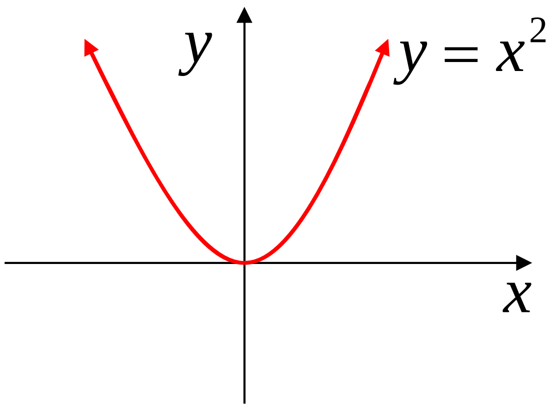
Quadratic Function



The linear function and the **quadratic function** are the building blocks of all polynomials

Every polynomial can be factorised down to a combination of linear and quadratic factors.

All quadratics can be transformed from the basic equation $y = x^2$ using translations, rotations, reflections or a combination of all three.

Recognising the quadratic function

$$y = ax^2 + bx + c$$

power `1′ (the y term)

power `2′ (the x^2 term)

- terms contain at most one variable, one variable is to the power of one, the other variable hs a term to the power of two

Quadratics and Completing the Square

a measures concavity

$$y = a(x-h)^2 + k$$

vertex is (h, k)

e.g. Sketch the parabola $y = x^2 + 8x + 12$

$$y = x^2 + 8x + 12$$
$$= (x+4)^2 - 4$$

$\therefore$ vertex is $(-4, -4)$

x intercepts

$$(x+4)^2 - 4 = 0$$
$$(x+4)^2 = 4$$
$$x + 4 = \pm 2$$
$$x = -4 \pm 2$$
$$x = -6 \text{ or } x = -2$$

$\therefore$ x intercepts are $(-6, 0)$ and $(-2, 0)$

(ii) Write down the quadratic with roots 2 and 8 and vertex $(5,3)$

$$y = k\left\{(x-5)^2\right\} + 3$$
$$(2,0):\ 0 = k\left\{(2-5)^2\right\} + 3$$
$$9k = -3$$
$$k = -\frac{1}{3}$$
$$y = -\frac{1}{3}\left\{(x-5)^2\right\} + 3$$
$$y = -\frac{1}{3}\left(x^2 - 10x + 16\right)$$

Quadratics and the Discriminant

$$\Delta = b^2 - 4ac$$

$$\text{vertex} = \left(\frac{-b}{2a}, \frac{-\Delta}{4a}\right)$$

$$\text{zeroes} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

Note: if $\Delta < 0,$ no x intercepts
$\Delta = 0,$ one x intercept
$\Delta > 0,$ two x intercepts

e.g. Sketch the parabola $y = x^2 + 8x + 12$

$$\Delta = 8^2 - 4(1)(12)$$

$$= 16$$

$$\therefore \text{ vertex} = \left(-\frac{8}{2}, -\frac{16}{4}\right)$$

$$= (-4, -4)$$

Exercise 3E;1a, 2a, 3ace, 4b, 5be, 6ac, 7bc, 8c, 9, 10ace, 11be, 12ac, 13

Exercise 3F; 1a, 3a, 5adf, 8a, 9, 10, 11, 12, 13a