7

(a) $\dfrac{3}{2+\sqrt{2}} + \dfrac{3}{\sqrt{2}}$

$= \dfrac{3\sqrt{2} + 6 + 3\sqrt{2}}{2\sqrt{2} + 2}$

$= \dfrac{6\sqrt{2} + 6}{2\sqrt{2} + 2}$

$= \dfrac{3\sqrt{2} + 3}{\sqrt{2} + 1} \times \dfrac{\sqrt{2} - 1}{\sqrt{2} - 1}$

$= \dfrac{6 - 3\sqrt{2} + 3\sqrt{2} - 3}{2 - 1}$

$= 3$ //

$\dfrac{6(\sqrt{2}+1)}{2(\sqrt{2}+1)}$

$= 3$

//

10 (b) $a = 2 - \sqrt{3}$

$$a + \frac{1}{a} = 2 - \sqrt{3} + \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$

$$= 2 - \sqrt{3} + \frac{2 + \sqrt{3}}{4 - 3}$$

$$= 2 - \sqrt{3} + 2 + \sqrt{3}$$

$$= \underline{4}$$

13 $\sqrt{17} \div 4.12$ (to 2 dp)

a) $\dfrac{1}{\sqrt{17}-4} = \dfrac{1}{4.12-4}$

$= \dfrac{1}{0.12}$

$= 8.\dot{3} = \dfrac{25}{3}$

b) $\left(\frac{1}{\sqrt{17}-4}\right) \times \frac{\sqrt{17}+4}{\sqrt{17}+4}$

$= \frac{\sqrt{17}+4}{17-16}$

$= \sqrt{17}+4$

$= 4+4.12$

$= 8.12$

$\frac{1}{\sqrt{17}-4}$

$= 8.123\ldots$

14

$$\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}} \times \frac{\sqrt{2}-\sqrt{3}-\sqrt{5}}{\sqrt{2}-\sqrt{3}-\sqrt{5}}$$

$$= \frac{\sqrt{2}-\sqrt{3}-\sqrt{5}}{2-\sqrt{6}-\sqrt{10}+\sqrt{6}-3-\sqrt{15}+\sqrt{10}-\sqrt{15}-5}$$

$$= \frac{\sqrt{2}-\sqrt{3}-\sqrt{5}}{-6-2\sqrt{15}} \times \frac{-6+2\sqrt{15}}{-6+2\sqrt{15}}$$

$6\sqrt{5}$ $10\sqrt{3}$

$$= \frac{-6\sqrt{2}+6\sqrt{3}+6\sqrt{5}+2\sqrt{30}-2\sqrt{45}-2\sqrt{75}}{36-60}$$

$$= \frac{-6\sqrt{2}-4\sqrt{3}+2\sqrt{30}}{-24}$$

$$= \frac{3\sqrt{2}+2\sqrt{3}-\sqrt{30}}{12}$$

15

$$\frac{a}{b+\sqrt{c}} + \frac{d}{\sqrt{c}}$$

$$= \frac{a\sqrt{c} + d(b+\sqrt{c})}{\sqrt{c}(b+\sqrt{c})}$$

$$= \frac{(a+d)\sqrt{c} + bd}{b\sqrt{c}+c} \times \frac{b\sqrt{c}-c}{b\sqrt{c}-c}$$

$$= \frac{(a+d)bc - c(a+d)\sqrt{c} + b^2d\sqrt{c} - bcd}{b^2c - c^2}$$

$$= \frac{(a+d)bc - bcd + (b^2d - c(a+d))\sqrt{c}}{b^2c - c^2}$$

to be rational

$$b^2d - c(a+d) = 0$$

$$b^2d = c(a+d)$$

If $\frac{a}{b+\sqrt{c}} + \frac{d}{\sqrt{c}}$ is rational

then $db^2 = c(a+d)$

If $\frac{a}{1+\sqrt{c}} + \frac{d}{\sqrt{c}}$ rational

then $d = c(a+d)$

$d = c(a+d)$

$= ac + cd$

$d - cd = ac$

$d = \frac{ac}{1-c}$

as c is a positive integer

$1 - c < 0$

$\therefore d < 0$

however $d > 0$

$\therefore \frac{a}{1+\sqrt{c}} + \frac{d}{\sqrt{c}}$ is not rational