Geometric Series

An geometric series is a sequence of numbers in which each term after the first is found by **multiplying** a constant amount to the previous term.

The constant amount is called the **common ratio**, symbolised, **_r_**.

$$r = \frac{T_2}{a}$$

$$= \frac{T_3}{T_2}$$

$$r = \frac{T_n}{T_{n-1}}$$

$$T_n = rT_{n-1}$$

$$T_1 = a$$
$$T_2 = ar$$
$$T_3 = ar^2$$
$$T_n = ar^{n-1}$$

$e.g.(i)$ Find r and the general term of $2, 8, 32, \ldots$

$$T_n = ar^{n-1}$$
$$= 2(4)^{n-1}$$
$$= 2\left(2^2\right)^{n-1}$$
$$= 2(2)^{2n-2}$$

$$a = 2, r = 4$$

$$\therefore T_n = 2^{2n-1}$$

(ii) If $T_2 = 7$ and $T_4 = 49$, find r

$$ar = 7$$
$$ar^3 = 49$$

$$r^2 = 7$$
$$r = \pm\sqrt{7}$$

(iii) find the first term of $1, 4, 16, \ldots$ to be greater than 500.

$$a = 1, r = 4 \qquad T_n = 1(4)^{n-1}$$

$$T_n > 500$$
$$4^{n-1} > 500$$
$$\log 4^{n-1} > \log 500$$
$$(n-1)\log 4 > \log 500$$
$$n - 1 > 4.48$$
$$n > 5.48$$
$$T_6 = 1024, \text{ is the first term} > 500$$

Exercise 6E; 1be, 2cf, 3ad, 5ac, 6c, 8bd, 9ac, 10ac, 15, 17, 18ab, 20a