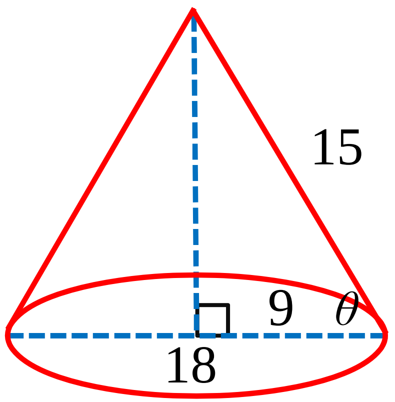
3D Trigonometry

When doing 3D trigonometry it is often useful to redraw all of the faces of the shape in 2D.

Sometimes you may need to draw triangles that are not drawn in the diagram.

Finding the angle between a line and a plane

e.g. A cone has a base diameter of 18 cm and a slant height of 15 cm. Find the angle the curved surface area makes with the base.



1. drop a perpendicular from any point on the line to the plane
2. join the point of intersection to the point where the line meets the plane

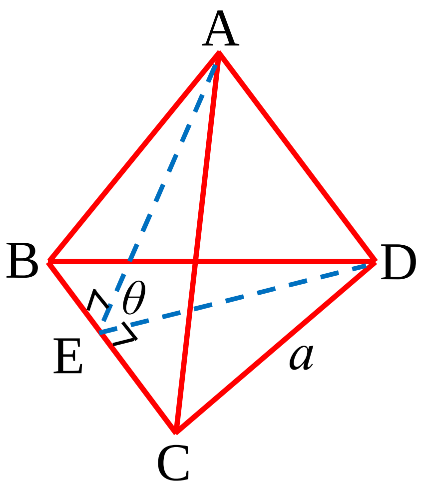
$$\cos\theta = \frac{9}{15}$$

$$\underline{\theta = 53^\circ}$$

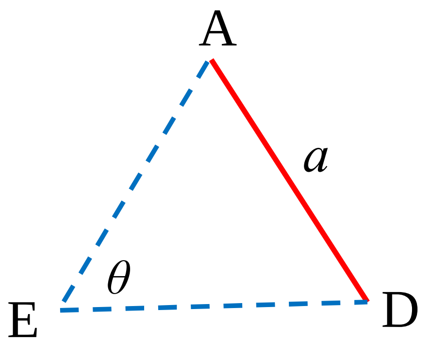
Finding the angle between two planes

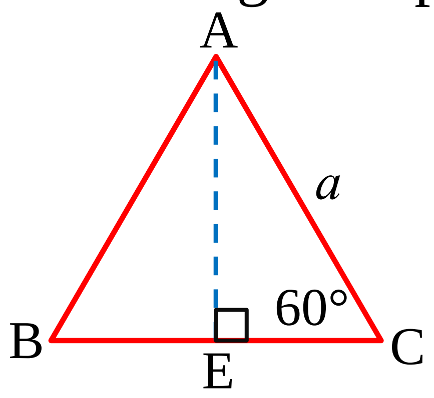
e.g. ABCD is a regular tetrahedron of side a cm each.

Find the angle between two adjacent faces.



From a point on the line of intersection of the two planes draw perpendiculars along both planes





$$AE = \frac{a\sqrt{3}}{2}$$

$30/60$ triangle, sides in ratio $1:2:\sqrt{3}$

Cosine rule is required to solve this problem

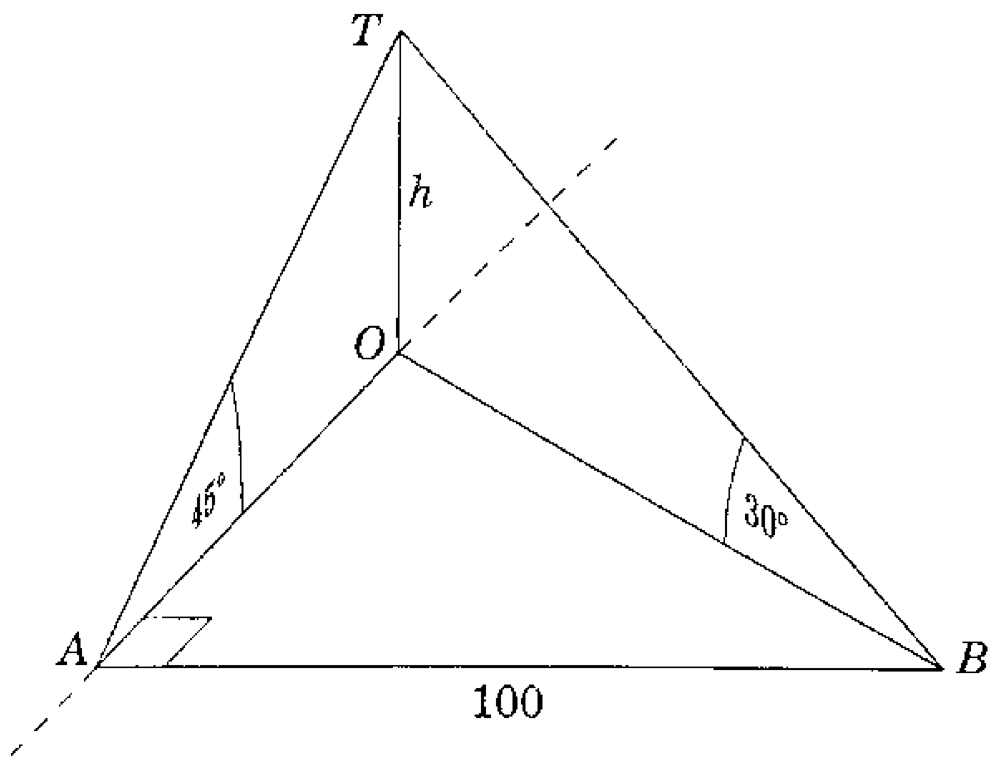
$$\cos\theta = \frac{\frac{3a^2}{4} + \frac{3a^2}{4} - a^2}{2 \times \frac{a\sqrt{3}}{2} \times \frac{a\sqrt{3}}{2}}$$

$$\cos\theta = \frac{2a^2}{6a^2} = \frac{1}{3}$$

$$\theta = 70^\circ 32'$$

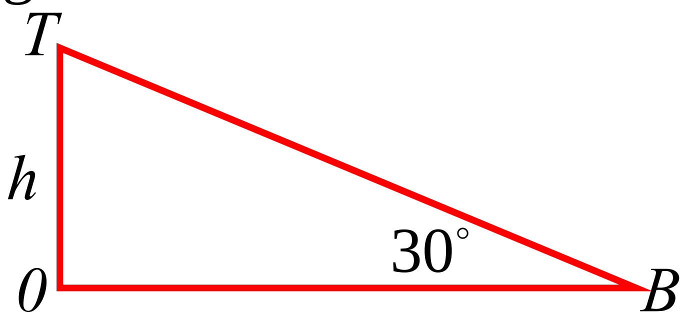
2000 Extension 1 HSC Q3c)

A surveyor stands at point *A*, which is due south of a tower *OT* of height *h* m. The angle of elevation of the top of the tower from *A* is 45°



The surveyor then walks 100 m due east to point *B*, from where she measures the angle of elevation of the top of the tower to be 30°

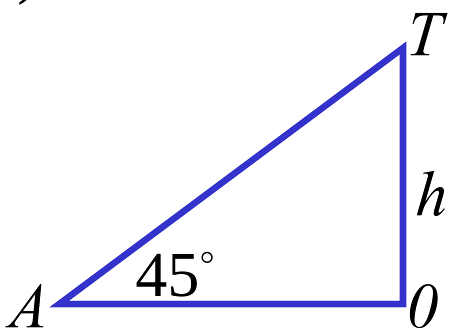
(i) Express the length of OB in terms of h.

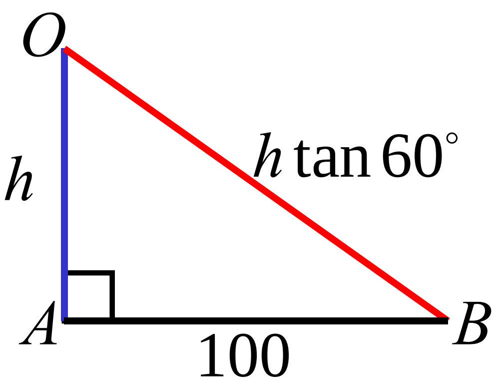


$$\frac{OB}{h} = \tan 60^\circ$$

$$\underline{OB = h\tan 60^\circ}$$

(ii) Show that $h = 50\sqrt{2}$





ΔATO is isosceles

$\therefore AO = h$

$$h^2 + 100^2 = h^2 \tan^2 60^\circ$$

$$h^2 + 100^2 = 3h^2$$

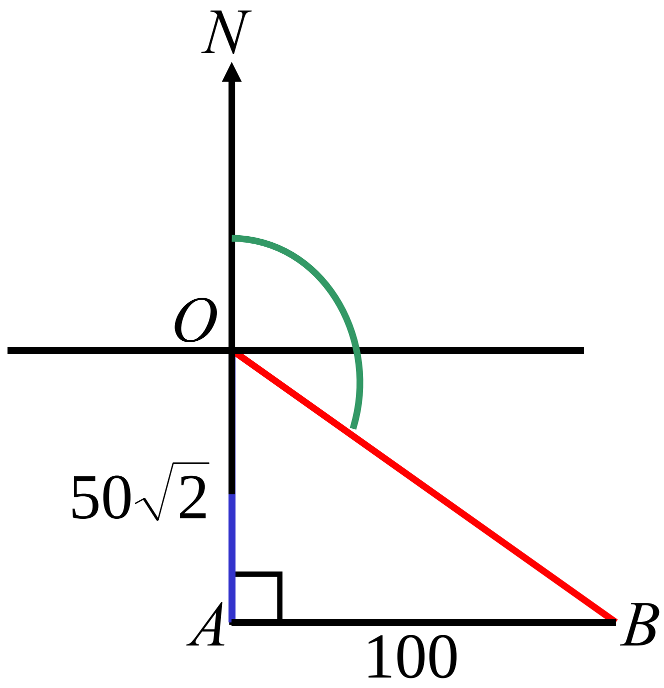
$$2h^2 = 100^2$$

$$h^2 = \frac{100^2}{2}$$

$$h = \frac{100}{\sqrt{2}}$$

$$\underline{h = 50\sqrt{2}}$$

(iii) Calculate the bearing of *B* from the base of the tower.



$$\tan \angle AOB = \frac{100}{50\sqrt{2}}$$

$$\angle AOB = 54^\circ 44'$$

$$\therefore \text{bearing} = 180^\circ - 54^\circ 44'$$

$$= 125^\circ 16'$$

Exercise 6C; 1, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15