NORTHERN BEACHES SECONDARY COLLEGE

MANLY SELECTIVE CAMPUS

HIGHER SCHOOL CERTIFICATE

TRIAL EXAMINATION

2017

Mathematics Extension I

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using **black** pen only
- Write your Student Number at the top of each page
- Answer Section I- Multiple Choice on Answer Sheet provided
- Answer Section II – Free Response in a separate booklet for each question.
- Board approved calculators and templates may be used.

Section I Multiple Choice
- 10 marks
- Attempt all questions
- Allow about 15 minutes for this section

Section II – Free Response
- 60 marks
- Each question is of equal value
- All necessary working should be shown in every question.
- Allow about 1 hour 45 minutes for this section

Weighting: 40%

Student Number: ___________________________

Section 1: Multiple Choice (10 marks)

Indicate your answer on answer sheet provided.
Allow approximately 15 minutes for this section.

Q1. Consider the polynomial $P(x) = 3x^3 + 3x + a$. If (x – 2) is a factor, what is the value of a ?

A -18

B -30

C 18

D 30

Q2. An asymptote for the curve $y = x^2 - \dfrac{3}{x^2 - 3} - 3$ is:

A $y = x^2$

B $y = -3$

C $x = -3$

D $x = -\sqrt{3}$

Q3. The point *P* (1, 2) divides the interval *AB* in the ratio k:1. If *A* is the point (-3, 6) and *B* is the point (7, -4), what is the value of *k?*

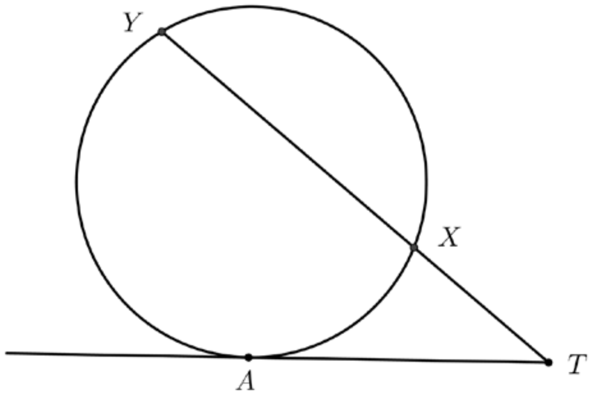
A $\dfrac{3}{2}$

B $\dfrac{3}{4}$

C $\dfrac{2}{3}$

D $\dfrac{6}{7}$

Q4. The diagram below shows a circle with tangent *TA*. *X* and *Y* are points on the circle and $XY = 2 \times TX$.



Which of the following is true?

A $TA = 3 \times TX$

B $TA^2 = 3 \times TX$

C $TA^2 = 2 \times TX^2$

D $TA^2 = 3 \times TX^2$

Q5. In the expansion of $(x - 2y)^{10}$, the middle term is:

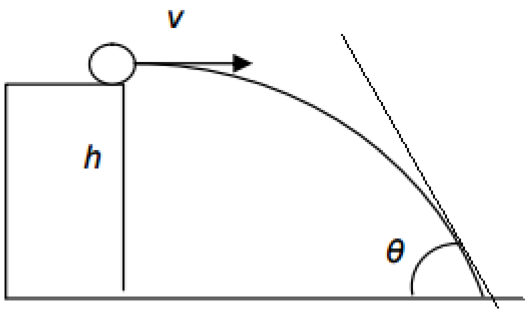
A $-8064x^5y^5$

B $6720\,x^5y^5$

C $3360\,x^6\,y^4$

D $13440\,x^4y^6$

Q6. The diagram below shows the path of a projectile launched with a horizontal velocity v from a cliff of height h.



Which of the following values of v and h will give the greatest value of θ.

A $v = 10$, $h = 30$

B $v = 30$, $h = 50$

C $v = 50$, $h = 10$

D $v = 10$, $h = 50$

Q7. Using the substitution $u = \sqrt{x}$, $\int \frac{dx}{x + \sqrt{x}}$ can be transformed to:

A $\int \frac{2du}{u + 1}$

B $\int \frac{du}{u^2 + u}$

C $\int \frac{2du}{u^2 + u}$

D $\frac{1}{2}\int \frac{du}{u^2 + u}$

Q8. The derivative of $y = x^2 \sin^{-1}(2x)$ is?

A $\quad 2x\sin^{-1}(2x) + \dfrac{2x^2}{\sqrt{1-4x^2}}$

B $\quad 2x\sin^{-1}(2x) + \dfrac{x^2}{\sqrt{1-4x^2}}$

C $\quad 2x\sin^{-1}(2x) - \dfrac{x^2}{\sqrt{1-4x^2}}$

D $\quad 2x\sin^{-1}(2x) + \dfrac{x^2}{\sqrt{1-2x^2}}$

Q9. The velocity of a particle moving in a straight line is given by $v = 2x + 5$, where x metres is the distance from a fixed point O and v is in metres per second. What is the acceleration of the particle when it is 1 metre to the right from O?

A $\quad a = 7ms^{-2}$

B $\quad a = 12ms^{-2}$

C $\quad a = 14ms^{-2}$

D $\quad a = 24ms^{-2}$

Q10. At a university, the probability that a student is studying part-time is $p = 0.2$.
Ten students from the university are randomly chosen to complete a survey.
The probability that 6 of the 10 students study part-time is given by:

A $\quad {}^{10}\mathbf{C}_6\,(0.2)^6$

B $\quad {}^{10}\mathbf{C}_6\,(0.2)^4(0.8)^6$

C $\quad {}^{10}\mathbf{C}_6\,(0.2)^{10}\,4^4$

D $\quad {}^{10}\mathbf{C}_6\,(0.2)^{10}\,4^6$

End of Multiple Choice

Student Number: ____

Section II Total Marks is 60

Attempt Questions 11 – 14.
Allow approximately 1 hour & 45 minutes for this section.

Answer all questions, starting each new question in a new booklet with your **student ID number** in the top right hand corner and the question number on the left hand side of your paper. All necessary working must be shown in each and every question.

Question 11 Start New Booklet **15 marks**

a) Find $\int x\sqrt{4-x}\,dx$ using the substitution $u = 4 - x$. **2**

b) When the polynomial $P(x)$ is divided by $(x+1)(x-2)$ the remainder is $18x + 17$. What is the remainder when $P(x)$ is divided by $(x-2)$. **2**

c) Solve $\dfrac{5}{4-x} \geq 1$. **2**

d) Find the general solution to $2\cos x = \sqrt{3}$.
Express your answer in terms of π. **2**

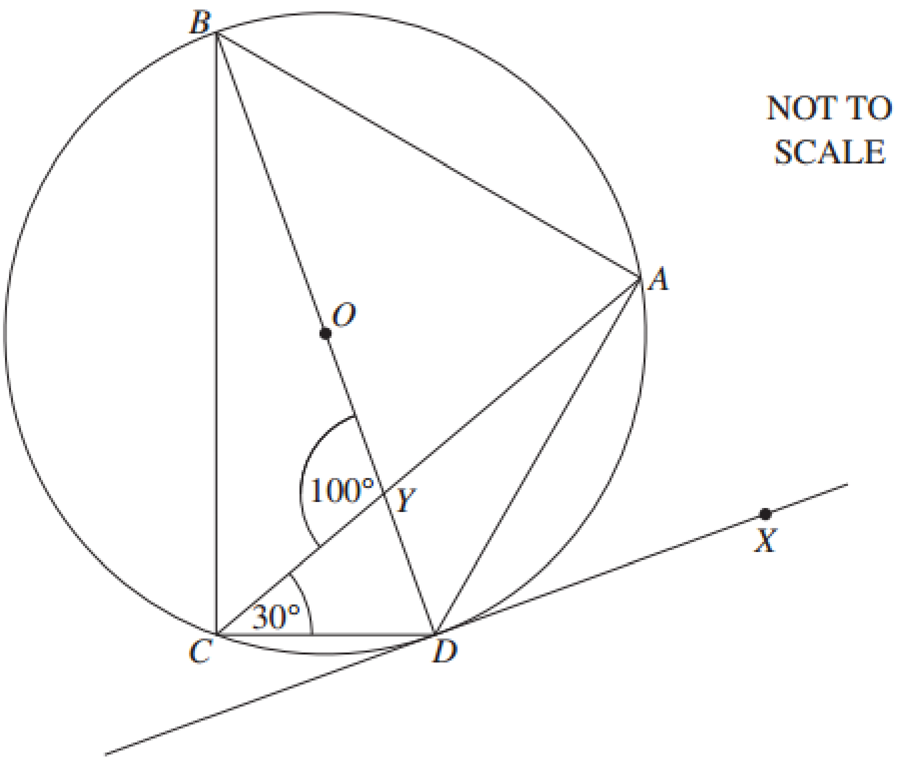
e) Evaluate $\displaystyle\int_0^{\frac{\pi}{6}} \frac{1}{\left(1+\frac{1}{2}x^2\right)}\,dx$ in simplest exact form. **3**

Question 11 continues on the next page

Question 11 continued

f) In the diagram, the points *A*, *B*, *C* and *D* are on the circumference of a circle, whose centre *O* lies on *BD*. The chord *AC* intersects the diameter *BD* at *Y*. The tangent at *D* passes through the point *X*.

It is given that $\angle CYB = 100°$ and $\angle DCY = 30°$.



Copy or trace the diagram into your writing booklet.

(i) What is the size of $\angle ACB$? **1**

(ii) What is the size of $\angle ADX$? **1**

(iii) Find, giving reasons, the size of $\angle CAB$. **2**

End of Question 11

Student Number: ______________________________

Question 12 Start New Booklet **15 Marks**

a) Without the use of calculus, sketch the graph of $y = \dfrac{x^2 - 9}{2x^2}$.

Show all intercepts on the coordinate axes and all asymptotes for the curve. **3**

b) A standard pack of 52 cards consists of 13 cards of four suits; hearts, diamonds, clubs and spades.

What is the probability that if five cards are selected without replacement, that at least four of the cards are of the same suit? **3**

c) Solve $\sin x + \sqrt{3}\cos x = \sqrt{2}$ for $0 \leq x \leq 2\pi$. **3**

d) By considering both sides of the identity $(1 + x)^m(1 + x)^n = (1 + x)^{m+n}$ and comparing coefficients, show that :

$$\binom{m+n}{3} = \binom{m}{3} + \binom{m}{2}\binom{n}{1} + \binom{m}{1}\binom{n}{2} + \binom{n}{3}$$ **2**

e) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$.

The equation of the chord PQ is $y = \left(\dfrac{p+q}{2}\right)x - apq$ (DO NOT PROVE THIS)

(i) If PQ passes through the point R $(2a, 3a)$, show that $pq = p + q - 3$. **1**

(ii) If M is the midpoint of PQ, show that the coordinates of M are:

$$\left(a\,(pq + 3), \frac{a}{2}(pq + 3)^2 - 2pq\right)$$ **1**

(iii) Hence, find the locus of M. **2**

End of Question 12

Student Number: ____

Question 13 – Start New Booklet **15 Marks**

a) Use the method of mathematical induction to prove that $3^{2n} - 2^{2n}$ is divisible by 5 for integers $n \geq 1$. **3**

b) Use the substitution $x = \cos\theta$ to evaluate $\displaystyle\int_{\frac{1}{2}}^{1} \frac{\sqrt{1-x^2}}{x^2}\,dx$.

State your answer in simplest exact form. **3**

c) The constant term in the expansion of $x^4\left(2x^2 + \dfrac{m}{x}\right)^7$ is 896.

What is the value of *m?* **3**

d) If α, β and ϒ are roots of the equation $2x^3 - 3x + 1 = 0$,
what is the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$? **2**

e) A camera, one kilometre away in the horizontal direction from where the Space Shuttle is being launched, is tracking the ascent of the shuttle. **4**

Assume the Space Shuttle ascends in a straight line in the vertical direction.

Thirty seconds after being launched, the Shuttle reaches a height *h* of 3240 metres and is travelling at a speed of 230 metres per second.

The angle θ is the angle of elevation of the camera as it tracks the shuttle.

At what rate is θ increasing 30 seconds after the shuttle is launched?

End of Question 13

Question 14 Start New Booklet **15 Marks**

a) The acceleration of a particle P is given by the equation $\ddot{x} = 8x(x^2 + 1)$ where x metres is the displacement of P from the origin O after t seconds. Initially, P moves from O with a velocity of -2 ms^{-1}.

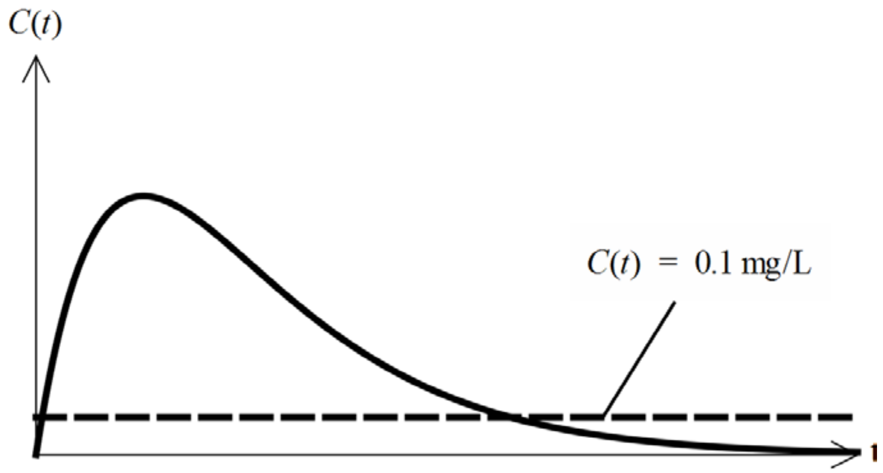
(i) Show that the velocity of P in any position x is given by

$$v = -2(x^2 + 1)\ ms^{-1}.$$ **2**

(ii) Find an expression for the displacement x in terms of time t. **2**

b) A patient was administered with a drug. The concentration of the drug in the patient's blood followed the rule $C(t) = 1.3t\, e^{-0.3t}$, where t is measured in hours and $C(t)$ is measured in mg/L.

The graph of $C(t)$ is shown below:



The doctor left instructions that the patient must not receive another dose of the medicine until the concentration of the drug has dropped below 0.1 mg/L.

Using t = 15 as a first approximation, use one application of Newton's method to find approximately when the concentration of the drug in the blood of the patient reaches 0.1 mg/L. **3**

Question 14 continues on the next page

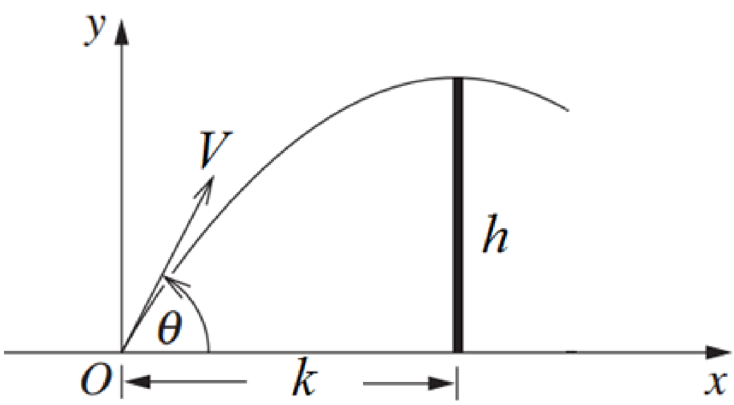
Question 14 continued.

c) A young man wishes to have a bowl of hot soup. He heats the soup to a temperature of 75°C. He then allows it to cool. The soup cools to 65°C in 5 minutes. Given that the surrounding temperature S °C is 22 °C and assuming Newton's Law of Cooling is given by $\frac{dT}{dt} = -k(T - S)$:

(i) Show that $T = S + Ae^{-kt}$ is a solution to the differential equation, where A is a constant. **1**

(ii) Find the temperature to the nearest degree of the soup 15 minutes from the time it begins to start cooling. **3**

d)



A vertical wall of height h metres stands on horizontal ground. When a projectile is launched, in a vertical plane which is perpendicular to the wall, from a point on the ground which is k metres from the wall, it just clears the wall at the highest point of its path.

The equations of motion for the projectile with angle of projection θ are:

$$x = (V\cos\theta)t \qquad y = (V\sin\theta)t - \frac{1}{2}gt^2$$

(DO NOT DERIVE THESE EQUATIONS)

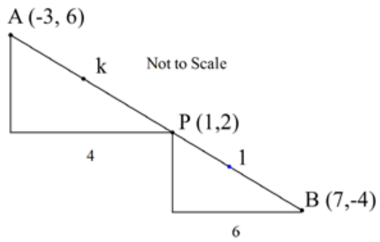
(i) Show that the particle reaches the highest point on its path when $t = \frac{V\sin\theta}{g}$ **1**

(ii) Hence, or otherwise, show $V^2 = \frac{g}{2h}(4h^2 + k^2)$. **3**

END OF EXAMINATION

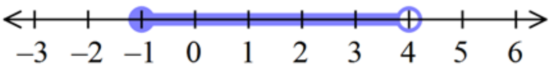
Student Number: ______________________________

Multiple Choice.

Q1	$P(2) = 3(2)^3 + 3 \times 2 + a = 0$ $30 + a = 0$ $a = -30$	B
Q2	Vertical Asymptote $x^2 - 3 = 0$ $x^2 = 3$ $x = \; = -\sqrt{3}$ $x = -\sqrt{3}$	D
Q3	 $k : 1 = 2 : 3$ $\frac{k}{1} = \frac{2}{3}$ $k = \frac{2}{3}$	C
Q4	$TA^2 = XT.YT$ $TY = XT + YX$ $= TX + 2TX$ $= 3TX$ $\therefore AT^2 = XT.3XT$ $= 3XT^2$	D
Q5	11 terms therefore sixth term is middle. ${}^{10}\mathbf{C}_5(x^5)(2y)^5) = \; = 8064x^5y^5$	A
Q6	Closest to vertical produced by maximum height and minimum horizontal velocity.	D

Q7	$u = \sqrt{x} \Rightarrow u^2 = x$ $2u.du = dx$ $\int \frac{2u\,du}{u^2 + u}$ $\int \frac{2du}{u+1}$	A
Q8	$y = x^2.\sin^{-1}(2x)$ $\frac{dy}{dx} = uv' + vu'$ $= 2x\sin^{-1}(2x) + x^2.\frac{2}{\sqrt{1-4x^2}}$	A
Q9	$v = (2x+5)$ $\frac{v^2}{2} = \frac{1}{2}(4x^2 + 20x + 25)$ $\ddot{x} = \frac{d\left(\frac{1}{2}v^2\right)}{dx}$ $= \frac{1}{2}(8x + 20)$ at $x = 1$ $\ddot{x} = \frac{1}{2}(8 + 20) = 14$	C
Q10	${}^{10}\mathbf{C}_4(0.2)^6(0.8)^4$ $= {}^{10}\mathbf{C}_4(0.2)^6(0.2)^4.\,4^4$ $= {}^{10}\mathbf{C}_4(0.2)^{10}.\,4^4$	C

Question 11

a	$\int x\sqrt{4-x}\ dx$ $u = 4-x$ $x = 4-u$ $dx = -\ du$ $\int (4-u)\ \sqrt{u}\ -du$ $= \int u^{\frac{3}{2}} - 4\sqrt{u}\ du$ $= \frac{2}{5}u^{\frac{5}{2}} - 4 \times \frac{2}{3} \times u^{\frac{3}{2}}$ $= \frac{2}{5}u^{\frac{5}{2}} - \frac{8}{3}u^{\frac{3}{2}} + C$ $= \frac{2}{5}(4-x)^{\frac{5}{2}} - \frac{8}{3}(4-x)^{\frac{3}{2}} + C$	2 marks – correct solution 1 mark – correct substitution.
b	$P(x) = (x-2)(x+1)Q(x) + 18x + 17$ $x = 2$ $18x + 17 = 53$ therefore remainder is 53	2 marks – correct solution 1 mark – correct expression for Division Transformation.
c	$\frac{5}{4-x} \geq$ $\therefore x \neq 4$ $5 = 4-x$ $x = -1$ $x = -2$: $\frac{5}{6} \geq 1$ False $x = 0$: $\frac{5}{4} \geq 1$ True $x = 5$: $\frac{5}{-1} \geq 1$ False  $-1 \leq x < 4$	2 marks – correct solution 1 mark – one correct inequality only

d	$2\cos x = \sqrt{3}$ $\cos x = \dfrac{\sqrt{3}}{2}$ $\cos\alpha = \dfrac{\sqrt{3}}{2} \Rightarrow \alpha = \dfrac{\pi}{6}$ $\therefore \quad x = 2n\pi \pm \dfrac{\pi}{6}$	2 marks – correct solution 1 mark – correct acute angle
e	$\displaystyle\int_0^{\frac{\pi}{6}} \frac{1}{1+\frac{1}{2}x^2}\,dx$ $\displaystyle= \int_0^{\frac{\pi}{6}} \frac{1}{\frac{1}{2}(2+x^2)\,dx}$ $\displaystyle= 2\int_0^{\frac{\pi}{6}} \frac{1}{(2+x^2)\,dx}$ $\displaystyle= 2 \times \left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]_0^{\frac{\pi}{6}}$ $\displaystyle= \sqrt{2}\left(\tan^{-1}\frac{\frac{\pi}{6}}{\sqrt{2}} - \tan^{-1}\frac{0}{\sqrt{2}}\right)$ $\displaystyle= \sqrt{2}\tan^{-1}\left(\frac{\pi}{6\sqrt{2}}\right)$ $\displaystyle= \sqrt{2}\tan^{-1}\frac{\sqrt{2}\,\pi}{12}$	3 marks – correct solution 2 marks – correct integration but incorrect substitution of values - Incorrect form of inverse tan but correct substitution of values 1 mark – a form of inverse tan as integral
f-i	$\angle ACD = 90°$ ($\angle$ in a semicircle) $\therefore \quad \angle ACB + \angle YCD = 90$ $\angle ACB = 90 - 30 = 60°$	1 mark – including correct reasons
f-ii	$\angle ADX = 30°$ (angle in alternate segment theorem) \	1 mark – including correct reasons

f-iii	$\angle CAB = \angle CDB$ ($\angle$'s standing on same chord are =) $100° = 30° + \angle CDB$ ($ext\ \angle$ of Δ = opp interior $\angle$) $\angle CDB = 70°$ $\therefore$ $\angle CAB = 70°$	2 marks – with FULL reasoning 1 mark – correct value with incomplete reasons

Markers Comments

Question 12

<table>
<tr><td>a</td><td>

$$y=\frac{x^2-9}{2x^2}$$
$$=\frac{1}{2}-\frac{9}{2x^2}$$

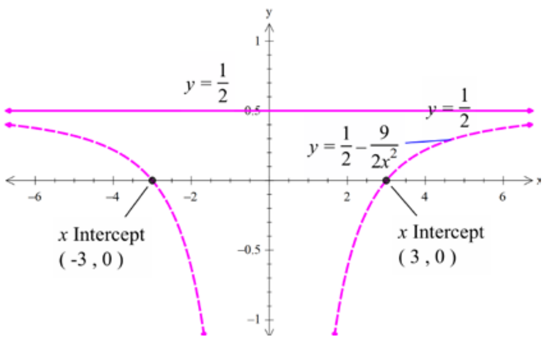
therefore hyperbola

$y=-\frac{9}{2x^2}$ which has been raised by $\frac{1}{2}$

x intercepts $y=0$

$\therefore x^2-9=0$

$x=\pm 3$



</td><td>3 marks - correct shape and all required features.

2 marks – correct shape and either intercepts or asymptote

1 mark – correct shape or intercepts or asymptote.</td></tr>
<tr><td>b</td><td>

Four cards of same suit.

$$\frac{{}^{13}\mathbf{C}_4 \times {}^{39}\mathbf{C}_1 \times 4}{{}^{52}\mathbf{C}_5}=\frac{143}{3332}=0.043$$

Five cards of same suit.

$$\frac{{}^{13}\mathbf{C}_5 \times 4}{{}^{52}\mathbf{C}_5}=\frac{5148}{3332}=1.980\times 10^{-3}$$

therefore

$P(5\text{ same})+P(4\text{ same})=0.44897\cong 0.0449$

</td><td>3 marks – correct solution

2 marks – one error but must be calculating the two cases

1 mark considers 2 cases and has correct denominator.</td></tr>
</table>

c	$\sin x + \sqrt{3}\cos x = \sqrt{2}$ $R\sin(x+\alpha) = R\sin x\cos\alpha + R\sin\alpha\cos x$ $\therefore\quad R\cos\alpha = 1$ $R\sin\alpha = \sqrt{3}$ $\therefore\quad \tan\alpha = \frac{\sqrt{3}}{1}$ $\alpha = \frac{\pi}{3}$ $R^2(\sin^2\alpha + \cos^2\alpha) = 1 + 3$ $R = 2$ $2\sin\left(x+\frac{\pi}{3}\right) = \sqrt{2} \qquad : \frac{\pi}{3} \le x + \frac{\pi}{3} \le \frac{7\pi}{3}$ $\sin\left(x+\frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$ $x + \frac{\pi}{3} = \frac{3\pi}{4}$ or $\frac{9\pi}{4}$ $x = \frac{5\pi}{12}$ or $\frac{23\pi}{12}$	3 marks – correct solution 2 marks – 2 solutions taking into consideration the domain 1 mark – one correct solution only.
d	coefficient of x^3 in $(1+x)^{m+n}$ $= {}^{m+n}\mathbf{C}_3 = \binom{m+n}{3}$ coefficient of x^3 in $(1+x)^m(1+x)^n$ $= \binom{m}{3}\binom{n}{0} + \binom{m}{2}\binom{n}{1} + \binom{m}{1}\binom{n}{2} + \binom{m}{0}\binom{n}{3}$ $= \binom{m}{3} + \binom{m}{2}\binom{n}{1} + \binom{m}{1}\binom{n}{2} + \binom{n}{3}$	2 marks – correctly identifying comparing coefficients of x^3 1 mark – incomplete explanation of identifying comparing coefficients of x^3

e-i	$y=\left(\frac{p+q}{2}\right)x-apq$ $x=2a \quad y=3a$ $3a=\left(\frac{p+q}{2}\right)2a-apq$ $3=p+q-pq$ $pq=p+q-3$	1 mark – correct demonstration
e-ii	$M=\left(\frac{2ap+2aq}{2},\frac{ap^2+aq^2}{2}\right)$ $=\left(a(p+q)\,,\frac{a(p^2+q^2)}{2}\right)$ $=\left(a\,(pq+3)\,,\frac{a(p^2+q^2)}{2}\right)$ $(p+q)^2=p^2+2pq+q^2$ $(p+q)^2-2pq=p^2+q^2$ $(pq+3)^2-2pq=p^2+q^2$ $\therefore$ $M=\left(a\,(pq+3)\,,\frac{a}{2}[(pq+3)^2-2pq]\right)$	1 mark correct calculation
	Solution for incorrect M $M=\left(a\,(pq+3)\,,\frac{a}{2}(pq+3)^2-2pq\right)$ $x=a(pq+3)$ $\Rightarrow\ pq+3=\frac{x}{a}$ $pq=\frac{x}{a}-3$ $y=\frac{a}{2}\left(\frac{x}{a}\right)^2-2\left(\frac{x}{a}-3\right)$ $y=\frac{x^2}{2a}-\frac{2x}{a}+6$	2 marks – correct solution 1 mark – correct substitution for either (pq+3) or pq.

	Solution for correct M	
	$M = \left(a(pq+3), \frac{a}{2}[(pq+3)^2 - 2pq] \right)$ $x = a(pq+3)$ $\Rightarrow\ pq + 3 = \frac{x}{a}$ $pq = \frac{x}{a} - 3$ $y = \frac{a}{2}\left(\frac{x}{a}\right)^2 - \frac{a}{2} \times 2\left(\frac{x}{a} - 3\right)$ $y = \frac{x^2}{2a} - x + 3a$	

Question 13

13 a)	For $n = 1$, $3^{2n} - 2^{2n} = 3^2 - 2^2 = 5$ $\therefore S(1)$ is true Assume $S(k): 3^{2k} - 2^{2k} = 5M,\ M \in \mathbb{Z}$ $\therefore \quad 3^{2k} = 5M + 2^{2k}$ Show $S(k) \Rightarrow S(k+1): 3^{2k+2} - 2^{2k+2} = 5N,\ N \in \mathbb{Z}$ $3^{2k+2} - 2^{2k+2} = 3^2(3^{2k}) - (2^2)2^{2k}$ $= 9[5M + 2^{2k}] - 4(2^{2k})$ $= 45M + 9(2^{2k}) - 4(2^{2k})$ $= 45M + 5(2^{2k}) = 5[9M + 2^{2k}]$ $= 5N$ where $N \in \mathbb{Z}$ $\therefore S(k) \Rightarrow S(k+1)$.	**3 Marks:** Correct solution **2 Marks:** Correct *S(k)* and correct *S(k+1)* **1 Mark:** Correct *S(1)* and *S(k)*.
13 b)	$x = \cos\theta \rightarrow \frac{dx}{d\theta} = -\sin\theta$ $x = \frac{1}{2} \rightarrow \cos\theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{3}$ $x = 1 \rightarrow \cos\theta = 1 \rightarrow \theta = 0$ $\int_{\frac{1}{2}}^{1} \frac{\sqrt{1-x^2}}{x^2}\,dx = \int_{\frac{\pi}{3}}^{0} \frac{\sqrt{1-\cos^2\theta}}{\cos^2\theta}.-\sin\theta\,d\theta$ $= \int_{\frac{\pi}{3}}^{0} \frac{\sqrt{\sin^2\theta}}{\cos^2\theta}(-\sin\theta)\,d\theta = \int_{\frac{\pi}{3}}^{0} \frac{-\sin^2\theta}{\cos^2\theta}\,d\theta$ $= \int_{0}^{\frac{\pi}{3}} \tan^2\theta\,d\theta = \int_{0}^{\frac{\pi}{3}} [\sec^2\theta - 1]\,d\theta$ $= \left[\tan\theta - \theta\right]_0^{\frac{\pi}{3}} = \tan\frac{\pi}{3} - \frac{\pi}{3}$ $= \sqrt{3} - \frac{\pi}{3}$	**3 Marks:** Correct solution. **2 Marks:** Correct integrand and correct primitive function. **1 Mark:** Correct substitution

13 c)	From $\left(2x^2+\frac{m}{x}\right)^7$: $a=2x^2$, $b=\frac{m}{x}$,$n=7$ $T_{k+1}={}^7\mathbf{C}_k(2x^2)^{7-k}\left(\frac{m}{k}\right)^k={}^7\mathbf{C}_k(2^{7-k})(m^k)(x^{14-3k})$ $x^4T_{k+1}={}^7\mathbf{C}_k(2^{7-k})(m^k)(x^{18-3k})\Rightarrow 18-3k=0$ $\therefore\quad k=6$ ${}^7\mathbf{C}_6\,(2\,)(m^6)=896\rightarrow 14m^6=896\rightarrow m^6=64$ $\therefore\quad m=2$	**3 Marks:** Correct solution **2 Marks:** Correct T_{k+1} and $k=6$ **OR** Correct k and m from incorrect T_{k+1} **1 Mark:** Correct expression for T_{k+1}
13 d)	$2x^3-3x+1=0$ $\rightarrow\quad \alpha+\beta+\gamma=0,\ \alpha\beta+\alpha\gamma+\beta\gamma=-\frac{3}{2},\ \alpha\beta\gamma=-\frac{1}{2}$ $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\alpha\beta+\alpha\gamma+\beta\gamma}{\alpha\beta\gamma}=\frac{-\frac{3}{2}}{-\frac{1}{2}}=3$	**2 Marks:** Correct solution **1 Mark:** Correct values for sums and products of roots
13 e)	$\tan\theta=\frac{y}{1000}\rightarrow y=1000\tan\theta$ $\frac{dy}{dt}=\frac{d}{dt}(1000\tan\theta)=\frac{d}{d\theta}(1000\tan\theta).\frac{d\theta}{dt}$ $\frac{dy}{dt}=(1000\sec^2\theta)\frac{d\theta}{dt}=1000[1+\tan^2\theta]\frac{d\theta}{dt}$ $\frac{dy}{dt}=1000\left[1+\left(\frac{y}{1000}\right)^2\right]\frac{d\theta}{dt}$ $\frac{dy}{dt}=230,\ y=3240\rightarrow 230=1000[1+3\cdot24^2]\frac{d\theta}{dt}$ $\frac{d\theta}{dt}=\frac{0\cdot23}{1+3\cdot24^2}=0\cdot02\text{ rad }s^{-1}$	**4 Marks:** Correct solution **3 Marks:** Correct expression for *dy/dt* and relevant attempt to find $\frac{d\theta}{dt}$ **2 Marks:** Correct expression for *dy/dt.* **1 Mark:** Obtains $\frac{d}{dt}(1000\tan\theta)$ or equivalent expression

Student Number: ____________________

Question 14

a (i)	$\dfrac{d\left(\frac{1}{2}v^2\right)}{dx} = 8x^3 + 8x$ $\frac{1}{2}v^2 = \int (8x^3 + 8x)\ dx$ $\frac{1}{2}v^2 = 2x^4 + 4x^2 + C$ $t = 0, \quad x = 0 \quad v = -2$ $\therefore C = 2$ $v^2 = 4(x^4 + 2x^2 + 1)$ $v^2 = 4(x^2 + 1)^2$ $v = \pm 2(x^2 + 1)$ $v = -2(x^2 + 1)$ $x = 0 \qquad v = -2$	2 marks correct solution and working 1 mark for C
(ii)	$\dfrac{dx}{dt} = -2(x^2 + 1)$ $\dfrac{dt}{dx} = \dfrac{1}{-2(x^2 + 1)}$ $t = -\frac{1}{2}\int \frac{1}{x^2 + 1} dx$ $t = -\frac{1}{2}\tan^{-1} x + C$ $x = 0 \qquad t = 0$ $C = 0$ $t = -\frac{1}{2}\tan^{-1} x$ $-2t = \tan^{-1} x$ $x = \tan(-2t)$ $x = -\tan(2t)$	2 marks for correct working and solution 1 mark for $t = -\frac{1}{2}\tan^{-1} x + C$ but incorrect solution.
b	$1.3te^{-0.3t} = 0.1$ $1.3te^{-0.3t} - 0.1 = 0$ $C(t) = 1.3te^{-0.3t} - 0.1$ $C'(t) = 1.3e^{-0.3t} - 0.39te^{-0.3t} \qquad product \quad rule$ $t = 15$ $\therefore t_1 = 15 - \dfrac{C(15)}{C'(15)} \approx 17.3$	3 marks for correct solution and working. 2 marks for correct C'(t) and obtaining an answer of approx. 19. However incorrect C(t) 1 mark for correct C(t) or C'(t), however

		incorrect approximation. No marks were given for making the question easier due to incorrect C(t) or C'(t).
c (i)	$T = S + Ae^{-kt}$ $\frac{dT}{dt} = -kAe^{-kt}$ $= -k(T - S)$ $\therefore$ *a* *solution*	1 mark for correct solution.
(ii)	$T = 22 + Ae^{-kt}$ $75 = 22 + Ae^{o}$ $A = 53$ $T = 22 + 53e^{-kt}$ $65 = 22 + 53e^{-5k}$ $\frac{43}{53} = e^{-5k}$ $-5k = \ln\left(\frac{43}{53}\right)$ $k \approx 0.041818...$ $T = 22 + 53e^{-15(0.041818)}$ $T \approx 50^{o}C$ *nearest* deg*ree*	3 marks for correct solution and working. 2 mark for correct A and k value but incorrect T. 1 mark for correct A value. Carry on marks were given for incorrect A value but correct k and T value.
d (i)	$y = Vt\sin\theta - \frac{1}{2}gt^{2}$ *highest* *po*int $\dot{y} = 0$ $\dot{y} = V\sin\theta - gt$ $t = \frac{V\sin\theta}{g}$	1 mark for correct solution.

(ii)	Method 1 At $t = \dfrac{V\sin\theta}{g}$ $x = k$ $y = h$ $k = V\cos\theta \dfrac{V\sin\theta}{g}$ $h = (V\sin\theta)\dfrac{V\sin\theta}{g} - \dfrac{1}{2g}\left(\dfrac{V\sin\theta}{g}\right)^2$ $k = \dfrac{V\sin\theta\cos\theta}{g}$ $h = \dfrac{V^2\sin^2\theta}{2g}$ $\dfrac{h}{k} = \dfrac{V^2\sin^2\theta}{2g} \times \dfrac{g}{V\sin\theta\cos\theta} = \dfrac{\tan\theta}{2}$ $\tan\theta = \dfrac{h}{2k}$ $\therefore$ $\sin\theta = \dfrac{2h}{\sqrt{h^2 + 4k^2}}$ $\cos\theta = \dfrac{k}{\sqrt{h^2 + 4k^2}}$ Given $h = \dfrac{V^2\sin^2\theta}{2g}$ $V^2 = \dfrac{2gh}{\sin^2\theta}$ $= 2gh \times \left(\dfrac{\sqrt{h^2 + 4k^2}}{2h}\right)^2$ $= 2gh \times \dfrac{h^2 + 4k^2}{4h^2}$ $= \dfrac{g}{2h}(h^2 + 4k^2)$	Method 1 1 mark for both k and h expressions 1 mark for correct sin and cos expressions 1 mark for obtaining required expression. Method 2 1 mark for sin2(theta) And cos2(theta) 1 mark for writing Pythagorean identity. 1 mark for obtaining correct expression

	Method 2 $x = V\cos\theta\ t \qquad y = Vi\sin\theta\ t - \frac{1}{2}gt^2$ At $t = \dfrac{V\sin\theta}{g} \Rightarrow x = k\ ,y \Rightarrow h$ therefore $k = V\cos\theta\dfrac{V\sin\theta}{g}$ $V^2 = \dfrac{\text{kg}}{\cos\theta\sin\theta}$ $= = \dfrac{2\text{kg}}{2\cos\theta\sin\theta}$ $= \dfrac{2\text{kg}}{\sin(2\theta)}$ $\therefore$ $\sin(2\theta) = \dfrac{2\text{kg}}{V^2}$ Eqn 1 $h = V\sin\theta\dfrac{V\sin\theta}{g} - \dfrac{g}{2}.\dfrac{v^2\sin^2\theta}{g^2}$ $= \dfrac{V^2\sin^2\theta}{2g}$ $\therefore$ $\sin^2\theta = \dfrac{2gh}{V^2}$ $\frac{1}{2}(1 - \cos 2\theta) = \dfrac{2gh}{V^2}$ $1 - \cos 2\theta = \dfrac{4gh}{V^2}$ $\cos 2\theta = 1 - \dfrac{4gh}{V^2}$ Eqn 2	

	$(\sin 2\theta)^2 + (\cos 2\theta)^2 = 1$ $\left(\frac{2\text{kg}}{V^2}\right)^2 + \left(1 - \frac{4gh}{V^2}\right)^2 = 1$ $\frac{4k^2g^2}{V^4} + \frac{(V^2 - 4gh)^2}{V^4} = 1$ $4k^2g^2 + (V^2 - 4gh)^2 = V^4$ $4k^2g^2 + v^4 - 8V^2gh + 16g^2h^2 = V^4$ $4k^2g^2 - 8V^2gh + 16g^2h^2 = 0$ $4k^2g^2 + 16g^2h^2 = 8V^2gh$ $\frac{4k^2g^2 + 16g^2h^2}{8gh} = V^2$ $\frac{4g^2(k^2 + 4h^2)}{8gh} = V^2$ $\frac{g(k^2 + 4h^2)}{2h} = V^2$ $V^2 = \frac{g}{2h}(4h^2 + k^2)$	