Student Number: ______________

Teacher: ______________

Class: ______________

FORT STREET HIGH SCHOOL

2017

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: TRIAL HSC

Mathematics Extension 1

Time allowed: 2 hours
(plus 5 minutes reading time)

Syllabus Outcomes	Assessment Area Description and Marking Guidelines	Questions
	Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1-10
HE2, HE4	Manipulates algebraic expressions to solve problems from topic areas such as inverse functions, trigonometry, polynomials, permutations and combinations.	11, 12
HE3, HE5 HE6	Uses a variety of methods from calculus to investigate mathematical models of real life situations, such as projectiles, kinematics and growth and decay	13
HE7	Synthesises mathematical solutions to harder problems and communicates them in appropriate form	14

Total Marks 70

Section I 10 marks
Multiple Choice, attempt all questions,
Allow about 15 minutes for this section

Section II 60 Marks
Attempt Questions 11-14,
Allow about 1 hour 45 minutes for this section

Section I	Total 10	Marks
Q1-Q10		
Section II	Total 60	Marks
Q11	/15	
Q12	/15	
Q13	/15	
Q14	/15	
	Percent	

General Instructions:

- Questions 11-14 are to be started in a new booklet.
- The marks allocated for each question are indicated.
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used.

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2017 Year 12 Trial Examination

MATHEMATICS EXTENSION 1

Multiple Choice Answer Sheet

Circle the correct answer in pen

Student Number: ________________________

Teacher's Name: ________________________

1	A	B	C	D
2	A	B	C	D
3	A	B	C	D
4	A	B	C	D
5	A	B	C	D
6	A	B	C	D
7	A	B	C	D
8	A	B	C	D
9	A	B	C	D
10	A	B	C	D

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SECTION I (One mark each)

Answer questions 1 to 10 on the multiple choice answer sheet.

1. When $P(x) = 3x^5 - 2x^3 + x - 5$ is divided by $2x - 1$ the remainder is

(A) -7

(B) -3

(C) $\dfrac{-149}{32}$

(D) $\dfrac{-171}{32}$

2. The point P divides the interval joining $A(2,3)$ and $B(-3,-5)$ externally in the ratio 3:4. What is the y-co-ordinate of P?

(A) 17

(B) $\dfrac{3}{7}$

(C) 27

(D) $-\dfrac{1}{7}$

3. The cartesian equation represented by $x = \cos t$ and $y = \sin t$ for $0 \le t \le \pi$ is:

(A) $y = x \tan t$

(B) $y = \sqrt{1 - x^2}$

(C) $x^2 - y^2 = 1$

(D) $x^2 + y^2 = 1$

4. What is the acute angle between the line $y = 2 - x$ and the tangent to the curve $y = x^2$ at $(2,4)$?

(A) $\tan^{-1}\left(\dfrac{3}{5}\right)$

(B) $\tan^{-1}\left(\dfrac{-5}{3}\right)$

(C) $\tan^{-1}\left(\dfrac{5}{3}\right)$

(D) $\tan^{-1}\left(\dfrac{-3}{5}\right)$

5. Find $\int \frac{3}{\sqrt{1-2x^2}}\,dx$

(A) $\frac{3}{\sqrt{2}}\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)+c$

(B) $\frac{3}{\sqrt{2}}\sin^{-1}\left(\sqrt{2}x\right)+c$

(C) $\frac{3}{2}\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)+c$

(D) $\frac{\sqrt{2}}{3}\sin^{-1}\left(\sqrt{2}x\right)+c$

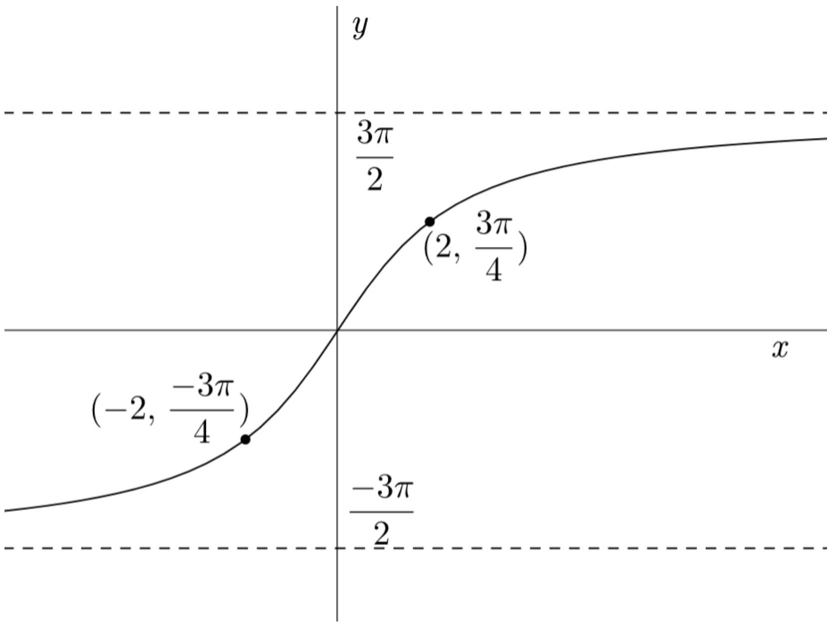
6. Which function best describes the following graph?

(A) $y=3\tan^{-1}x$

(B) $y=3\tan^{-1}\left(\frac{x}{2}\right)$

(C) $y=x^{\frac{1}{3}}$

(D) $y=3x^{\frac{1}{3}}$



7. How many permutations of the letters of the word *arrange* are possible if the 2 letters *r* are not together?

(A) 1260
(B) 900
(C) 1080
(D) 180

8. If n is an integer, the general solution of $\cos x = \frac{\sqrt{3}}{2}$ is

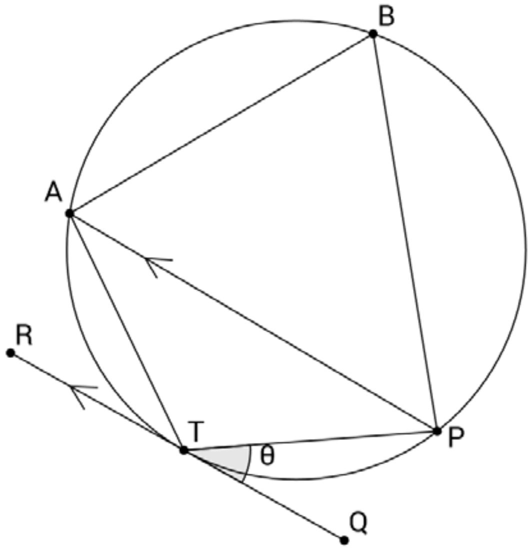
(A) $x = 2n\pi \pm \frac{\pi}{6}$

(B) $x = n\pi + (-1)^n \times \frac{\pi}{6}$

(C) $x = n\pi \pm \frac{\pi}{6}$

(D) $x = 2n\pi + (-1)^n \times \frac{\pi}{6}$

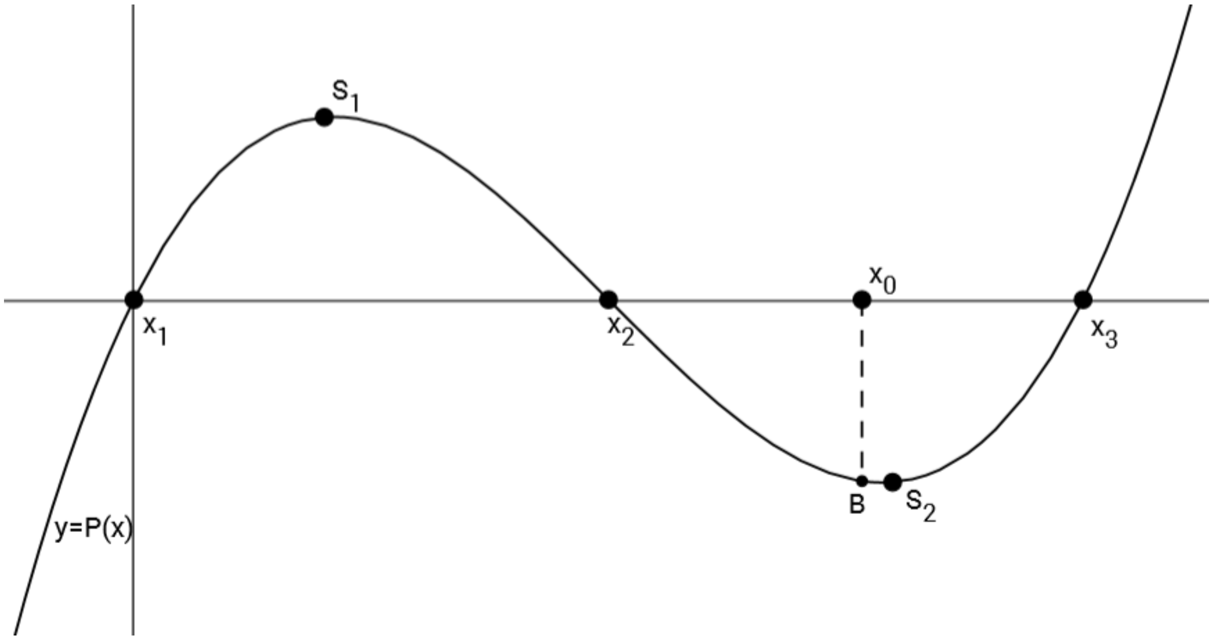
9. The points A, B, P and T lie on a circle. Tangent RQ has point of contact T. Chord AP is parallel to Tangent RQ.



What is $\angle ABP$ in terms of θ

(A) θ
(B) $90-\theta$
(C) 2θ
(D) $180-2\theta$

10. The polynomial $y = P(x)$, with roots x_1, x_2 and x_3, is sketched below. The point at x_2 is also a point of inflexion. Stationary points exist at S_1 and S_2. There are no other stationary points or points of inflexion. A first approximation for a root of $P(x)$ is taken as x_0. Point B is close but not equal to S_2 and has the same *x*-value as x_0. Starting with x_0, Newton's Method for approximating the roots of an equation is applied 3 times. To which root would the approximations produced by Newton's Method converge?



(A) x_1

(B) x_2

(C) x_3

(D) None of these

SECTION II (15 marks for each question.)

Answer each question in the appropriate booklet. Extra writing booklets are available.

Question 11: Use a separate writing booklet

(a) Evaluate $\lim_{x\to 0}\dfrac{\sin 3x}{2x}$ **1**

(b) Solve $\dfrac{x+1}{x-2}\le 2$ **3**

(c) The equation $\tan x = e^x$ has a root near $x = 1\cdot 3$. Use Newton's Method once to find another approximation to this equation. Write your answer correct to 2 decimal places. **2**

(d) Evaluate $\int_{\frac{\pi}{16}}^{\frac{\pi}{8}}\cos^2 2x\,dx$ leaving your answer in exact form. **3**

(e) Differentiate $y=\sin^3 x\cos x$ writing your answer in terms of powers of $\sin x$ **3**

(f) Use the substitution $x = u-2$ to evaluate $\int_0^1 \dfrac{2-x}{(2+x)^2}\,dx$ **3**

Question 12: Use a separate writing booklet

(a) Prove by mathematical induction that for positive integers *n*, $n^2 > n-1$ **3**

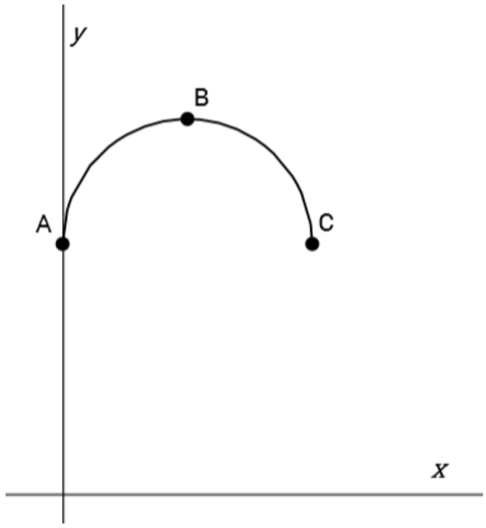
(b)

i. How many combinations are there when choosing 4 letters, without replacement, from the word PECULIAR? **1**

ii. What is the probability that the 4 letters chosen in i) are all consonants. **1**

(c) Without using calculus, sketch $y = \dfrac{x-1}{x(x+3)}$. Show any asymptotes and intercepts with the axes. **3**

(d) Consider the function $f(x) = 2 + \sqrt{1-(x-1)^2}$. The graph of $y = f(x)$ is a semi-circle as shown below.



i. State the co-ordinates of points *A*, *B* and *C*. **3**

ii. Let $g(x) = 2 + \sqrt{1-(x-1)^2}$ *for* $0 \le x \le 1$. On the same set of axes, sketch $y = f(x)$ and $y = g^{-1}(x)$ showing the endpoints of the curve $y = g^{-1}(x)$. **3**

iii. Let ϕ be a real number in the domain $1 \le x \le 2$. Find $g^{-1}(f(\phi))$ **1**

Question 13: Use a separate writing booklet

(a) The velocity, of a particle moving in simple harmonic motion, is given by the equation $v^2 = 4x - x^2$ where x is in metres and v is in metres per second.

i. Find the two x-values between which it is oscillating **1**

ii. Find the maximum speed of the particle **1**

iii. Find the acceleration of the particle in terms of x. **1**

(b) Show that $\cos\left(2\sin^{-1}\left(\frac{3}{4}\right)\right)$ can be written in the form $\frac{a}{b}$, where a and b are integers. **2**

(c) The population of a city is given by $\frac{dP}{dt} = k(P-110)$.

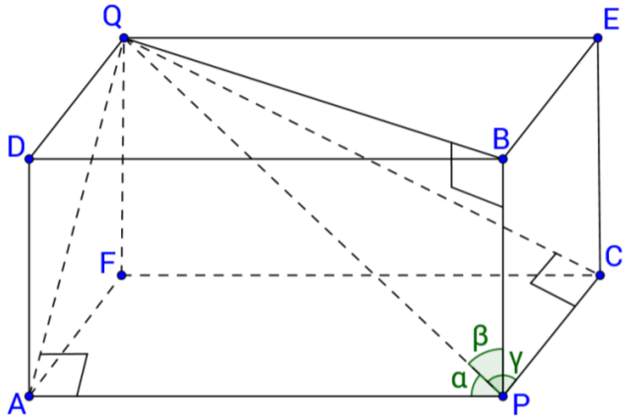
i. Show that $P = 110 + Ae^{kt}$ is a solution of the differential equation. **2**

ii. If the population is originally 18500 and after 10 years it is 23500, find the population after 9 years. **2**

iii. When will the population reach 75000 (to 3 signicant figures) **2**

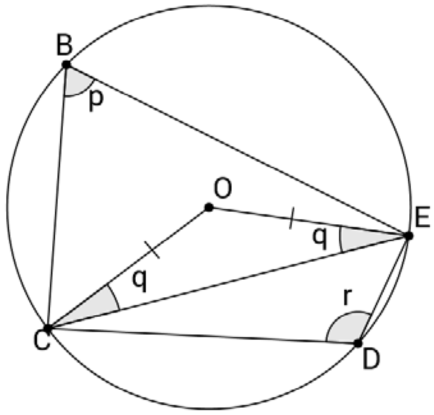
(d) The diagonal PQ, of the rectangular prism in the diagram, makes angles of α, β and γ with PA, PB and PC.

Prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ **4**



Question 14: Use a separate writing booklet

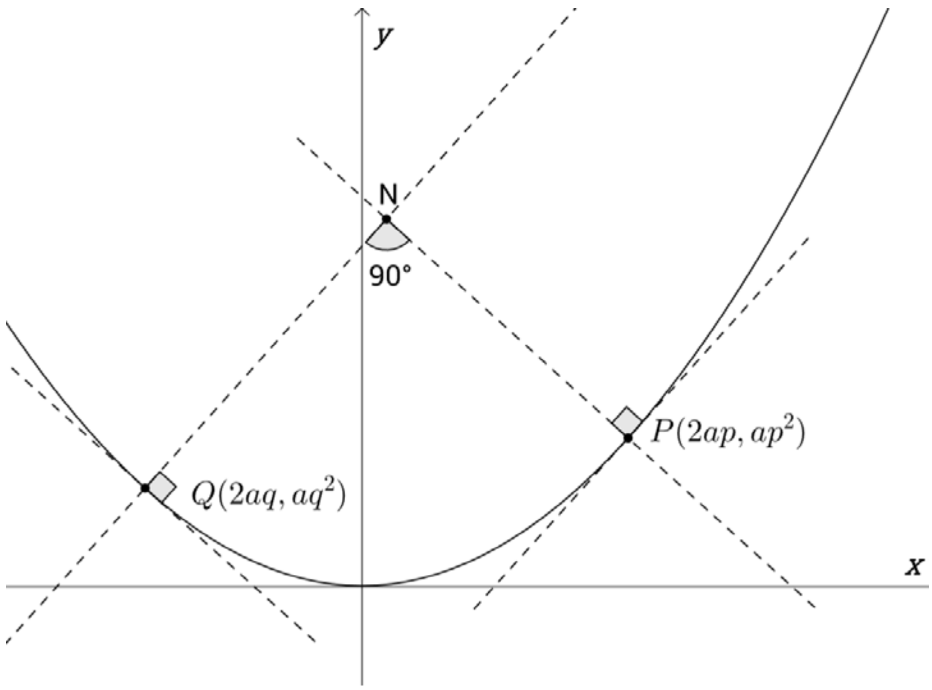
(a) O is the centre of the circle $BCDE$. Prove that:



i. $p+q=90$ **2**

ii $r-p=2q$ **3**

(b) $P\left(2ap,ap^2\right)$ and $Q\left(2aq,aq^2\right)$ are variable points on the parabola $x^2=4ay$. The normal to the parabola at P and Q are perpendicular to one another and intersect at N. You are given that $pq=-1$ and that the equation of the normals at P and Q are $x+py=2ap+ap^3$ and $x+qy=2aq+aq^3$ (Do not prove these results).

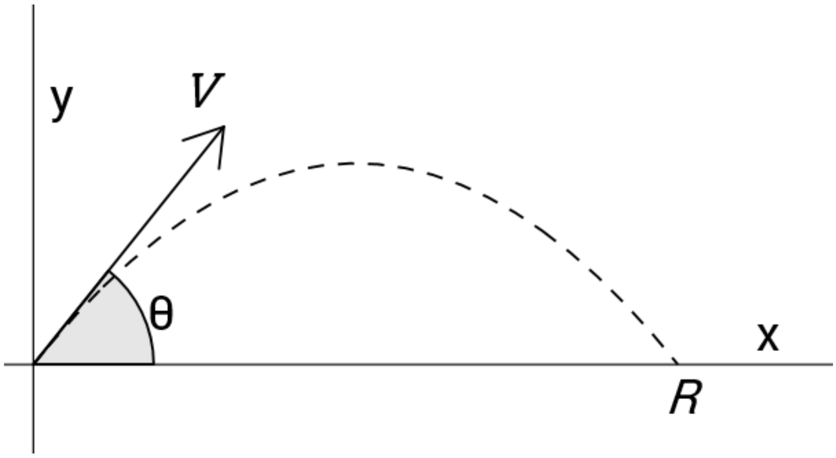


i. Show that N has co-ordinates $\left(a\left(p+q\right),a\left(p^2+q^2+1\right)\right)$ **3**

ii. Hence find the cartesian equation of the locus of N. **2**

Question 14 continues on the next page

(c) A projectile is fired on the *x*-*y* plane with initial velocity *V* and angle of projection θ . You are given that the cartesian equation of a projectile is $y = x\tan\theta - \dfrac{gx^2}{2V^2}\sec^2\theta$ (Do not prove this)

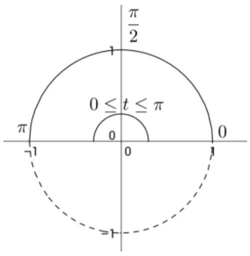


i. By substituting $y = 0$ into the cartesian equation, show that the range of the projectile is $R = \dfrac{V^2 \sin 2\theta}{g}$ **2**

ii. A projectile falls 100 metres short of a target when the angle of projection is 15° and lands 558.8 metres past the same target when the angle of projection is 30°. Find the angle of projection required to hit the target giving your answer to the nearest minute. **3**

End of examination

2017 Mathematics Extension 1 Trial Solutions

1	$P\left(\frac{1}{2}\right)=\frac{-149}{32}$	C
2	$y=\frac{-3\times-5+4\times3}{-3+4}=27$	C
3	$\cos^2 t+\sin^2 t=1$ $\therefore x^2+y^2=1$ $y=\pm\sqrt{1-x^2}$ $but\ 0\le t\le\pi$ $\therefore y=\sqrt{1-x^2}$ 	B
4	$m_1=$ Gradient of $y=2-x$ $=-1$ $m_2=$ Gradient of tangent $f(x)=x^2$ $f'(x)=2x$ $f'(2)=4$ $\therefore m_2=4$ $\tan\theta=\left\|\frac{4-{}^-1}{1+4\times{}^-1}\right\|$ $\tan\theta=\left\|\frac{5}{{}^-3}\right\|$ $\theta=\tan^{-1}\left(\frac{5}{3}\right)$	C
5	$\int\frac{3}{\sqrt{1-2x^2}}\,dx=\frac{3}{\sqrt{2}}\int\frac{1}{\sqrt{\frac{1}{2}-x^2}}\,dx$ $=\frac{3}{\sqrt{2}}\int\frac{1}{\sqrt{\frac{1}{\left(\sqrt{2}\right)^2}-x^2}}dx$ $=\frac{3}{\sqrt{2}}\sin^{-1}\left(\frac{x}{1/\sqrt{2}}\right)+c$ $=\frac{3}{\sqrt{2}}\sin^{-1}\left(\sqrt{2}\,x\right)+c$	B

6	By substitution of the points into the equation or An inverse tan graph that has been dilated vertically by a factor of 3 and horizontally by a factor of 2.	B
7	Number of possible arrangements $=\frac{7!}{2!2!}$ $=1260$ Arrangements with *r*s together $=\frac{6!}{2!}$ $=360$ Arrangements with *r*s not together $=1260-360$ $=900$	B
8		A
9	$\angle APT=\theta\left(Adjacent\ angles\ on\ parrallel\ lines\right)$ $\angle TAP=\theta\left(\begin{matrix}Angle\ between\ a\ tangent\ and\ chord\ equals\\ angle\ in\ the\ alternate\ segment\end{matrix}\right)$ $\therefore\angle ATP=180-2\theta\left(Angle\ sum\ of\ \triangle ATP\right)$ $\angle ABP=2\theta\left(\begin{matrix}Opposite\ angles\ of\ cyclic\ quadrilateral\\ are\ supplementary\end{matrix}\right)$	C
10		A

Question 11

(a) $\lim\limits_{x\to 0}\dfrac{\sin 3x}{2x}=\dfrac{1}{2}\lim\limits_{x\to 0}\dfrac{\sin 3x}{x}$

$=\dfrac{3}{2}\lim\limits_{x\to 0}\dfrac{\sin 3x}{3x}$

$=\dfrac{3}{2}\times 1$

$=\dfrac{3}{2}$ ❶

Mostly well done

(b) $\dfrac{x+1}{x-2}\le 2 \quad \boxed{note: x\ne 2}$

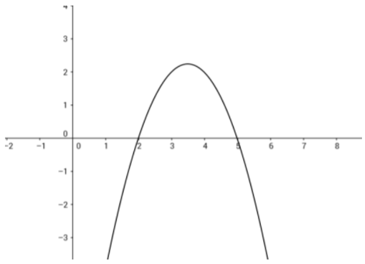
$\dfrac{x+1}{x-2}\times(x-2)^2\le 2\times(x-2)^2$ ❶

$(x+1)(x-2)-2(x-2)^2\le 0$

$(x-2)\left[x+1-2(x-2)\right]\le 0$

$(x-2)(5-x)\le 0$ ❶

$x<2\, and\ x\ge 5$ ❶



Students included x=2

Some drawings were not detailed enough or just careless.

(c) Let $f(x)=\tan x-e^x$

$f'(x)=\sec^2 x-e^x$

$f(1.3)=-0.0672(4d.p)$

$f'(1.3)=10.3058(4d.p.)$ ❶

Calculators were in degree mode instead of radians.

Students got the formula for Newton's method wrong.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$
$$= 1.3 + \frac{0.0672}{10.3058}$$
$$= 1.31 (2d.p.) ❶$$

(d) $$\int_{\frac{\pi}{16}}^{\frac{\pi}{8}} \cos^2 2x \, dx = \frac{1}{2}\int_{\frac{\pi}{16}}^{\frac{\pi}{8}} \cos 4x + 1 \, dx ❶$$

$$= \frac{1}{2}\left[\frac{1}{4}\sin 4x + x\right]_{\frac{\pi}{16}}^{\frac{\pi}{8}} ❶$$
$$= \frac{1}{2}\left[\frac{1}{4}\sin\frac{\pi}{2} + \frac{\pi}{8} - \left(\frac{1}{4}\sin\frac{\pi}{4} + \frac{\pi}{16}\right)\right]$$
$$= \frac{1}{2}\left[\frac{1}{4} + \frac{\pi}{8} - \left(\frac{1}{4\sqrt{2}} + \frac{\pi}{16}\right)\right]$$
$$= \frac{1}{2}\left[\frac{1}{4} - \frac{1}{4\sqrt{2}} + \frac{\pi}{16}\right]$$
$$= \frac{1}{2}\left[\frac{1}{4} - \frac{\sqrt{2}}{8} + \frac{\pi}{16}\right]$$
$$= \frac{4 - 2\sqrt{2} + \pi}{32} ❶$$

Mostly well done

(e) $y = \sin^3 x \cos x$

$u = \sin^3 x$ $\quad\quad$ $v = \cos x$

$u' = 3\sin^2 x \cos x$❶ $\quad\quad$ $v' = -\sin x$❶

$y' = \cos x \times 3\sin^2 x \cos x + \sin^3 x \times -\sin x$

$y' = 3\cos^2 x \sin^2 x - \sin^4 x$

$y' = 3\left(1 - \sin^2 x\right)\sin^2 x - \sin^4 x$

$y' = 3\sin^2 x - 4\sin^4 x$❶

Chain rule was often incorrect when differentiating $\sin^3 x$

Also students forgot the negative sign in front of sin*x*

(f) $x = u - 2$

$dx = du$

$when\, x = 1, u = 3$

$when\, x = 0, u = 2$❶

$$\int_0^1 \frac{2-x}{(2+x)^2}\,dx$$

$$= \int_2^3 \frac{2-(u-2)}{(2+u-2)^2}\,du$$

$$= \int_2^3 \frac{4-u}{u^2}\,du$$ ❶

$$= \int_2^3 4u^{-2} - \frac{1}{u}\,du$$

$$= \left[\frac{-4}{u} - \ln u\right]_2^3$$

$$= \frac{-4}{3} - \ln 3 - (-2 - \ln 2)$$

$$= \frac{2}{3} + \ln\frac{2}{3}$$ ❶

Students forgot to change the sign of the -2 in the bracket so instead of $\dfrac{4-u}{u^2}$ they got $\dfrac{-u}{u^2}$

Question 12

(a) For $n=1$,

$$LHS = 1^2$$
$$= 1$$
$$RHS = 1-1$$
$$= 0$$

$\therefore$ true for $n=1$.

Many students forgot to mention the assumption and hence lost a mark

Assume true for $n = k$

i.e. assume that $k^2 > k-1$

For $n = k+1$,

It is required to prove that

$(k+1)^2 > k$ ❶

$$LHS = (k+1)^2$$
$$= k^2 + 2k + 1$$
$$> k - 1 + 2k + 1 (by\, the\, assumption)$$ ❶
$$= 3k$$
$$> k$$ ❶
$$= RHS$$

So the result is true for $n = k+1$ if it is true for $n = k$.

Hence, the result is proven by mathematical induction

(b) i) ${}^8C_4 = 70$

Some students did ${}^8P_4$ instead.

ii) ${}^4C_4 = 1$

$\therefore P(\text{all consonants}) = \dfrac{1}{70}$

Well done

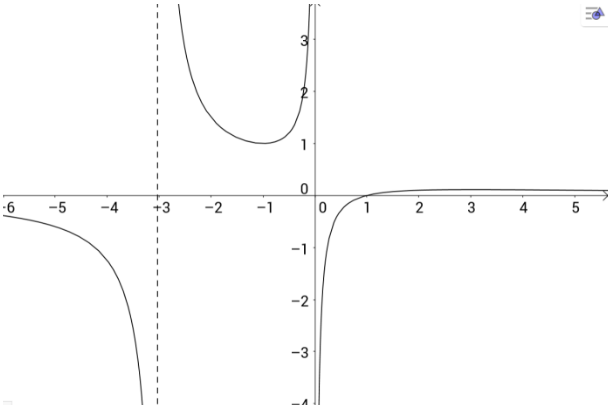
(c) $y = \dfrac{x-1}{x(x+3)}$

➊ for x-intercept at x=1 and correct shape

➊for vertical asymptotes at $x=-3$ and 0

➊for horizontal asymptote at y=0

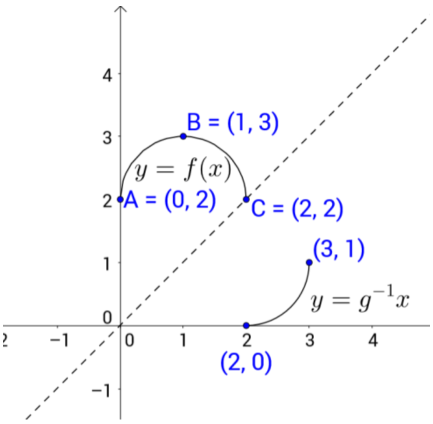
-➊ for an error



(d) i) $A(0,2),\ B(1,3),\ C(2,2)$

Very well done

ii)



Well done

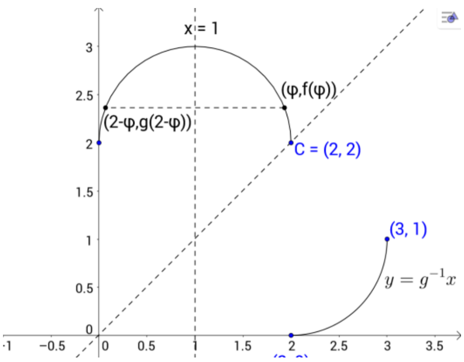
iii)

$g^{-1}\left(f(\phi)\right) \neq \phi$, as $f(x)$ and $g^{-1}(x)$ are not inverses.

However $y = f(x)$ is symmetrical about $x = 1$.

Note $f(\phi) = f(2-\phi)$ and $0 \leq 2-\phi \leq 1$

Hence, $g^{-1}\left(f(\phi)\right) = g^{-1}\left(f(2-\phi)\right) = g^{-1}\left(g(2-\phi)\right) = 2-\phi$



12diii had a typing error, so all students awarded 1 mark

Question 13

(a) i) $v^2 = x(4-x)$

As velocity = 0 at the extremities of motion,

$0 = x(4-x)$

$x = 0 \text{ or } 4$

The particle oscillates between 0 and 4 metres.

Generally done well

ii) Maximum speed occurs at the centre of motion where x=2.

$v^2 = 2(4-2)$

$v^2 = 4$

$v = \pm 2$

therefore maximum speed is 2 metres per second.

Some students left their answer in terms of $\pm v$ instead of $|v|$ for speed. A few forgot take the square of v^2

iii) $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \frac{d}{dx}\left(\frac{1}{2}x(4-x)\right)$

$= \frac{1}{2}\frac{d}{dx}(4x-x^2)$

$= \frac{1}{2}\times(4-2x)$

$= 2-x$

Generally done well

(b) Let $\theta = \sin^{-1}\frac{3}{4}$

$\therefore \sin\theta = \frac{3}{4}$

$\cos\theta = \frac{\sqrt{7}}{4}$ ❶

$\cos\left(2\sin^{-1}\frac{3}{4}\right)$

$= \cos 2\theta$

$= \cos^2\theta - \sin^2\theta$

$= \frac{7}{16} - \frac{9}{16}$

$= \frac{-1}{8}$ ❶

Generally done well – some students used sin identities instead of cos when solving

(c) i) $LHS = \frac{dP}{dt}$

$= \frac{d}{dt}\left(110 + Ae^{kt}\right)$

$= kAe^{kt}$ ❶

$RHS = k\left(P - 110\right)$

$= k\left(110 + Ae^{kt} - 110\right)$ ❶

$= kAe^{kt}$

$= LHS$

Many students had poor setting out. Some also failed to show link between derivative and the primitive

ii) When $t = 0, P = 18500$

$18500 = 110 + Ae^{0}$

$A = 18390$

$\therefore \boxed{P = 110 + 18390e^{kt}}$

When $t = 10, P = 23500$

$23500 = 110 + 18390e^{10k}$

$23390 = 18390e^{10k}$

$e^{10k} = \frac{23390}{18390}$

$10k = \ln\left(\frac{23390}{18390}\right)$

$k = \frac{1}{10}\ln\left(\frac{23390}{18390}\right)$ ❶

$k = \cdot 024050$ (5 sig. fig.)

When $t = 9$,

$P = 110 + 18390e^{\cdot 024051 \times 9}$

$P = 22944$ ❶

Generally done well but careless errors made when calculating k & A

iii) $75000 = 110 + 18390e^{\cdot 024051t}$

$\frac{74890}{18390} = e^{\cdot 024051t}$

$\cdot 024051t = \ln\left(\frac{74890}{18390}\right)$

$t = \frac{1}{\cdot 024051} \times \ln\left(\frac{74890}{18390}\right)$

$t = 58.4$ years $(3\,sig.fig.)$ ❶

Done well, however, students who rounded k off to only 2 decimal places were far off the final answer

(d) In $\triangle AQP$, $\cos\alpha = \dfrac{AP}{PQ}$

In $\triangle BQP$, $\cos\beta = \dfrac{BP}{PQ}$

In $\triangle CQP$, $\cos\gamma = \dfrac{CP}{PQ}$ ❶

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma$

$= \left(\dfrac{AP}{PQ}\right)^2 + \left(\dfrac{BP}{PQ}\right)^2 + \left(\dfrac{CP}{PQ}\right)^2$

$= \dfrac{AP^2 + BP^2 + CP^2}{PQ^2}$ ❶

but by pythagoras in right triangle ABP,

$AP^2 + BP^2 = AB^2$

$\therefore \dfrac{AP^2 + BP^2 + CP^2}{PQ^2}$

$= \dfrac{AB^2 + CP^2}{PQ^2}$ ❶

and $AB = PD = CQ$ (*Diagonals of a rectangle are equal and opposite faces of a rectangular prism are congruent*)

$\therefore AB^2 = CQ^2$

$\therefore \dfrac{AB^2 + CP^2}{PQ^2}$

$= \dfrac{CQ^2 + CP^2}{PQ^2}$ ❶

and using pythagoras in right triangle CQP

$= \dfrac{PQ^2}{PQ^2}$

$= 1$

- Students who drew diagrams tended to receive more marks.
- Many were able to come up with the cos ratios and the equation for $\cos^2\alpha + \cos^2\beta + \cos^2\gamma$ but had difficulty completing the proof
- Rather than using the vertices to label sides a few students renamed as x, y and z which made it difficult to follow working out.

Question 14

a)i) $\angle COE = 180^\circ - 2q \left(Angle\ sum\ of\ \triangle\ is\ 180 \right)$ ❶

$p = \frac{1}{2}\left(180^\circ - 2q\right)\left(\begin{array}{l} Angle\ at\ centre\ is\ twice\ angle\ at \\ circumference\ subtended\ by\ same \\ arc \end{array}\right)$ ❶

$p = 90^\circ - q$

$p + q = 90^\circ ...①$

14a)i) Some students needed to give more detail in their reasoning e.g. giving as a reason 'cyclic quadrilateral' is not good enough.

a)ii) $Reflex \angle COE = 2r \left(\begin{array}{l} Angle\ at\ centre\ is\ twice\ angle\ at \\ circumference\ subtended\ by\ same \\ arc \end{array}\right)$

$Obtuse \angle COE = 360 - 2r \left(Angles\ at\ a\ point\ add\ to\ 360 \right)$ ❶

$\therefore 180^\circ - 2q = 360^\circ - 2r$

$90^\circ - q = 180^\circ - r$

$r - q = 90^\circ ...②$ ❶

$Equating\ ...①\ and\ ...②$

$r - q = p + q$

$r - p = 2q$ ❶

14a)ii) There were alternative methods for this proof. It is incorrect to combine the sides of the result you are meant to prove.

b)i)

$x+py=2ap+ap^3\ldots$①

$x+qy=2aq+aq^3\ldots$②

①–②

$(p-q)y=2a(p-q)+a\left(p^3-q^3\right)$

$(p-q)y=2a(p-q)+a(p-q)\left(p^2+pq+q^2\right)$

$y=2a+a\left(p^2+pq+q^2\right)$❶

$but\ pq=-1$

$y=2a+a\left(p^2-1+q^2\right)$

$y=2a+ap^2-a+aq^2$

$y=ap^2+aq^2+a$

$y=a\left(p^2+q^2+1\right)\ldots$③❶

*sub*③*in*①

$x+ap\left(p^2+q^2+1\right)=2ap+ap^3$

$x+ap^3+apq^2+ap=2ap+ap^3$

$x=ap-apq^2$

$but\ pq=-1$

$x=ap+aq$

$x=a(p+q)$❶

$\therefore N\left(a(p+q),a\left(p^2+q^2+1\right)\right)$

14b)i) Overall, well done.

b)ii)

$x=a(p+q)\ldots$①

$y=a\left(p^2+q^2+1\right)\ldots$②

①^2 *gives*

$x^2=a^2(p+q)^2$

$x^2=a^2\left(p^2+2pq+q^2\right)$❶

$but\ pq=-1$

$x^2=a^2\left(p^2-2+q^2\right)$

$x^2=a^2\left(p^2+1+q^2-3\right)$

$x^2=a^2\left(p^2+q^2+1\right)-3a^2$

$x^2=ay-3a^2$

$x^2=a(y-3a)$❶

14b)ii) Overall, well done.

ci)

$$0 = x\tan\theta - \frac{gx^2\sec^2\theta}{2V^2}$$

$$= x\left(\tan\theta - \frac{gx}{2V^2\cos^2\theta}\right)$$

$$x = 0\, or\, \frac{\tan\theta}{g/2V^2\cos^2\theta}$$ ❶

$$Range = \frac{\tan\theta}{g/2V^2\cos^2\theta}$$

$$= \frac{\sin\theta}{\cos\theta} \times \frac{2V^2\cos^2\theta}{g}$$

$$= \frac{2V^2\sin\theta\cos\theta}{g}$$ ❶

$$= \frac{V^2\sin 2\theta}{g}$$

14c)i) The main error here was to cancel x which resulted in losing a solution. One mark was deducted.

cii)

$$\frac{V^2\sin(2\times 15^{\circ})}{g} + 100 = \frac{V^2\sin(2\times 30^{\circ})}{g} - 558\cdot 8$$

$$\frac{V^2\sin 30^{\circ}}{g} + 100 = \frac{V^2\sin 60^{\circ}}{g} - 558\cdot 8$$

$$658.8 = \frac{V^2}{g}\left(\frac{\sqrt{3}-1}{2}\right)$$

$$\frac{V^2}{g} = \frac{1317\cdot 6}{\sqrt{3}-1}$$ ❶

Let θ be the required angle to hit the target

$$\frac{V^2\sin 2\theta}{g} = \frac{V^2\sin 30^{\circ}}{g} + 100$$ ❶

$$\sin 2\theta = \frac{1}{2} + \frac{100g}{V^2}$$

$$\sin 2\theta = \frac{1}{2} + \frac{100\left(\sqrt{3}-1\right)}{1317\cdot 6}$$

$$2\theta = \sin^{-1}\left(\frac{1}{2} + \frac{100\left(\sqrt{3}-1\right)}{1317\cdot 6}\right)$$

$$\theta = \frac{1}{2}\sin^{-1}\left(\frac{1}{2} + \frac{100\left(\sqrt{3}-1\right)}{1317\cdot 6}\right)$$ ❶

$$\theta = 16^{\circ}52'$$

14c)ii) Poorly set out. Students should explain what the pronumerals they are using represent so that it is easier to follow working.

Many students found this question difficult.

Please note that to hit target, the angle of projection should lie between 15° & 30°.