10

12b) $(x+y+z)^2 - 2(xy + xz + yz)$

$= \underline{x^2 + y^2 + z^2}$

$\left[\sum x^2 = \left(\sum x \right)^2 - 2 \sum xy \right]$

10

12d)

$$(a+b+c)(a^2+b^2+c^2-ab-ac-bc)$$

$$= a^3 + \cancel{ab^2} + \cancel{ac^2} - \cancel{a^2b} - \cancel{a^2c} - abc$$
$$+ \cancel{a^2b} + b^3 + \cancel{bc^2} - \cancel{ab^2} - abc - \cancel{b^2c}$$
$$+ \cancel{a^2c} + \cancel{b^2c} + c^3 - abc - \cancel{ac^2} - \cancel{bc^2}$$

$$= \underline{a^3+b^3+c^3-3abc}$$

10

~~12~~d) $(a+b+c)(a^2+b^2+c^2-ab-ac-bc)$

$= \underline{a^3+b^3+c^3-3abc}$

13 ~~1~~ a) Prove the identity.

$$(a+b+c)(ab+bc+ca)-abc = (a+b)(b+c)(c+a)$$

$$LHS = (a+b+c)(ab+bc+ca) - abc$$

$$= a^2b + abc + a^2c + ab^2 + b^2c + abc + abc + bc^2 + ac^2 - abc$$

$$= a^2b + a^2c + ab^2 + b^2c + ac^2 + bc^2 + 2abc$$

$$RHS = (a+b)(b+c)(c+a)$$

$$= (ab + ac + b^2 + bc)(c+a)$$

$$= abc + ac^2 + b^2c + b^2c^2 + a^2b + a^2c + ab^2 + abc$$

$$= LHS$$

14 If $(a+b)^2+(b+c)^2+(c+d)^2=4(ab+bc+cd)$

Prove $a=b=c=d$

$$(a-b)^2+(b-c)^2+(c-d)^2=0$$

but $x^2 \geq 0$

$\therefore (a-b)^2=0 \quad (b-c)^2=0 \quad (c-d)^2=0$

$a=b \quad b=c \quad c=d$

ie $a=b=c=d$