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a, b irrational $a < b$

choose integer n , $\frac{1}{n} < b - a$

p is greatest integer $\frac{p}{n} < a$

a)

$$\frac{1}{n} < b - a$$

$$\frac{p}{n} < a$$

$$\frac{p}{n} + \frac{1}{n} < a + b - a$$

$$\frac{p+1}{n} < b$$

$\frac{p}{n} < a$ but p is greatest integer
Such that $\frac{p}{n} < a$.

$\frac{p+1}{n} \not< a$ ($\because p+1$ is a greater integer
than p)

$$\therefore \frac{p+1}{n} > a$$

$$a < \frac{p+1}{n} < b$$

$$a = \frac{1}{\sqrt{1001}}$$

$$b = \frac{1}{\sqrt{1000}}$$

$$\frac{1}{n} < \frac{1}{\sqrt{1000}} - \frac{1}{\sqrt{1001}}$$

$$\frac{1}{n} < \frac{\sqrt{1001} - \sqrt{1000}}{\sqrt{(1000)(1001)}}$$

$$n > \frac{\sqrt{(1000)(1001)}}{\sqrt{1001} - \sqrt{1000}} \quad 63292.9\dots$$

$$\underline{n = 63293}$$

$$\frac{P}{n} < a$$

$$\frac{P}{63293} < \frac{1}{\sqrt{1001}}$$

$$P < \frac{63293}{\sqrt{1001}} \quad (2000.5..)$$

$$P = 2000$$

$$\frac{1}{\sqrt{1001}} < \frac{P+1}{n} < \frac{1}{\sqrt{1000}}$$
$$\frac{1}{\sqrt{1001}} < \frac{2001}{63293} < \frac{1}{\sqrt{1000}}$$

0.031607...

0.031615...

0.0316228

✓