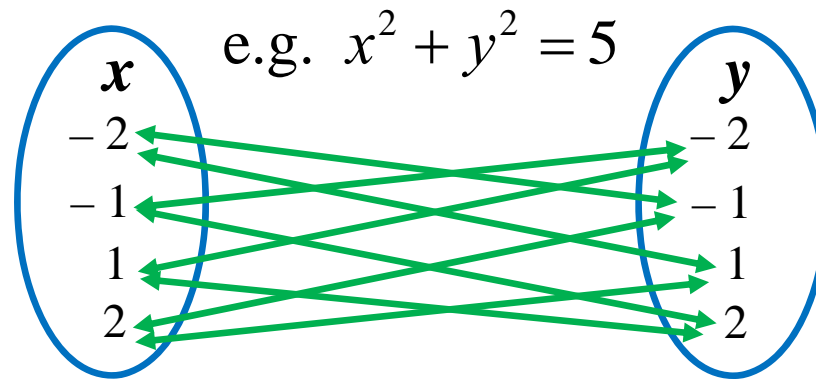


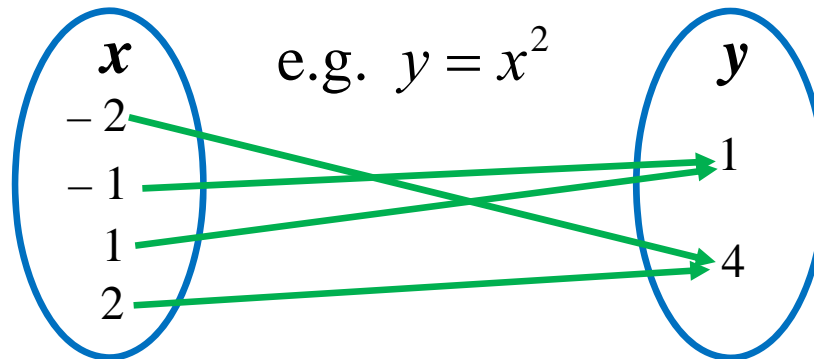
Functions

Definitions:

relation – a rule that maps between two sets of values



function – a type of relation that uniquely maps from one set of values to another set of values



independent variable – the “*input*” of the function

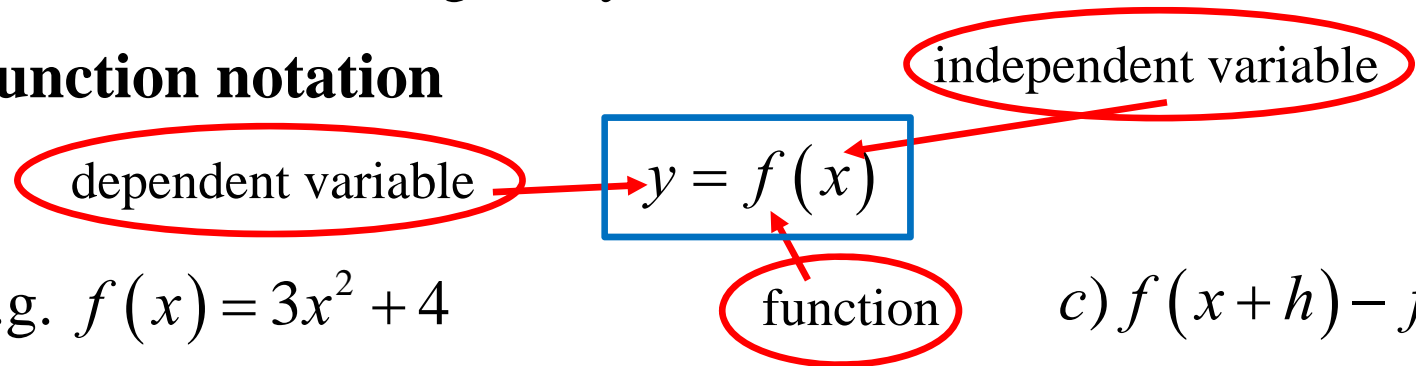
domain– *natural domain*: all possible values of the independent variable that can be substituted into the function

restricted domain: domains are restricted when some values are not required e.g. if independent variable represents time, then only $t \geq 0$ would be needed

dependent variable– the “*output*” of the function, its value *depends* upon the value of the independent variable

range– all possible values the dependent variable can take; obtained by substituting every value in the domain

function notation



e.g. $f(x) = 3x^2 + 4$

a) $f(5) = 3(5)^2 + 4$
 $= 75 + 4$
 $= 79$

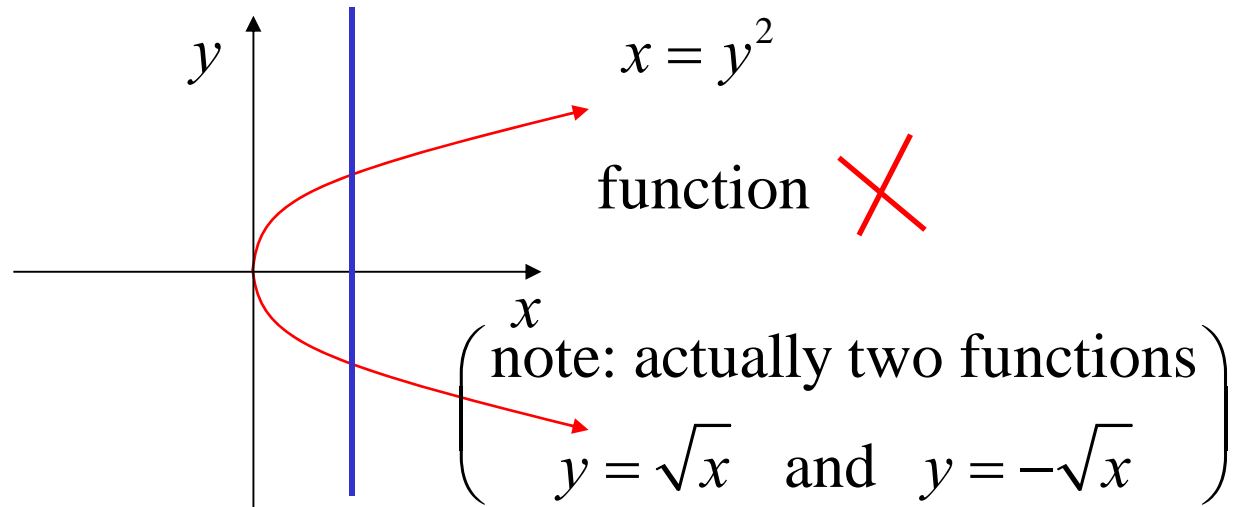
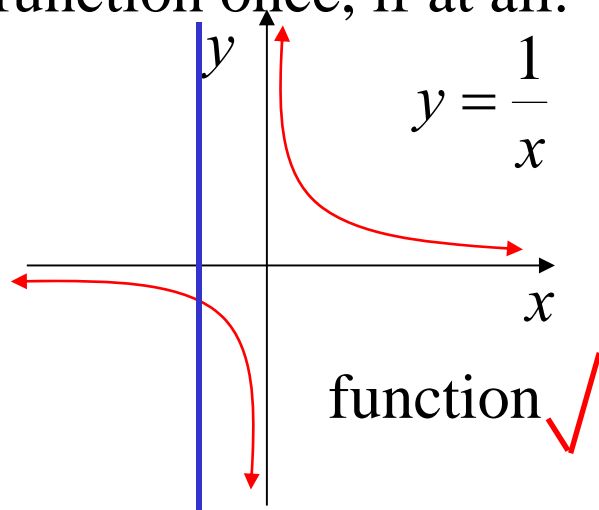
b) $f(a) = \underline{3a^2 + 4}$

c) $f(x+h) - f(x)$

$= 3(x+h)^2 + 4 - (3x^2 + 4)$
 $= 3x^2 + 6xh + 3h^2 + 4 - 3x^2 - 4$
 $= 6xh + 3h^2$

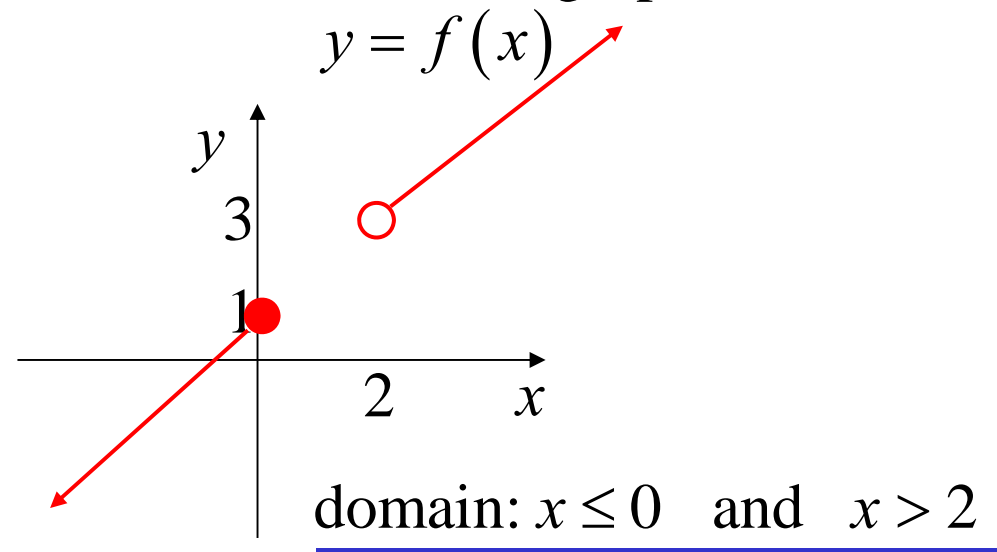
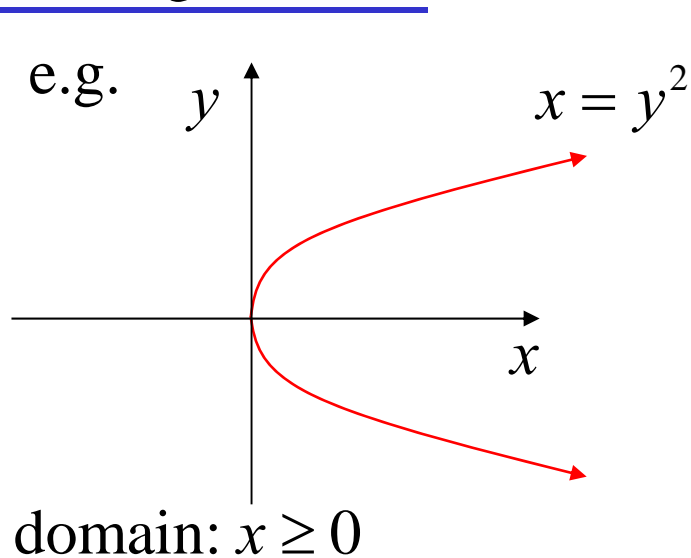
Vertical Line Test for Functions

If a straight line is drawn parallel to the y axis, it will only cross a function once, if at all.



Domain and Range $y = f(x)$

Finding Domain: Geometrically it can be read from the graph



Algebraically; things to look for:

1. Fractions: bottom of fraction $\neq 0$

e.g. (i) $y = \frac{1}{x}$
 $x \neq 0$

domain: all real x except $x = 0$

(iii) $y = \frac{4x}{x-1} + \frac{3}{7-x}$

$$x-1 \neq 0 \quad 7-x \neq 0$$

$$x \neq 1 \quad x \neq 7$$

domain: all real x except $x = 1$ or 7

(ii) $y = \frac{1}{x^2-1}$

$$x^2-1 \neq 0$$

$$x^2 \neq 1$$

$$x \neq \pm 1$$

domain: all real x except $x = \pm 1$

2. Root Signs: you can't find the square root of a negative number.

e.g. (i) $y = \sqrt{4 - x^2}$

$$4 - x^2 \geq 0$$

$$x^2 \leq 4$$

domain: $-2 \leq x \leq 2$

(ii) $y = \sqrt{x + 3} - \sqrt{5 - x}$

$$x + 3 \geq 0 \quad 5 - x \geq 0$$

$$x \geq -3 \quad x \leq 5$$

domain: $-3 \leq x \leq 5$

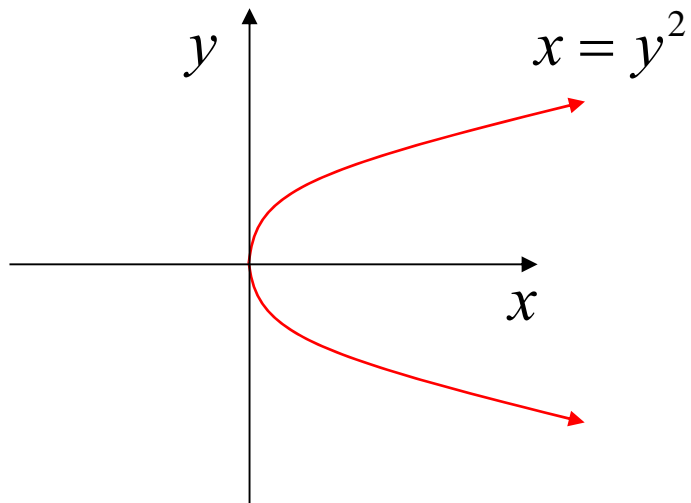
(iii) $y = \frac{1}{\sqrt{x + 2}}$

$$x + 2 > 0$$

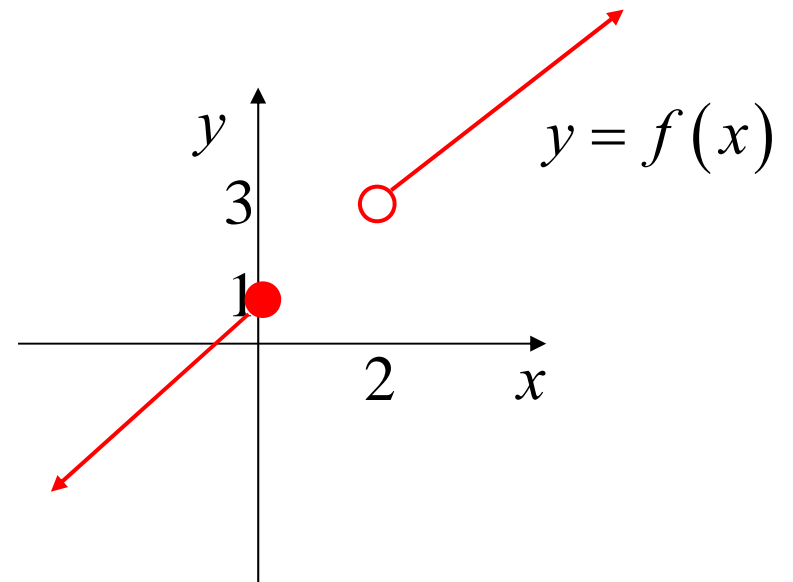
domain: $x > -2$

Range: Geometrically

e.g.



range: all real y



range: $y \leq 1$ and $y > 3$

Algebraically; things to look for:

1. Maximum/Minimum values: even powers and absolute values
are always ≥ 0

e.g. (i) $y = x^2$

range: $y \geq 0$

(ii) $y = x^2 + 3$

$y \geq 0 + 3$

range: $y \geq 3$

(iii) $y = 5 - x^2$

$y \leq 5 - 0$

range: $y \leq 5$

(iv) $y = |x + 2|$

range: $y \geq 0$

(v) $y = |x + 2| - 5$

$y \geq 0 - 5$

range: $y \geq -5$

2. Restrictions on Domain: sub in endpoints and centre of domain

e.g. $y = \sqrt{4 - x^2}$ when $x = 2, y = \sqrt{4 - 2^2} = 0$ when $x = 0, y = \sqrt{4 - 0^2} = 2$
domain: $-2 \leq x \leq 2$ range: $0 \leq y \leq 2$

3. Fractions: If you have a constant on the top of the fraction, fraction $\neq 0$

e.g. (i) $y = \frac{1}{x}$
 $y \neq 0$

range: all real y except $y = 0$

(ii) $y = 5 + \frac{1}{x}$
 $y \neq 5 + 0$

range: all real y except $y = 5$

(iii) $y = \frac{x+7}{x+4}$ $x+4 \overline{) \frac{x+7}{x+4}} \frac{1}{3}$
 $y = 1 + \frac{3}{x+4}$
 $y \neq 1 + 0$

range: all real y except $y = 1$

**Exercise 3A; 1a, 2b, 3c, 4d, 5d,
7aceg, 8c, 9b, 10bd, 11, 12bc, 13a,
14bd, 15ac, 16ac, 17**

**Exercise 3B; 2, 3, 4b, 5b, 6d, 8adk,
9a, 10, 11, 12, 14adf, 15**