

7
a)

$$\frac{3}{2+\sqrt{2}} + \frac{3}{\sqrt{2}}$$

$$= \frac{3\sqrt{2} + 6 + 3\sqrt{2}}{2\sqrt{2} + 2}$$

$$= \frac{6\sqrt{2} + 6}{2\sqrt{2} + 2}$$

$$= \frac{3\sqrt{2} + 3}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1}$$

$$= \frac{6 - 3\sqrt{2} + 3\sqrt{2} - 3}{2 - 1}$$

$$= 3$$

$$\frac{6(\sqrt{2} + 1)}{2(\sqrt{2} + 1)}$$

$$= 3$$

10 ~~11~~ b) $a = 2 - \sqrt{3}$

$$\begin{aligned} a + \frac{1}{a} &= 2 - \sqrt{3} + \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} \\ &= 2 - \sqrt{3} + \frac{2 + \sqrt{3}}{4 - 3} \\ &= 2 - \sqrt{3} + 2 + \sqrt{3} \\ &= \underline{\underline{4}} \end{aligned}$$

~~13~~¹³ $\sqrt{17} \doteq 4.12$ (to 2 dp)

$$\begin{aligned} \text{a) } \frac{1}{\sqrt{17} - 4} &= \frac{1}{4.12 - 4} \\ &= \frac{1}{0.12} \\ &= 8.\dot{3} = \frac{25}{3} \end{aligned}$$

$$\begin{aligned} b) & \left(\frac{-1}{\sqrt{17}-4} \right) \times \frac{\sqrt{17}+4}{\sqrt{17}+4} \\ &= \frac{\sqrt{17}+4}{17-16} \\ &= \frac{\sqrt{17}+4}{1} \\ &= 4+4 \cdot 1,2 \\ &= \underline{\underline{8 \cdot 1,2}} \end{aligned}$$

$$\begin{aligned} & \frac{1}{\sqrt{17}-4} \\ &= 8 \cdot 1,23 \dots - \end{aligned}$$

14/

$$\begin{aligned} & \frac{1}{\sqrt{2} + \sqrt{3} + \sqrt{5}} \times \frac{\sqrt{2} - \sqrt{3} - \sqrt{5}}{\sqrt{2} - \sqrt{3} - \sqrt{5}} \\ &= \frac{\sqrt{2} - \sqrt{3} - \sqrt{5}}{2 - \sqrt{6} - \sqrt{10} + \sqrt{6} - 3 - \sqrt{15} + \sqrt{10} - \sqrt{15} - 5} \\ &= \frac{\sqrt{2} - \sqrt{3} - \sqrt{5}}{-6 - 2\sqrt{15}} \times \frac{-6 + 2\sqrt{15}}{-6 + 2\sqrt{15}} \\ &= \frac{-6\sqrt{2} + 6\sqrt{3} + 6\sqrt{5} + 2\sqrt{30} - 2\sqrt{45} - 2\sqrt{75}}{36 - 60} \\ &= \frac{-6\sqrt{2} - 4\sqrt{3} + 2\sqrt{30}}{-24} \\ &= \frac{3\sqrt{2} + 2\sqrt{3} - \sqrt{30}}{12} \end{aligned}$$

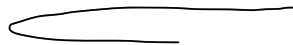
15

$$\begin{aligned} & \frac{a}{b+\sqrt{c}} + \frac{d}{\sqrt{c}} \\ = & \frac{a\sqrt{c} + d(b+\sqrt{c})}{\sqrt{c}(b+\sqrt{c})} \\ = & \frac{(a+d)\sqrt{c} + bd}{b\sqrt{c} + c} \times \frac{b\sqrt{c} - c}{b\sqrt{c} - c} \\ = & \frac{(a+d)bc - c(a+d)\sqrt{c} + b^2d\sqrt{c} - bcd}{b^2c - c^2} \\ = & \frac{(a+d)bc - bcd + (b^2d - c(a+d))\sqrt{c}}{b^2c - c^2} \end{aligned}$$

to be rational

$$b^2d - c(a+d) = 0$$

$$b^2d = c(a+d)$$



If $\frac{a}{b+\sqrt{c}} + \frac{d}{\sqrt{c}}$ is rational
then $db^2 = c(a+d)$

If $\frac{a}{1+\sqrt{c}} + \frac{d}{\sqrt{c}}$ rational
then $d = c(a+d)$

$$d = c(a+d)$$
$$= ac + cd$$

$$d - cd = ac$$

$$d = \frac{ac}{1-c}$$

as c is a positive integer

$$1-c < 0$$

$$\therefore d < 0$$

however $d > 0$

$\therefore \frac{a}{1+\sqrt{c}} + \frac{d}{\sqrt{c}}$ is not rational