Representing Real Numbers

All real numbers can be placed on the number line and described;

- geometrically *(using a picture of the number line)*
- algebraically *(using an inequation or equation)*
- using interval notation *(often used when describing domain & range)*
- using set notation *(formal way of describing all possible numbers)*

Types of Intervals

(i) bounded: interval has two endpoints

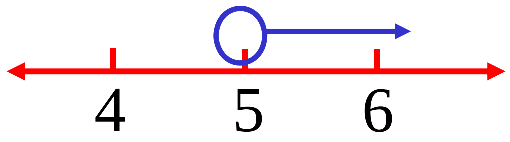
(ii) unbounded: interval has one endpoint

(iii) closed: all endpoints are included

(iv) open: an endpoint is not included

(v) degenerate: a single point

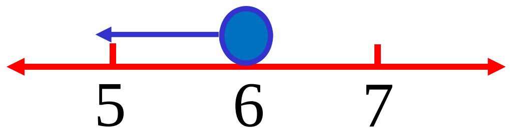
e.g. $(i)\ x > 5$



open unbounded interval

$(5, \infty) \ = \{x : x > 5\}$

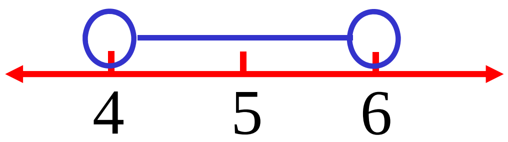
$(ii)\ x \leq 6$



closed unbounded interval

$(-\infty, 6] = \{x : x \leq 6\}$

$(iii)\, 4 < x < 6$



open bounded interval

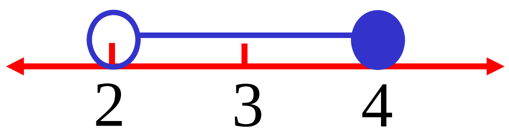
$(4, 6) = \{x : 4 < x < 6\}$

$(iv)\, -2 \leq x \leq 1$

closed bounded interval

$[-2, 1] = \{x : -2 \leq x \leq 1\}$

$(v)\, 2 < x \leq 4$



bounded interval

$(2, 4] = \{x : 2 < x \leq 4\}$

Rational Numbers

Rational numbers can be expressed in the form $\frac{a}{b}$ where a and b are integers.

Irrational Numbers

Irrational numbers are numbers which are not rational.

All irrational numbers can be expressed as a unique infinite decimal.

$e.g.$ Prove $\sqrt{2}$ is irrational

"Proof by contradiction"

Assume $\sqrt{2}$ is rational

$\therefore \sqrt{2} = \dfrac{a}{b}$ where a and b are integers with no common factors

$b\sqrt{2} = a$

$2b^2 = a^2$

Thus a^2 must be divisible by 2

As prime factors of squares must appear in pairs, any square that is divisible by 2 is also divisible by 4

Thus a^2 must be divisible by 4

$\therefore \; 2b^2 = 4k$ where k is an integer

$b^2 = 2k$

So a^2 and b^2 are both divisible by 2 and must have a common factor

However, a and b have no common factors

so $\sqrt{2}$ is not rational

$\therefore \sqrt{2}$ is irrational

Significant Figures

Irrational numbers cannot be calculated exactly, so sometimes an approximation is required.

When approximating we write a number correct to either;

- a certain number of decimal places; ***OR***
- a certain number of significant figures

Rounding off to a given number of significant figures

Start at the first **non-zero** digit and count to the required number and round *(if the answer is ambiguous, scientific notation should be used)*

e.g. Write correct to the given number of significant figures

$(i)\ 0.050703\ (2) = \underline{0.051}$

$(ii)\ 0.050703\ (3) = \underline{0.0507}$

$(iii)\ 0.050703\ (4) = \underline{0.05070}$

$(iv)\ 3000\ (2) = \underline{3000} \quad = \underline{3.0 \times 10^3}$

$(vii)\ 3000\ (3) = \underline{3000} \quad = \underline{3.00 \times 10^3}$

$(vi)\ 3000\ (4) = \underline{3000} \quad = \underline{3.000 \times 10^3}$

Finding the number of significant figures

Start at the first **non-zero** digit and count the number of digits until the end of the number

e.g. (i) 0.050703 (5 significant figures)

(ii) 0.010031 (5 significant figures)

(iii) 0.0100310 (6 significant figures)

(iv) 1200 (2, 3 or 4 significant figures)

(4 significant figures)

*If the answer is ambiguous use the **largest** answer*

Exercise 2B; 1cdfikl, 3, 6, 7, 8, 11hkl, 14, 15