_{j=1}^{k+1} j \times 2^{j-1}$ $= \sum_{j=1}^{k} j \times 2^{j-1} + (k+1) \times 2^{(k+1)-1}$ $= 1 + (k-1)2^{k} + (k+1) \times 2^{k}$ from assumption $= 1 + 2^{k}((k-1) + (k+1))$ $= 1 + 2^{k}(2k)$ $= 1 + 2^{k+1}(k)$ $= 1 + ((k+1)-1)2^{k+1}$ $= \text{RHS}$	✓ substitution from assumption ✓ LHS=RHS
	Therefore the proposition is true by the process of mathematical induction.	

(c)	(i)  	
	(ii) Angle between a chord and the tangent and the point of contact is equal to the angle subtended by the chord at the circumference in the alternate segment.	✓correct reason
	(iii) $\angle TBA = \angle SBY$ (vertically opposite angles) $\angle SBY = \angle BPY$ (shown in part (i)) $\angle BPY = \angle APX$ (vertically opposite angles) $\angle APX = \angle RAX$ (alternate segment theorem) $\angle RAX = \angle TAB$ (vertically opposite angles) $\therefore\ \angle TAB = \angle TBA$ $\therefore$ AT = TB (equal sides opposite equal angles in ΔABT)	✓2 relevant applications of vertically opposite angles ✓second application of alternate segment theorem ✓base angles of isosceles triangle equal

Question 13		
(a)	(i)	
	$T = 23 + Ae^{-kt}$ $\dfrac{dT}{dt} = -kAe^{-kt}$ $= -k\,(23 + Ae^{-kt} - 23)$ $= -k\,(T - S)$ (as $T = 23 + Ae^{-kt}$ and $S = 23$)	✓differentiation and substitution
	$\therefore$ $T = 23 + Ae^{-kt}$ is a solution to the differential equation	
	(ii) $T = 23 + Ae^{-kt}$ when $t = 0$ $70 = 23 + Ae^{-0k}$ $70 = 23 + A$ $\therefore$ $A = 47$	✓finding A
	$T = 23 + 47e^{-kt}$ when $t = 5$, $A = 60$ $60 = 23 + 47e^{-5k}$ $37 = 47e^{-5k}$ $-5k = \log_e\left(\dfrac{37}{47}\right)$ $\therefore$ $k = -\dfrac{\log_e\left(\dfrac{37}{47}\right)}{5}$	✓finding k
	when $t = 15$ $T = 23 + 47e^{-15k}$ $= 46°C$ (to the nearest degree Celcius)	✓final answer

(b)	$P(p) = p$ (remainder theorem and given information) $P(q) = q$ (remainder theorem and given information) $P(x)$ can be expressed as: $P(x) = (x-p)(x-q)Q(x) + (ax+b)$ sub in $x = p$ $P(p) = 0 + ap + b$ $\therefore \quad p = ap + b \quad \ldots$ ① sub in $x = q$ $P(q) = 0 + aq + b$ $\therefore \quad q = aq + b \quad \ldots$ ② ① – ② $p - q = (ap+b) - (aq+b)$ $p - q = ap - aq$ $p - q = a(p-q)$ $\therefore \quad a = 1$ sub back into ① $p = 1p + b$ $\therefore \quad b = 0$ since a = 1 and $b = 0$ $ax + b = x$	✓ using the remainder theorem to create at least one of the two equations ✓ solving simultaneous equations to find values for a and b

(c)	(i) $$v = -\frac{1}{8}x^3$$ $$v^2 = \frac{1}{64}x^6$$ $$\frac{1}{2}v^2 = \frac{1}{128}x^6$$ $$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{6}{128}x^5$$ $$= \frac{3}{64}x^5$$ $$\therefore \quad a = \frac{3}{64}x^5$$	✓ correct application of formula ✓ correct differentiation to get to the answer
	(ii) $$\frac{dx}{dt} = -\frac{1}{8}x^3$$ $$\frac{dt}{dx} = -8x^{-3}$$ $$t = 4x^{-2} + C$$ when $t = 0$, $x = 2$ $$0 = 4(2)^{-2} + C$$ $$C = -\frac{4}{(2)^2}$$ $$= -1$$ $$t = \frac{4}{x^2} - 1$$ $$\frac{4}{x^2} = t + 1$$ $$\frac{x^2}{4} = \frac{1}{t+1}$$ $$x^2 = \frac{4}{t+1}$$ $$x = \sqrt{\frac{4}{t+1}}$$ (as x > 0 and the particle cannot cross the origin as it will stop there) $$x = \frac{2}{\sqrt{(t+1)}}$$	✓ expression for t ✓ evaluating C ✓ expression for x
	(iii) As time approaches infinity the particle will approach 0 from the positive direction. Velocity will be negative but approaching 0. Acceleration will be positive but also approaching 0. So the particle will be continually slowing down but never quite stopping and also never quite reaching the origin.	✓ correct description

(d)	 Let A, B, C and D be as shown in ΔADO: $\tan(62°) = \frac{OD}{AD}$ $\therefore \quad OD = AD(\tan(62°))$ $= 45\tan(62°)$ in ΔBCO: $\tan(37°) = \frac{OC}{BC}$ $\therefore \quad OC = BC(\tan(37°))$ $= 45\tan(37°)$ in ΔDOC: $DC^2 = OD^2 + OC^2 - 2(OD)(OC)\cos(\angle DOC)$ $= (45\tan(62°))^2 + (45\tan(37°))^2 - 2 \times (45\tan(62°)) \times (45\tan(37°)) \times \cos(50°)$ $\therefore$ AB = 68 m (to the nearest metre)	✓ expression for OD and/or OC ✓ correct use of cosine rule in triangle DOC ✓ correct answer

	Question 14	
(a)	(i) Because when $x \geq 2$ each *y* value on *f(x)* has only one corresponding *x* value (i.e. it passes the horizontal line test for inverse functions). (ii) $x \geq 0$ and $y \geq 2$ (iii) Since $f(x)$ and $f^{-1}(x)$ meet on the line $y = x$ $f(x) = x$ $(x-2)^2 = x$ $x^2 - 4x + 4 = x$ $x^2 - 5x + 4 = 0$ $(x-1)(x-4) = 0$ since $x \geq 2$ $x = 4$ (iv) $4 - k$	✓ Correct answer ✓ Correct answer ✓ Correct answer ✓ Correct answer
(b)	(i) $m = -\frac{p}{q}$ $b = q$ $\therefore\ y = -\frac{q}{p}x + q$ (ii) $x^2 = 4ay$ $y = \frac{x^2}{4a}$ ... ② ① = ② $-\frac{q}{p}x + q = \frac{x^2}{4a}$ $-4aqx + 4apq = px^2$ $px^2 + 4aqx - 4apq = 0$ for tangent $\Delta = 0$ $(4aq)^2 - 4(p)(-4apq) = 0$ $16a^2q^2 + 16ap^2q = 0$ $\therefore\ aq + p^2 = 0$ (iii) $aq + p^2 = 0$ $p^2 = -aq$ $x^2 = -ay$ This is a concave down parabola with vertex at (0,0) and a focal length of $\frac{a}{4}$	✓ Correct answer (any format) ✓ Solving simultaneous equations ✓ Discriminant = 0 ✓ Correct result ✓ Equation ✓ Description

(c)	(i) $\angle AOP = 90 - \theta$ (complementary adjacent angles) $\angle APO = 180 - \angle PAO - \angle AOP$ $= 180 - \alpha - (90 - \theta)$ $= 90 - a + \theta$ $= 90 - (\alpha - \theta)$ $\dfrac{\text{OP}}{\sin \angle PAO} = \dfrac{\text{AO}}{\sin \angle APO}$ $\dfrac{k}{\sin\alpha} = \dfrac{1}{\sin(90 - (\alpha - \theta))}$ $\therefore \quad k = \dfrac{\sin\alpha}{\cos(\alpha - \theta)}$ (ii) – (1) $\dfrac{dk}{d\alpha} = \dfrac{\cos\alpha(\cos(\alpha - \theta)) - (-\sin(\alpha - \theta)(\sin\alpha))}{\cos^2(\alpha - \theta)}$ $\dfrac{d\alpha}{dk} = \dfrac{\cos^2(\alpha - \theta)}{\cos\alpha(\cos(\alpha - \theta)) - (-\sin(\alpha - \theta)(\sin\alpha))}$ $= \dfrac{(\cos(\alpha - \theta))^2}{\cos\alpha(\cos(\alpha - \theta) + \sin\alpha(\sin(\alpha - \theta))}$ $= \dfrac{(\cos(\alpha - \theta))^2}{\cos(\alpha - (\alpha - \theta))}$ $= \dfrac{(\cos(\alpha - \theta))^2}{\cos\theta}$ $\dfrac{d\alpha}{dt} = \dfrac{dk}{dt} \times \dfrac{d\alpha}{dk}$ $= S \times \dfrac{(\cos(\alpha - \theta))^2}{\cos\theta}$ $= \dfrac{S(\cos(\alpha - \theta))^2}{\cos\theta}$ when $\alpha = 2\theta$ $\dfrac{d\alpha}{dt} = \dfrac{S(\cos(2\theta - \theta))^2}{\cos\theta}$ $= \dfrac{S\cos^2(\theta)}{\cos\theta}$ $= S\cos\theta$	✓ Correct solution ✓ Correct application of chain rule ✓ Correct answer

(ii) - (2)

$$\dot{\alpha} = \frac{S\cos(\alpha - \theta)^2}{\cos\theta} \quad \text{(from part 1)}$$

$$= \frac{S}{\cos\theta} \times \cos(\alpha - \theta)^2$$

$$\frac{d\dot{\alpha}}{d\alpha} = \frac{S}{\cos\theta} \times 2(- \sin(\alpha - \theta)(\cos(\alpha - \theta)$$

$$= \frac{-2S(\sin(\alpha - \theta))(\cos(\alpha - \theta))}{\cos\theta}$$

$$\ddot{\alpha} = \frac{d\dot{\alpha}}{dt}$$

$$= \frac{d\alpha}{dt} \times \frac{d\dot{\alpha}}{d\alpha}$$

$$= \dot{\alpha} \times \frac{-2S(\sin(\alpha - \theta))(\cos(\alpha - \theta))}{\cos\theta}$$

✓ Correct application of chain rule

$$= -\alpha\dot{S} \times \frac{2\,(\sin(\alpha - \theta))(\cos(\alpha - \theta))}{\cos\theta}$$

$$= -\dot{\alpha}S \times \frac{\sin(2(\alpha - \theta))}{\cos\theta}$$

when $\alpha = 2\theta$

$$k = \frac{\sin 2\theta}{\cos\,(2\theta - \theta)}$$

$$= \frac{\sin 2\theta}{\cos\theta}$$

$$\ddot{\alpha} = -\dot{\alpha}S \times \frac{\sin(2(2\theta - \theta))}{\cos\theta}$$

$$= -\dot{\alpha}S \times \frac{\sin(2\theta)}{\cos\theta}$$

$$= -\dot{\alpha} \times S \times k$$

✓ Correct answer