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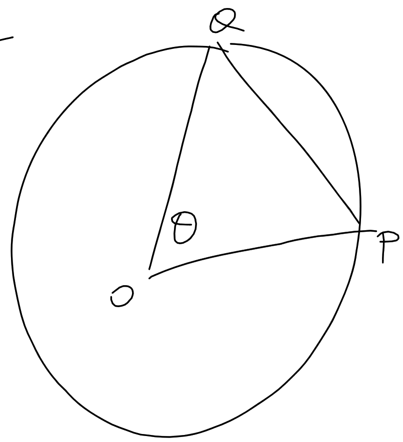

$$\frac{d\theta}{dt} = 0.1 \text{ rad/min}$$

$$A = 18\sin\theta$$

$$\frac{dA}{d\theta} = 18\cos\theta$$

$$\frac{dA}{dt} = \frac{dA}{d\theta} \times \frac{d\theta}{dt}$$

$$= 1.8\cos\theta$$

when $\theta = \frac{\pi}{4}$, $\frac{dA}{dt} = 1.8 \times \frac{1}{\sqrt{2}}$

b) $A_{\triangle} = \frac{1}{2} r^2 \sin\theta$

$$\frac{dA_{\triangle}}{dt} = \frac{dA_{\triangle}}{d\theta} \times \frac{d\theta}{dt}$$

$$= \frac{1}{2} r^2 \cos\theta \times 0.1$$

when $r = 6$, $\theta = \frac{\pi}{4}$, $\frac{dA_{\triangle}}{dt} = \frac{1}{2} \times 36 \times \frac{1}{\sqrt{2}} \times 0.1$

$$= \frac{1.8}{\sqrt{2}}$$

$$= \frac{9}{5\sqrt{2}} \text{ cm}^2/\text{min}$$

c) $A = 18(\theta - \sin\theta)$

$$\frac{dA}{d\theta} = 18(1 - \cos\theta)$$

$$\frac{dA}{dt} = \frac{9}{5}(1 - \cos\theta)$$

$$\frac{d\left(\frac{dA}{dt}\right)}{d\theta} = \frac{9}{5}\sin\theta$$

$$\frac{d}{d\theta}\left(\frac{dA}{dt}\right) = 0$$

$$\sin\theta = 0$$

$$\theta = 0, \pi, 2\pi$$

$$\frac{d^2}{d\theta^2}\left(\frac{dA}{dt}\right) = \frac{9}{5}\cos\theta$$

when $\theta = \pi$, $\frac{d^2}{d\theta^2}\left(\frac{dA}{dt}\right) = -\frac{9}{5} < 0$

$\therefore$ when $\theta = \pi$, $\frac{dA}{dt}$ is max

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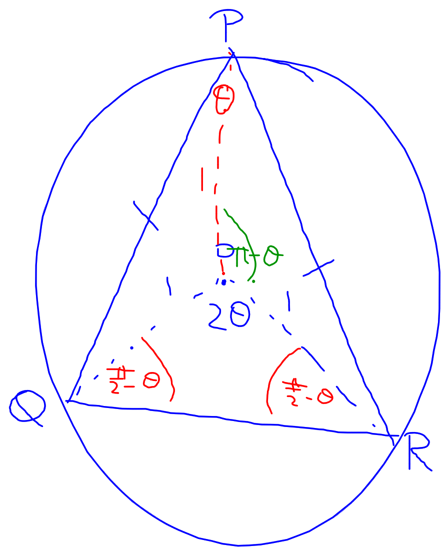


$$PR^2 = 1^2 + 1^2 - 2\cos(\pi-\theta)$$
$$= 2 - 2\cos(\pi-\theta)$$

$$A_{\Delta PQR} = \frac{1}{2} \times PR^2 \times \sin\theta$$
$$= \frac{1}{2}\left(2 - 2\cos(\pi-\theta)\right)\sin\theta$$
$$= \left(1 - \cos(\pi-\theta)\right)\sin\theta$$
$$= \underline{(1+\cos\theta)\sin\theta}$$

$$\frac{dA}{d\theta} = (1+\cos\theta)(\cos\theta) + (\sin\theta)(-\sin\theta)$$

$$= \cos\theta + \cos^2\theta - \sin^2\theta$$

$$= \cos\theta + \cos^2\theta - (1-\cos^2\theta)$$

$$= 2\cos^2\theta + \cos\theta - 1$$

$$= (2\cos\theta - 1)(\cos\theta + 1)$$

stationary pts occur when $\frac{dA}{d\theta} = 0$

$$(2\cos\theta - 1)(\cos\theta + 1) = 0$$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = -1$

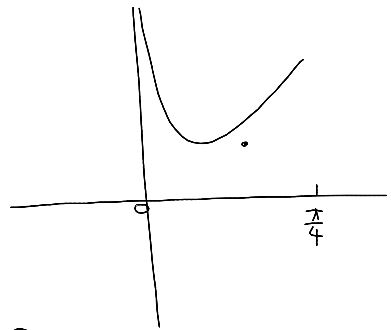
$\theta = \frac{\pi}{3}$ no solutions (θ must be acute)

when $\theta = \frac{\pi}{3}$, $\angle PQR = \angle PRQ = \frac{\pi}{3}$ $(2\sin\theta)$

$\therefore \triangle PQR$ is equilateral

20/ $f(\theta) = \dfrac{2 - \sin\theta}{\cos\theta}$, $\quad 0 \le \theta \le \frac{\pi}{4}$

min tp when $\theta = \frac{\pi}{6}$, $f\left(\frac{\pi}{6}\right) = \sqrt{3}$



$f(0) = \dfrac{2}{1}$
$= 2$

$f\left(\frac{\pi}{4}\right) = \dfrac{2 - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$
$= 2\sqrt{2} - 1$

$\therefore$ minimum is $\sqrt{3}$, maximum is 2