2017	Hurlstone Agricultural High School **HSC Assessment Task4 – Trial**

Mathematics Extension 1

Examiners

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- **Ms P Biczo**

General Instructions

- Reading time – 5 minutes
- Working time – 120 minutes
- Write using black or blue pen
- NESA-approved calculators may be used
- A Reference sheet is provided for your use

In Questions 11 to 14, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks (pages 2–4)

- Attempt Questions 1 to 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 5–8)

- Attempt Questions 11 to 14
- Allow about 105 minutes for this section

Student Name: ______________________________

Teacher: ___________________________________

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

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1. $\lim_{x \to 0} \frac{1-\cos 2x}{4x^2} =$

(A) $\frac{1}{2}$

(B) 1

(C) 2

(D) 4

2. The angle θ satisfies $\sin\theta = \frac{5}{13}$ and $\frac{\pi}{2} < \theta < \pi$.
What is the value of $\sin 2\theta$?

(A) $\frac{10}{13}$

(B) $-\frac{10}{13}$

(C) $\frac{120}{169}$

(D) $-\frac{120}{169}$

3. If $x = e^y + 4$ then $\frac{dy}{dx}$ is

(A) e^y

(B) $\frac{1}{x-4}$

(C) $x-4$

(D) $\frac{1}{e^y + 4}$

4. The polynomial $P(x)=x^3+2x+k$ has $(x-2)$ as a factor.

What is the value of k?

(A) -12

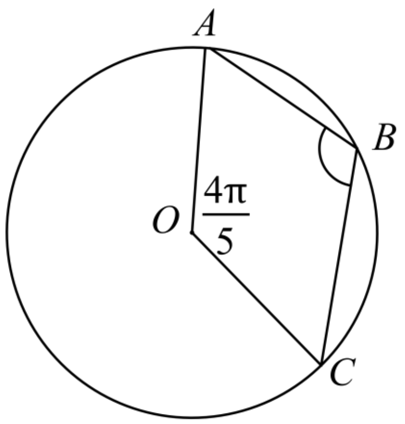
(B) -10

(C) 10

(D) 12

5. The points A, B and C lie on a circle with centre O, as shown in the diagram.

The size of $\angle AOC$ is $\dfrac{4\pi}{5}$ radians.



Not to scale

What is the size of $\angle ABC$ in radians?

(A) $\dfrac{3\pi}{10}$

(B) $\dfrac{\pi}{2}$

(C) $\dfrac{3\pi}{5}$

(D) $\dfrac{4\pi}{5}$

6. A curve has parametric equations $x=t-3$ and $y=t^2+2$. What is the Cartesian equation of this curve?

(A) $y=x^2-x-1$

(B) $y=x^2+x-1$

(C) $y=x^2-6x+11$

(D) $y=x^2+6x+11$

7. Let $|a| \le 1$. What is the general solution of $\sin 2x = a$?

(A) $x = n\pi + (-1)^n \dfrac{\sin^{-1} a}{2}$, n is an integer

(B) $x = \dfrac{n\pi + (-1)^n \sin^{-1} a}{2}$, n is an integer

(C) $x = 2n\pi \pm \dfrac{\sin^{-1} a}{2}$, n is an integer

(D) $x = \dfrac{2n\pi \pm \sin^{-1} a}{2}$, n is an integer

8. At a dinner party, the host, hostess and their six guests sit at a round table. In how many ways can they be arranged if the host and hostess are separated?

(A) 720

(B) 1440

(C) 3600

(D) 5040

9. Which of the following is an expression for $\int \dfrac{e^{-2x}}{e^{-x}+1} dx$ in terms of u?

Use the substitution $u = e^{-x} + 1$.

(A) $\int \dfrac{1-u}{u} du$

(B) $\int \dfrac{u-1}{u} du$

(C) $\int \dfrac{(1-u)^3}{u} du$

(D) $\int \dfrac{(u-1)^3}{u} du$

10. The functions $y = x$ and $y = x^3$ meet at the point $(1,1)$.

What is the acute angle between the tangents to these functions at this point?
Answer to the nearest degree.

(A) 10°

(B) 27°

(C) 45°

(D) 63°

End of Section I

Section II

60 marks

Attempt Questions 11 – 14

Allow about 105 minutes for this section

Answer each question in a new answer booklet.

All necessary working should be shown in every question.

Question 11 (15 marks) **Start a new answer booklet.** **Marks**

(a) Solve the inequality: $\dfrac{4x}{x-3} \le 1$ **3**

(b) Solve the inequality: $|5x-1| < \sqrt{2x(1-x)}$ **3**

(c) A total of five players is selected at random from four sporting teams.
Each of the teams consists of ten players numbered from 1 to 10.

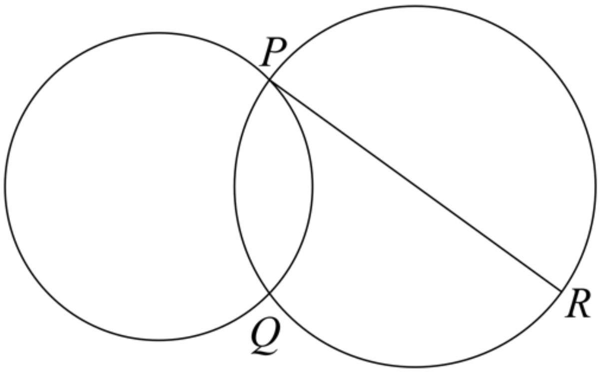
(i) What is the probability that of the five selected players, three are numbered '6' and two are numbered '8'? **2**

(ii) What is the probability that the five selected players contain at least four players from the same team? **3**

(d) Evaluate $\displaystyle\int_0^1 \frac{2x}{(2x+1)^2}dx$ by using the substitution $u = 2x+1$ **4**

Question 12 (15 marks) **Start a new answer booklet.** **Marks**

(a) Two circles intersect at P and Q.
The diameter of one circle is PR.



Copy, or trace, this diagram into your answer booklet.

(i) Draw a straight line through P, parallel to QR to meet the other circle at S. Prove that QS is a diameter of the second circle. **2**

(ii) Prove that the circles have equal radii if QS is parallel to PR. **2**

(b) $P\left(2at, at^2\right)$ is a variable point on the parabola $x^2 = 4ay$, whose focus is S.

$Q\left(x, y\right)$ divides the interval from P to S in the ratio $t^2:1$ [i.e., $PQ:QS$ = $t^2:1$].

(i) Find the coordinates of Q in terms of a and t. **2**

(ii) Show that $\dfrac{y}{x} = t$. **1**

(iii) Prove that, as P moves on the parabola, Q moves on a circle, and state its centre and radius. **3**

(c) It is given that $P\left(x\right) = \left(x-a\right)^3 + \left(x-b\right)^3$, where $a \neq b$.

(i) Prove that $x = \dfrac{a+b}{2}$ is a zero of $P\left(x\right)$. **2**

(ii) Prove that $P\left(x\right)$ has no stationary points. **3**

Question 13 (15 marks) **Start a new answer booklet.** **Marks**

(a)

(i) Express $3\sin x + 4\cos x$ in the form $A\sin(x+\alpha)$, where $0 \le \alpha \le \frac{\pi}{2}$ and $A > 0$. **2**

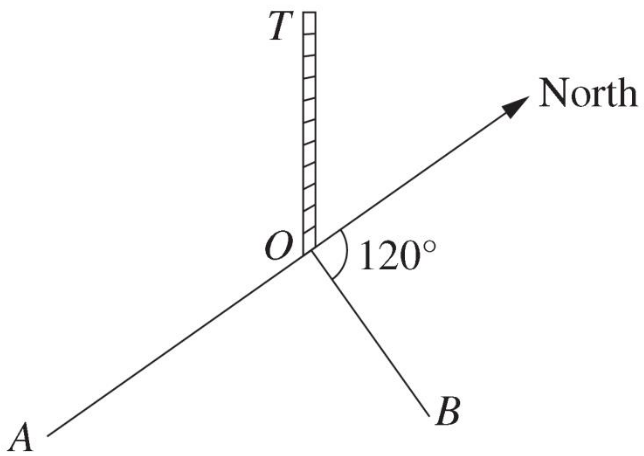
(ii) Hence, or otherwise, solve $3\sin x + 4\cos x = 5$ for $0 \le x \le 2\pi$.
Give your answer, or answers, correct to two decimal places. **2**

(b)

(i) Prove that $\tan^2\theta = \dfrac{1-\cos 2\theta}{1+\cos 2\theta}$, provided that $\cos 2\theta \ne -1$. **2**

(ii) Hence find the exact value of $\tan\dfrac{\pi}{8}$. **2**

(c) From a point *A* due south of a tower, the angle of elevation of the top of the tower *T*, is 23°. From another point *B*, on a bearing of 120° from the tower, the angle of elevation of *T* is 32°. The distance *AB* is 200 metres.



Let the height of the tower *OT* be *h*.

(i) Show that $OA = h\tan 67^o$ and $OB = h\tan 58^o$ **1**

(ii) Hence, find the height of the tower *OT*. Give your answer to the nearest metre. **3**

(d) Use the principle of mathematical induction to prove that
$4^n + 14$ is a multiple of 6 for all integers $n \ge 1$. **3**

Question 14 (15 marks) **Start a new answer booklet.** **Marks**

(a) Consider the curves $y = \sin x$ and $y = \cos 2x$ for $-\pi \le x \le \pi$.

(i) Find any points of intersection of the curves in the domain $-\pi \le x \le \pi$. **3**

(ii) On the same number plane, sketch $y = \sin x$ and $y = \cos 2x$ for $-\pi \le x \le \pi$, showing these points of intersection. **2**

(iii) Calculate the area of the region bounded by the curves $y = \sin x$ and $y = \cos 2x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{6}$. **2**

(b) Consider the curve $f(x) = x^2 - 4x + 5$

(i) Find the largest possible domain containing only positive numbers for which $f(x)$ has an inverse function $f^{-1}(x)$. **1**

(ii) Find the point(s) of intersection of $y = f(x)$ and $y = f^{-1}(x)$ in the domain determined in part (i) **2**

(iii) State the domain of $y = f^{-1}(x)$? **1**

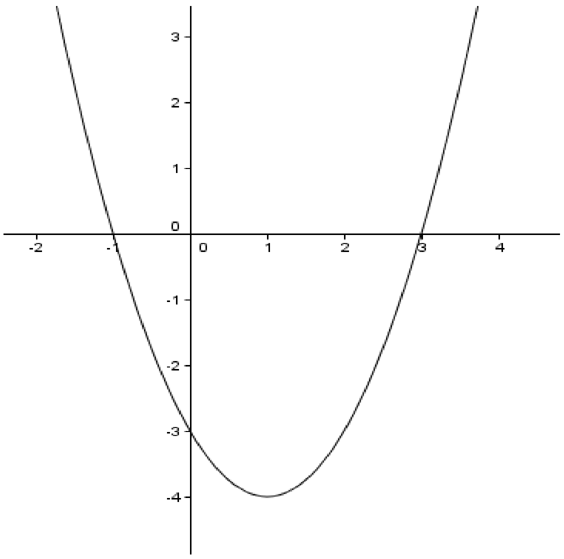
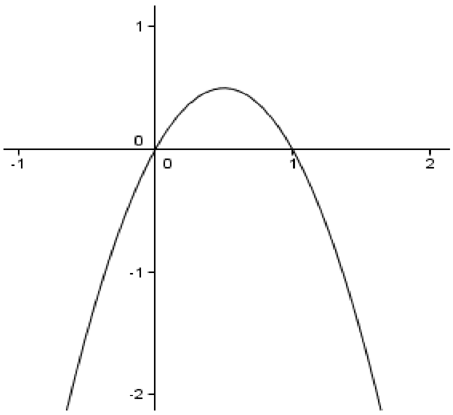
(iv) What is the equation of $y = f^{-1}(x)$? **2**

(v) Sketch the parabola $y = f(x)$ for the restricted domain and sketch the inverse function $y = f^{-1}(x)$ on the same diagram, clearly showing any points of intersection. Clearly label each graph. **2**

END OF EXAMINATION

Outcomes Addressed in this Question

PE3 solves problems involving permutations and combinations, inequalities
HE6 determines integrals by reduction to a standard form through a given substitution

Part	Solutions	Marking Guidelines
(a)	$\frac{4x}{x-3} \le 1$ $4x(x-3) \le (x-3)^2$ $4x(x-3)-(x-3)^2 \le 0$ $4x^2-12x-x^2+6x-9 \le 0$ $3x^2-6x-9 \le 0$ $x^2-2x-3 \le 0$ $(x-3)(x+1) \le 0$  From the graph and the condition $x \ne 3$, $-1 \le x < 3$	**Award 3** marks for the correct answer. **Award 2** mark for substantial progress towards the correct solution. **Award 1** mark for some progress towards the correct solution.
(b)	$\lvert 5x-1 \rvert < \sqrt{2x(1-x)}$ Condition: Restricted domain for $2x(1-x)$  RHS only exist for $0 \le x \le 1$	**Award 3** marks for the correct answer. **Award 2** mark for substantial progress towards the correct solution. **Award 1** mark for some progress towards the correct solution.

	$\left\|5x-1\right\| < \sqrt{2x(1-x)}$ Squaring both sides $(5x-1)^2 < 2x(1-x)$ $25x^2 - 10x + 1 < 2x - 2x^2$ $27x^2 - 12x + 1 < 0$ $(9x-1)(3x-1) < 0$ $\therefore \frac{1}{9} < x < \frac{1}{3}$ $\therefore$ With the domain applied, $\frac{1}{9} < x < \frac{1}{3}$ is the solution	
(c)	(i) There are 4 players numbered '6' and 4 players numbered '8' from a total of forty players. Three '6''s can be selected in ${}^{4}C_{3}$ ways. Two '8''s can be selected in ${}^{4}C_{2}$ ways. Five players can be selected in ${}^{40}C_{5}$ ways. $\therefore$ Required probability $= \frac{{}^{4}C_{3} \times {}^{4}C_{2}}{{}^{40}C_{5}}$ $= \frac{4 \times 6}{658008}$ $= \frac{1}{27417}$	**Award 2** marks for the correct answer. **Award 1** mark for substantial progress towards the solution
	(ii) "At least 4 players" means 4 or 5players: 5 players from one team can be selected in ${}^{10}C_{5}$ ways. But there are 4 teams, hence 5 players from the same team can be selected in ${}^{4}C_{1} \times {}^{10}C_{5}$ ways. 4 players from one team and one player from the remaining teams (30 players) can be selected in ${}^{4}C_{1} \times {}^{10}C_{4} \times {}^{30}C_{1}$ ways. $\therefore$ Required probability $= \frac{{}^{4}C_{1} \times {}^{10}C_{5}}{{}^{40}C_{5}} + \frac{{}^{4}C_{1} \times {}^{10}C_{4} \times {}^{30}C_{1}}{{}^{40}C_{5}}$ $= \frac{4(252 + 210 \times 30)}{658008}$ $= \frac{28}{703}$	**Award 3** marks for the correct answer. **Award 2** mark for substantial progress towards the correct solution. **Award 1** mark for some progress towards the correct solution.

(d)

$u = 2x + 1$

$\dfrac{du}{dx} = 2$

$\therefore\ du = 2dx$

$\text{and } x = \dfrac{u-1}{2}$

$\text{When } x = 0, u = 1$

$\text{When } x = 1, u = 3$

$$\int_0^1 \frac{2x}{(2x+1)^2}dx$$

$$= \int_0^1 x \times \frac{1}{(2x+1)^2} 2dx$$

$$= \int_1^3 \frac{u-1}{2u^2}du$$

$$= \frac{1}{2}\int_1^3 \frac{u}{u^2} - \frac{1}{u^2}\,du$$

$$= \frac{1}{2}\int_1^3 \frac{1}{u} - \frac{1}{u^2}\,du$$

$$= \frac{1}{2}\left[\ln u + u^{-1}\right]_1^3$$

$$= \frac{1}{2}\left[\left(\ln 3 + \frac{1}{3}\right) - \left(\ln 1 + 1\right)\right]$$

$$= \frac{1}{2}\left[\ln 3 - \frac{2}{3}\right]$$

Award 4 marks for the correct answer.

Award 3 mark for the correct solution with minor errors.

Award 2 mark for substantial progress towards the correct solution.

Award 1 mark for some progress towards the correct solution.

Multiple Choice Answers	
1	A
2	D
3	B
4	A
5	C
6	D
7	B
8	C
9	A
10	B

Outcome Addressed in this Question

PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

Part	Solutions	Marking Guidelines
(a) (i)	Draw PQ $\angle PQR = 90^0$ $\left(\begin{array}{l}\text{the angle at the circumeference subtended}\\ \text{by a diameter equals } 90^0\end{array}\right)$ $\angle SPQ = \angle PQR$ (alternate angles are equal, $SP \parallel QR$) $= 90^0$ $\therefore SQ$ is a diameter $\left(\begin{array}{l}\text{a right angle at the circuference subtends}\\ \text{a diameter}\end{array}\right)$	**Award 2** for correct solution **Award 1** for substantial progress towards solution
(ii)	If $QS \parallel SR$ then $PQRS$ is a parallelogram (both pairs of opposite sides are parallel). $\therefore PR = QS$ (opposite sides of a parallelogram are equal) $\therefore$ Both circles have equal diameters and hence, equal radii.	**Award 2** for correct solution **Award 1** for substantial progress towards solution
(b) (i)	$Q \equiv \left(\dfrac{1.2at + 0.t^2}{t^2+1}, \dfrac{1.at^2 + a.t^2}{t^2+1}\right) = \left(\dfrac{2at}{t^2+1}, \dfrac{2at^2}{t^2+1}\right)$	**Award 2** for both coordinates correct **Award 1** for only one correct coordinate (or equivalent merit)
(ii)	From (i), $x = \dfrac{2at}{t^2+1}$ and $y = \dfrac{2at^2}{t^2+1}$ $\dfrac{y}{x} = \dfrac{\frac{2at^2}{t^2+1}}{\frac{2at}{t^2+1}} = \dfrac{2at^2}{2at} = t$	**Award 1** for correct solution
(iii)	From (i) and (ii), $x = \dfrac{2at}{t^2+1} = \dfrac{2a\left(\frac{y}{x}\right)}{\left(\frac{y}{x}\right)^2 + 1}$ $x\left(\left(\dfrac{y}{x}\right)^2 + 1\right) = 2a\left(\dfrac{y}{x}\right)$ $\dfrac{y^2}{x} + x = \dfrac{2ay}{x}$ $x^2 + y^2 = 2ay$ $x^2 + y^2 - 2ay = 0$ $x^2 + y^2 - 2ay + a^2 = a^2$ $\therefore x^2 + (y-a)^2 = a^2$ Which is a circle, centre $= (0, a)$ and radius $= a$	**Award 3** for correct solution **Award 2** for substantial progress towards solution **Award 1** for limited progress towards solution

(c) (i)	$P(x)=(x-a)^3+(x-b)^3$ $\therefore P\left(\frac{a+b}{2}\right)=\left(\frac{a+b}{2}-a\right)^3+\left(\frac{a+b}{2}-b\right)^3$ $=\left(\frac{b-a}{2}\right)^3+\left(\frac{a-b}{2}\right)^3$ $=(-1)^3\left(\frac{a-b}{2}\right)^3+\left(\frac{a-b}{2}\right)^3$ $=0$ $\therefore x=\frac{a+b}{2}$ is a zero of the polynomial	**Award 2** for correct solution **Award 1** for substantial progress towards solution
(c) (ii)	$P(x)=(x-a)^3+(x-b)^3$ $P'(x)=3(x-a)^2+3(x-b)^2$ Stationary points occur where $P'(x)=0$ $\therefore 3(x-a)^2+3(x-b)^2=0$ $(x-a)^2+(x-b)^2=0$ $x^2-2ax+a^2+x^2-2bx+b^2=0$ $2x^2-(2a+2b)x+(a^2+b^2)=0$ $\Delta=(-(2a+2b))^2-4.2.(a^2+b^2)$ $=4a^2+8ab+4b^2-8a^2-8b^2$ $=-4a^2+8ab-4b^2$ $=-4(a-b)^2$ <0 for all a and b, $a\neq b$ $\therefore P'(x)\neq 0$ for any real values of x $\therefore P(x)$ has no stationary points.	**Award 3** for correct solution **Award 2** for substantial progress towards solution **Award 1** for limited progress towards solution

Year 12	Mathematics Extension 1 2017	TRIAL
Question No. 13	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

PE2 - uses multi-step deductive reasoning in a variety of contexts
HE2 - uses inductive reasoning in the construction of proofs

Part / Outcome	Solutions	Marking Guidelines

(a)

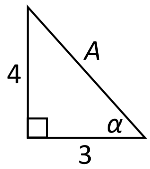
(i) $A\sin(x+\alpha) = A\sin x\cos\alpha + A\cos x\sin\alpha$

$= 3\sin x + 4\cos x$

so, $A\cos\alpha = 3$ and $A\sin\alpha = 4$

$$\frac{A\sin\alpha}{A\cos\alpha} = \frac{4}{3}$$

$$\tan\alpha = \frac{4}{3}$$



$$\alpha = \tan^{-1}\left(\frac{4}{3}\right) \text{ and } A = \sqrt{4^2+3^2} = 5$$

so, $3\sin x + 4\cos x = 5\sin\left[x + \tan^{-1}\left(\frac{4}{3}\right)\right]$

2 marks – Correct solution

1 mark – Substantially correct

Note: working in degrees gives values which are outside the stated domain

(ii) $3\sin x + 4\cos x = 5$

$$5\sin\left[x+\tan^{-1}\left(\tfrac{4}{3}\right)\right] = 5$$

$$\sin\left[x+\tan^{-1}\left(\tfrac{4}{3}\right)\right] = 1$$

$$x+\tan^{-1}\left(\tfrac{4}{3}\right) = \tfrac{\pi}{2}$$

$$x = \tfrac{\pi}{2} - \tan^{-1}\left(\tfrac{4}{3}\right)$$

$$= \tfrac{\pi}{2} - 0\cdot92720\ldots$$

$$= 0\cdot64 \ (\text{to 2 dec pl.})$$

2 marks – Correct solution

1 mark – Substantially correct

(b)

(i)

note $\begin{cases} \cos 2\theta = 1 - 2\sin^2\theta \\ \sin^2\theta = \dfrac{1-\cos 2\theta}{2} \end{cases}$ and $\begin{cases} \cos 2\theta = 2\cos^2\theta - 1 \\ \cos^2\theta = \dfrac{1+\cos 2\theta}{2} \end{cases}$

LHS $= \tan^2\theta$

$$= \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\frac{1-\cos 2\theta}{2}}{\frac{1+\cos 2\theta}{2}}$$

$$= \frac{1-\cos 2\theta}{1+\cos 2\theta} = \text{RHS}$$

2 marks – Correct solution

1 mark – Substantially correct

(ii) Let $\theta = \dfrac{\pi}{8}$

$$\tan^2\frac{\pi}{8} = \frac{1-\cos 2\left(\frac{\pi}{8}\right)}{1+\cos 2\left(\frac{\pi}{8}\right)}$$

$$= \frac{1-\cos\frac{\pi}{4}}{1+\cos\frac{\pi}{4}}$$

$$= \frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$$

$$= \frac{\sqrt{2}-1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$= \left(\sqrt{2}-1\right)^2$$

$$\tan\frac{\pi}{8} = \sqrt{2}-1 \quad \left(\tan\frac{\pi}{8} > 0 \text{ as } \frac{\pi}{8} \text{ is in } 1^{\text{st}} \text{ quad}\right)$$

2 marks – Correct solution

1 mark – Substantially correct

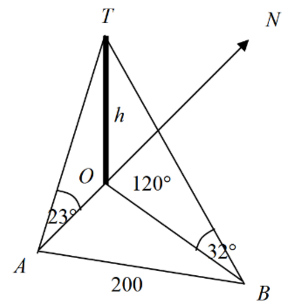
(c)

(i) in ΔOAT,

$$\angle OTA = 90° - 23°$$
$$= 67°$$

$$\tan 67° = \frac{OA}{h}$$

$$OA = h\tan 67°$$



in ΔOBT,

$$\angle OTB = 90° - 32°$$
$$= 58°$$

$$\tan 58° = \frac{OA}{h}$$

$$OB = h\tan 58°$$

1 mark – Correct solution

(ii) from diagram, $\angle AOB = 180° - 120°$
$= 60°$

Using the cosine rule in ΔAOB

$$AB^2 = OA^2 + OB^2 - 2(OA)(OB)\cos 60°$$

$$200^2 = h^2\tan^2 67° + h^2\tan^2 58° - 2h^2\tan 67°\tan 58° \times \frac{1}{2}$$

$$= h^2\left(\tan^2 67° + \tan^2 58° - \tan 67°\tan 58°\right)$$

$$h^2 = \frac{200^2}{\tan^2 67° + \tan^2 58° - \tan 67°\tan 58°}$$

$$h = \sqrt{\frac{40\,000}{4\cdot 3409\ldots}}$$

$$= \sqrt{9214.5535\ldots}$$

$$= 96\text{m} \ \left(\text{to nearest m}\right)$$

3 marks – Correct solution

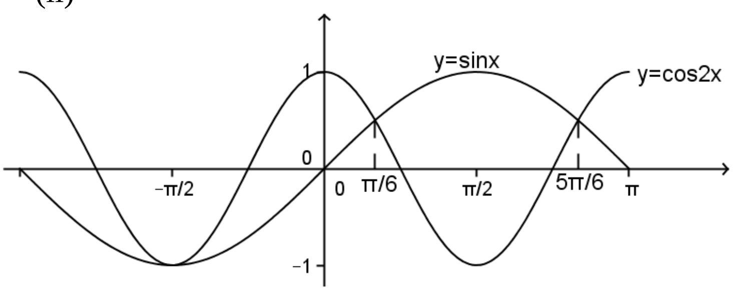
2 marks – Substantially correct solution

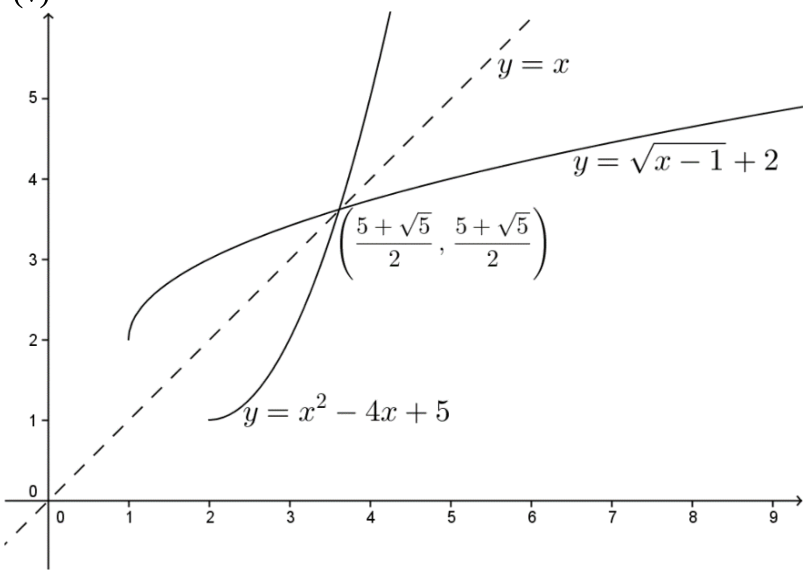
(d)	Show true for $n = 1$ $4^n + 14 = 4^1 + 14$ $= 18$ $= 6(3) \quad \therefore$ true for $n = 1$ Assume true for $n = k$ ie, $4^k + 14 = 6M$, where M is an integer Prove true for $n = k + 1$ $4^{k+1} + 14 = 4^k \times 4 + 14$ $= (6M - 14) \times 4 + 14$ $= 6 \times 4M - 4 \times 14 + 14$ $= 6 \times 4M - 42$ $= 6(4M - 7)$ $= 6N$, where N is an integer $\therefore$ true by the Principle of Maffamadikal Inducement	**1 mark** – significant progress towards correct solution **3 marks** – Correct solution **2 marks** – Substantially correct solution **1 mark** – significant progress towards correct solution

Year 12 Trial Higher School Certificate Extension 1 Mathematics Examination 2017
Question No. 14 Solutions and Marking Guidelines

Outcomes Addressed in this Question

HE4 Uses the relationship between functions, inverse functions and their derivatives.
H5 Applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems
H8 Uses techniques of integration to calculate areas and volumes.
H9 Communicates using mathematical language, notation, diagrams and graphs.

Outcome	Solutions	Marking Guidelines
H5	(a)(i) $y=\sin x$ and $y=\cos 2x$ meet when $\sin x=\cos 2x$. $\therefore \sin x = 1-2\sin^2 x$ $2\sin^2 x+\sin x-1=0$ $2\sin^2 x+2\sin x-\sin x-1=0$ $2\sin x(\sin x+1)-(\sin x+1)=0$ $(\sin x+1)(2\sin x-1)=0$ $\sin x=-1$ and $\sin x=\frac{1}{2}$ For $-\pi \le x \le \pi$, $x=-\frac{\pi}{2},\frac{\pi}{6}$ and $\pi-\frac{\pi}{6}$. $\therefore x=-\frac{\pi}{2},\frac{\pi}{6},\frac{5\pi}{6}$.	3 marks : correct solution 2 marks : substantially correct solution 1 mark : significant progress towards correct solution
H9	(ii)  	2 marks : correct graph 1 mark : significant progress towards correct graph
H8	(iii) $A=\int_{-\frac{\pi}{2}}^{\frac{\pi}{6}}(\cos 2x-\sin x)dx$ $=\left[\frac{1}{2}\sin 2x-(-\cos x)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{6}}$ $=\left[\frac{1}{2}\sin 2x+\cos x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{6}}$ $=\frac{1}{2}\sin\frac{\pi}{3}+\cos\frac{\pi}{6}-\left(\frac{1}{2}\sin(-\pi)+\cos\left(-\frac{\pi}{2}\right)\right)$ $=\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{2}$ $=\frac{3\sqrt{3}}{4}$ square units.	2 marks : correct solution 1 mark : significant progress towards correct solution

HE4	(b) (i) $y = x^2 - 4x + 5$ has axis of symmetry $x = \frac{-(-4)}{2} = 2$. $\therefore$ largest domain containing positive numbers is $x \geq 2$.	1 mark : correct answer
HE4	(ii) $y = x^2 - 4x + 5$ and the inverse function intersect on the line $y = x$. $y = x^2 - 4x + 5$ and $y = x$ meet when $x = x^2 - 4x + 5$ Solving $x^2 - 5x + 5 = 0$, $x = \frac{5 \pm \sqrt{25 - 4.5}}{2}$ $\therefore x = \frac{5 \pm \sqrt{5}}{2}$. But $x \geq 2$, so $x = \frac{5 + \sqrt{5}}{2}$. $\therefore y = f(x)$ and $\therefore y = f^{-1}(x)$ intersect at $\left(\frac{5+\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right)$.	2 marks : correct solution 1 marks : substantial progress towards correct solution
HE4	(iii) For the original function, when $x = 2$, $y = 2^2 - 4.2 + 5 = 1$ $\therefore$ is range is $y \geq 1$. $\therefore$ domain of the inverse function is $x \geq 1$.	1 mark : correct answer
HE4	(iv) Interchanging *x* and *y* in $y = x^2 - 4x + 5$, $x = y^2 - 4y + 5$. $x - 1 = y^2 - 4y + 4$ $x - 1 = (y - 2)^2$ $y - 2 = \pm\sqrt{x - 1}$ $y = \pm\sqrt{x - 1} + 2$ But, $y \geq 2$ (range of inverse), $\therefore y = \sqrt{x - 1} + 2$.	2 marks : correct solution 1 mark : substantial progress towards correct solution
HE4	(v)  	2 marks : correct graph 1 mark : significant progress towards correct graph