Probability

Definitions

Probability: the chance of something happening

Experimental Probability: (relative frequency); probability based upon collected data

e.g. 5 coins are tossed and they land **HHTTH**

Experimental probability of tossing a head would be $\frac{3}{5}$

Theoretical probability of tossing a head is $\frac{1}{2}$

As the number of times an event is repeated $\rightarrow \infty$

experimental probability $\rightarrow$ theoretical probability

Sample Space: all possible outcomes

Event Space: all possible ways a particular event can occur

Equally Likely Events: events which have an equal chance of happening

Mutually Exclusive Events: only one possible outcome can occur at any one time.

e.g. a coin can be either a head or a tail, not both

Non-Mutually Exclusive Events: more than one outcome could possibly happen at any one time

e.g. a number could be both even and a multiple of three

$P(E)$: probability of E happening

$P(\overline{E})$: probability of E not happening

$\overline{E}$ is called the **complementary event**

Probability Theory

$$0 \le P(E) \le 1$$

P(E)=0: impossible.
E never happens

P(E)=1: a certain event.
E must happen

If all events are equally likely
(sample space is uniform)

$$P(E) = \frac{|E|}{|S|}$$

$|E|$: the number of times E occurs (size of the event space)

$|S|$: total number of possibilities (size of the sample space)

$$P(\overline{E}) = 1 - P(E)$$

Listing Sample Spaces

In multi-stage experiments, listing a sample space helps when calculating all the possible outcomes

(1) create a list – sometimes simply writing down every possibility is the simplest way of working out a sample space

2004 Mathematics HSC Q6c)

In a game, a turn involves rolling two dice, each with faces marked 0, 1, 2, 3, 4 and 5. The score for each turn is calculated by multiplying the two numbers uppermost on the dice.

0 X 0 = 0	1 X 0 = 0	2 X 0 = 0	3 X 0 = 0	4 X 0 = 0	5 X 0 = 0
0 X 1 = 0	1 X 1 = 1	2 X 1 = 2	3 X 1 = 3	4 X 1 = 4	5 X 1 = 5
0 X 2 = 0	1 X 2 = 2	2 X 2 = 4	3 X 2 = 6	4 X 2 = 8	5 X 2 = 10
0 X 3 = 0	1 X 3 = 3	2 X 3 = 6	3 X 3 = 9	4 X 3 = 12	5 X 3 = 15
0 X 4 = 0	1 X 4 = 4	2 X 4 = 8	3 X 4 = 12	4 X 4 = 16	5 X 4 = 20
0 X 5 = 0	1 X 5 = 5	2 X 5 = 10	3 X 5 = 15	4 X 5 = 20	5 X 5 = 25

(i) What is the probability of scoring zero on the first turn?

$$P(=0) = \frac{11}{36}$$

(ii) What is the probability of scoring 16 or more on the first turn?

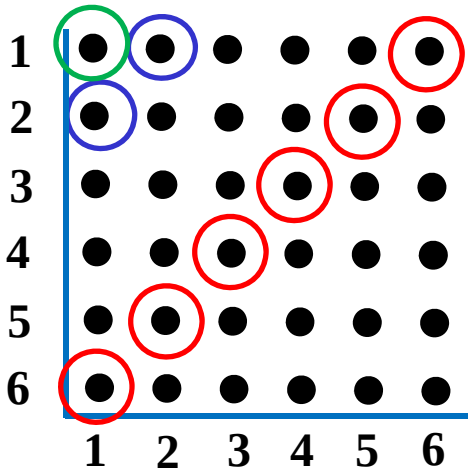
$$P(\geq 16) = \frac{4}{36} = \frac{1}{9}$$

(2) dot diagram– useful for listing sample space of a two-stage experiment

e.g. A pair of dice are thrown. What is the probability that they;

(*i*) total 3?

(*ii*) total 7?



$$(i)\ P(=3)=\frac{2}{36}=\frac{1}{18}$$

$$(ii)\ P(=7)=\frac{6}{36}=\frac{1}{6}$$

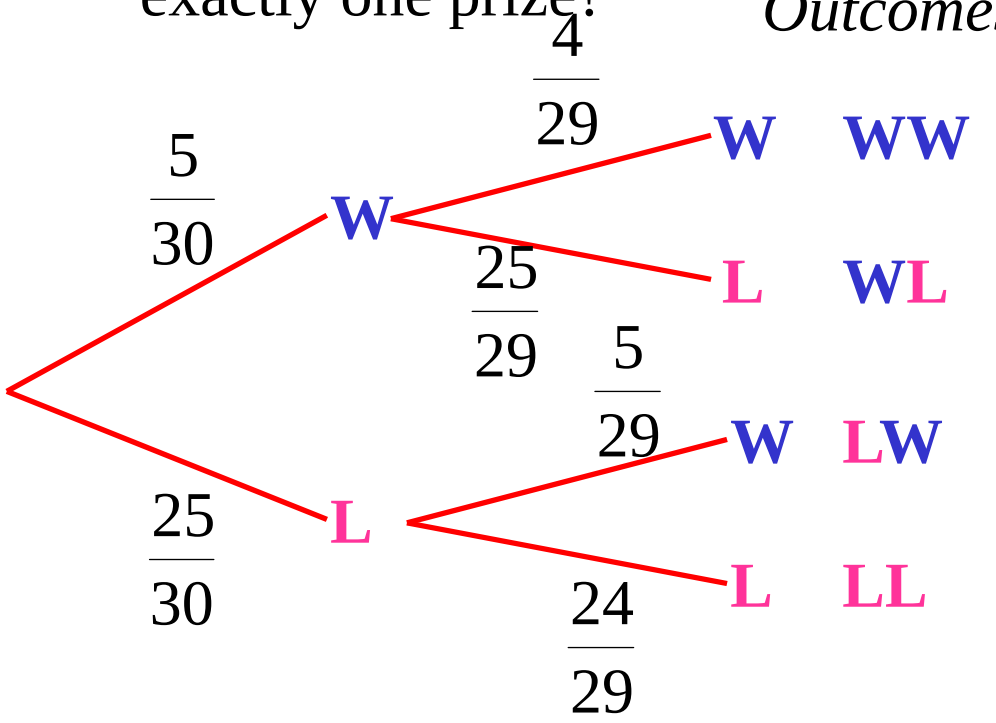
(iii) What is the probability of totaling at least 3?

$$P(\geq 3)=1-P(1\text{ or }2)$$

$$=1-\frac{1}{36}\quad=\frac{35}{36}$$

(3) tree diagram– useful for listing sample space of any multi-stage experiment, also when events are not equally likely

e.g. In a raffle 30 tickets are sold and there are 2 prizes.
What is the probability that someone buying 5 tickets wins exactly one prize?



Outcomes *Probabilities*

WW: $\frac{2}{87}$

WL: $\frac{25}{174}$

LW: $\frac{25}{174}$

LL: $\frac{20}{29}$

$P(1\,prize) = P(WL) + P(LW)$

$= \frac{25}{87}$

(*ii*) What is the probability of winning at least one prize?

$P(\geq 1\,prize) = 1 - P(0\,prizes)$

$= \frac{9}{29}$

Exercise 12A; 3, 5, 6, 7, 8, 10, 12, 13,
14 to 17 ace etc,
18, 19, 21

Exercise 12B; 3, 4, 5, 6, 7, 8,
11, 12, 13