8b) $x^6 - 64$

$$= (x^3 - 8)(x^3 + 8)$$

$$= (x - 2)(x^2 + 2x + 4)(x + 2)(x^2 - 2x + 4)$$

$x^6 - 64$

$$= (x^2 - 4)(x^4 + 4x^2 + 16)$$

$$= (x + 2)(x - 2)\left[x^4 + 8x^2 + 16 - 4x^2\right]$$

8d) $(x+y)^3 - (x-y)^3$

$= (x+y-x+y)\left[(x+y)^2 + (x+y)(x-y) + (x-y)^2\right]$

$= \underline{2y(3x^2+y^2)}$

8 i)

$$u^7 + u^6 + u + 1$$

$$= u^6(u+1) + 1(u+1)$$

$$= (u+1)(u^6+1)$$

$$= \underline{(u+1)(u^2+1)(u^4 - u^2 + 1)}$$

8k) $x^7 - x^3 + 8x^4 - 8$

$= x^3(x^4-1) + 8(x^4-1)$

$= (x^4-1)(x^3+8)$

$= (x^2-1)(x^2+1)(x+2)(x^2-2x+4)$

$= \underline{(x-1)(x+1)(x^2+1)(x+2)(x^2-2x+4)}$

8 l)

$$a^5 + a^4 + a^3 + a^2 + a + 1$$

$$= a^3(a^2 + a + 1) + 1(a^2 + a + 1)$$

$$= (a^2 + a + 1)(a^3 + 1)$$

$$= \underline{(a^2 + a + 1)(a + 1)(a^2 - a + 1)}$$

9f)

$$\frac{1+x+x^2}{1-x^3} + \frac{x-x^2}{(1-x)^3}$$

$$= \frac{(1+x+x^2)}{(1-x)(1+x+x^2)} + \frac{x(1-x)}{(1-x)^3}$$

$$= \frac{1}{(1-x)} + \frac{x}{(1-x)^2}$$

$$= \frac{1-x+x}{(1-x)^2}$$

$$= \underline{\underline{\frac{1}{(1-x)^2}}}$$

11a)

$$x^7 + x$$

$$= x(x^6+1)$$

$$= x((x^2)^3+1)$$

$$= x(x^2+1)(x^4 - x^2+1)$$

$$= x(x^2+1)[x^4+2x^2+1-3x^2]$$

$$= x(x^2+1)[(x^2+1)^2-(\sqrt{3}x)^2]$$

$$= x(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)$$

11b)

$$x^{12} - y^{12}$$

$$= (x^4)^3 - (y^4)^3$$

$$= (x^4 - y^4)(x^8 + x^4y^4 + y^8)$$

$$= (x^2+y^2)(x+y)(x-y)(x^8+2x^4y^4+y^8 - x^4y^4)$$

$$= (x^2+y^2)(x+y)(x-y)\left[(x^4+y^4)^2 - (x^2y^2)^2\right]$$

$$= (x^2+y^2)(x+y)(x-y)(x^4-x^2y^2+y^4)(x^4+x^2y^2+y^4)$$

$$= (x^2+y^2)(x+y)(x-y)\left[(x^2+y^2)^2 - 3x^2y^2\right]\left[(x^2+y^2)^2 - x^2y^2\right]$$

$$= \underline{(x^2+y^2)(x+y)(x-y)(x^2+\sqrt{3}xy+y^2)(x^2-\sqrt{3}xy+y^2)(x^2+xy+y^2)(x^2-xy+y^2)}$$

12/ $x+y=1$ $\quad$ $x^3+y^3=19$

$(x+y)(x^2-xy+y^2)=19$

$x^2-xy+y^2=19$

$x^2+2xy+y^2=1$

$-3xy=18$

$xy=-6$

$\therefore x^2+6+y^2=19$

$\underline{\underline{x^2+y^2=13}}$

13/ $(x-y)^3 + (x+y)^3 + 3(x-y)^2(x+y) + 3(x+y)^2(x-y)$

$= (x-y)^3 + 3(x-y)^2(x+y) + 3(x-y)(x+y)^2 + (x+y)^3$

$= (x-y+x+y)^3$

$= 8x^3$

14 Show $a+b+c=0$ let $A=2a-b$ $B=2b-c$ $C=2c-a$

$$(2a-b)^3+(2b-c)^3+(2c-a)^3 = 3(2a-b)(2b-c)(2c-a)$$

Show

$$A^3+B^3+C^3 = 3ABC$$

$A+B+C$
$= 2a-b+2b-c+2c-a$
$= a+b+c$
$= 0$

$$A+B+C=0$$
$$A+B=-C$$
$$(A+B)^3=-C^3$$
$$A^3+B^3+3AB(A+B)=-C^3$$
$$A^3+B^3+C^3=-3AB(A+B)$$
$$=3ABC$$

$$\therefore (2a-b)^3+(2b-c)^3+(2c-a)^3=3(2a-b)(2b-c)(2c-a)$$

15 $$\frac{a^4-b^4}{a^2-2ab+b^2} \div \frac{a^2b+b^3}{a^3-b^3} \times \frac{a^2b-ab^2+b^3}{a^4+a^2b^2+b^4}$$

$$= \frac{\cancel{(a^2+b^2)}(a+b)\cancel{(a-b)}}{\cancel{(a-b)^2}} \times \frac{\cancel{(a-b)}\cancel{(a^2+ab+b^2)}}{\cancel{b}\cancel{(a^2+b^2)}} \times \frac{\cancel{b}\cancel{(a^2-ab+b^2)}}{\cancel{(a^2+ab+b^2)}\cancel{(a^2-ab+b^2)}}$$

$$= \underline{a+b}$$

1b

$$(1+a)^2 \div \left(1 + \frac{a}{1-a+\frac{a}{1+a+a^2}}\right)$$

$(1-a)(1+a+a^2)$

$$= (1+a)^2 \div \left(1 + \frac{a}{\frac{1-a^3+a}{1+a+a^2}}\right)$$

$$= (1+a)^2 \div \left(1 + \frac{a+a^2+a^3}{1+a-a^3}\right)$$

$$= (1+a)^2 \div \frac{1+a-a^3+a+a^2+a^3}{1+a-a^3}$$

$$= \frac{(1+a)^2}{1} \times \frac{(1+a-a^3)}{(1+a)^2}$$

$$= 1+a-a^3$$