The Quadratic Polynomial and the Parabola

Quadratic polynomial – $ax^2 + bx + c$

Quadratic function – $y = ax^2 + bx + c$

Quadratic equation – $ax^2 + bx + c = 0$

Coefficients – a, b, c

Indeterminate – x

Roots – Solutions to the quadratic equation

Zeroes – x intercepts of the quadratic function

a quadratic with
$a = 1$
is called **monic**

e.g. Find the roots of $x^2 - 1 = 0$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

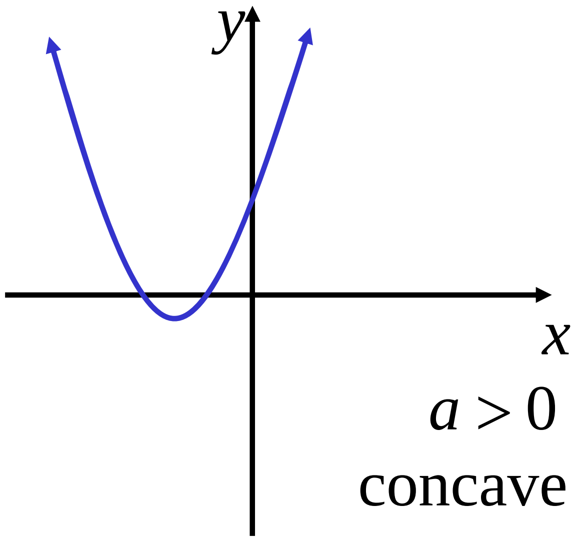
$\therefore$ the roots are $x = -1$ and $x = 1$

Graphing Quadratics

The graph of a quadratic function is a parabola.

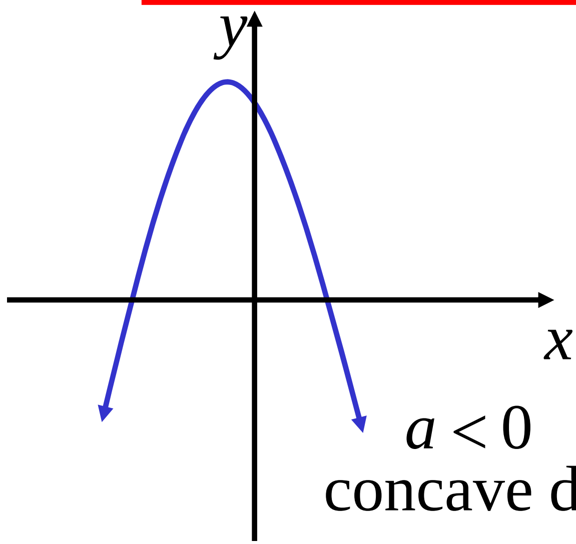
$$y = ax^2 + bx + c$$

a



$a > 0$

concave up



$a < 0$

concave down

c = y intercept

zeroes (roots) = x intercepts

$x = \dfrac{-b}{2a}$ = axis of symmetry

Note: AOS is the average of the zeroes

vertex x value is the AOS

y value is found by substituting AOS into the function.

(It is the maximum/minimum value of the function)

e.g. Graph $y = x^2 + 8x + 12$

a = 1 > 0 ∴ concave up

zeroes $x^2 + 8x + 12 = 0$

$(x+6)(x+2) = 0$

$x = -6$ or $x = -2$

∴ x intercepts are

$(-6, 0)$ and $(-2, 0)$

AOS $x = \frac{-b}{2a}$ ***OR*** $x = \frac{-6-2}{2}$

$= \frac{-8}{2}$ $= -4$

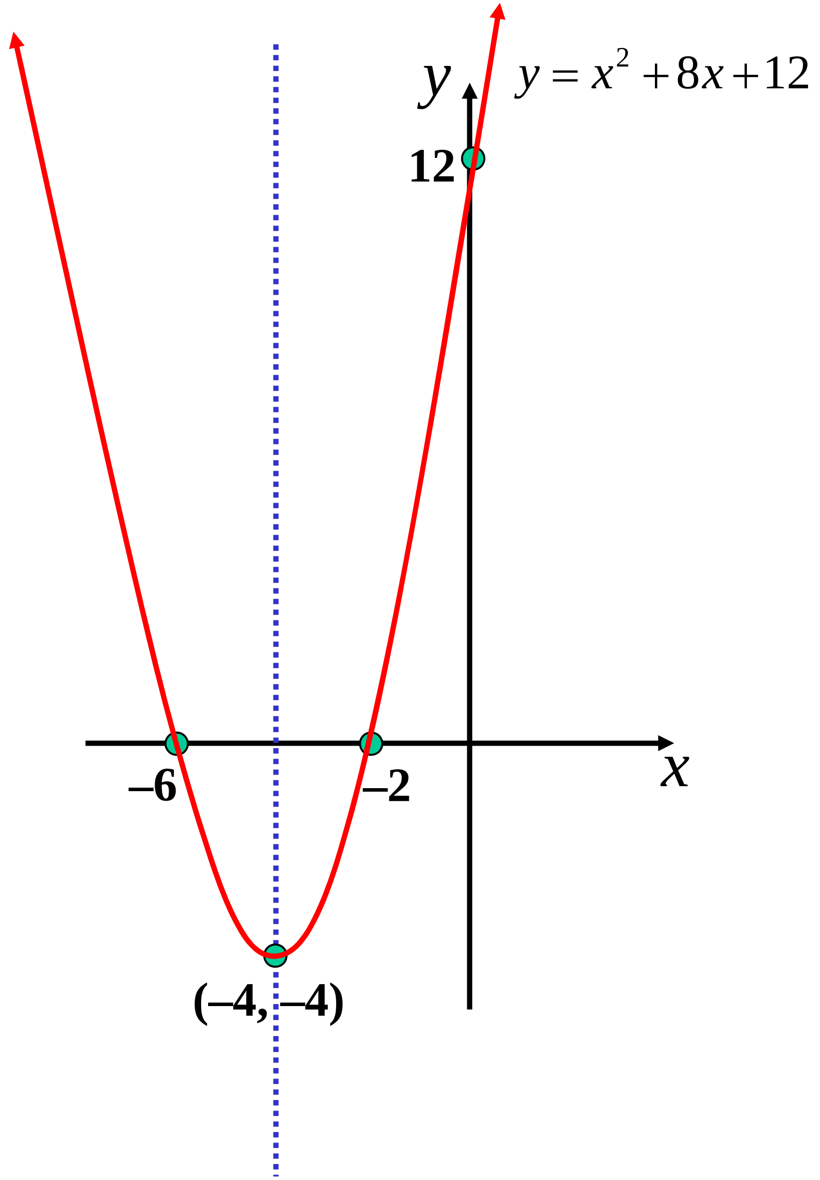
$= -4$

vertex $y = (-4)^2 + 8(-4) + 12$

$= -4$

∴ vertex is $(-4, -4)$

c = 12 ∴ y intercept is $(0, 12)$



(ii) Find the quadratic with;

a) roots 3 and 6

$$y = a\left(x^2 - 9x + 18\right)$$

$-(6+3)$ 6×3

b) monic roots $3+\sqrt{2}$ and $3-\sqrt{2}$

$$y = x^2 - 6x + 7$$

$-\left(3+\sqrt{2}+3-\sqrt{2}\right)$ $\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)$

c) roots 2 and 8 and vertex $(5,3)$

$$y = a\left(x^2 - 10x + 16\right)$$

$$(5,3):\ 3 = a\left(5^2 - 10(5) + 16\right)$$

$$3 = -9a$$

$$a = -\frac{1}{3}$$

$$\therefore y = -\frac{1}{3}\left(x^2 - 10x + 16\right)$$

Exercise 3D; 1a, 2b(ii), 3a,4bd, 6b, 7c, 8a, 9c, 10c,11ad, 12 (*use desmos*), 13b, 14, 15