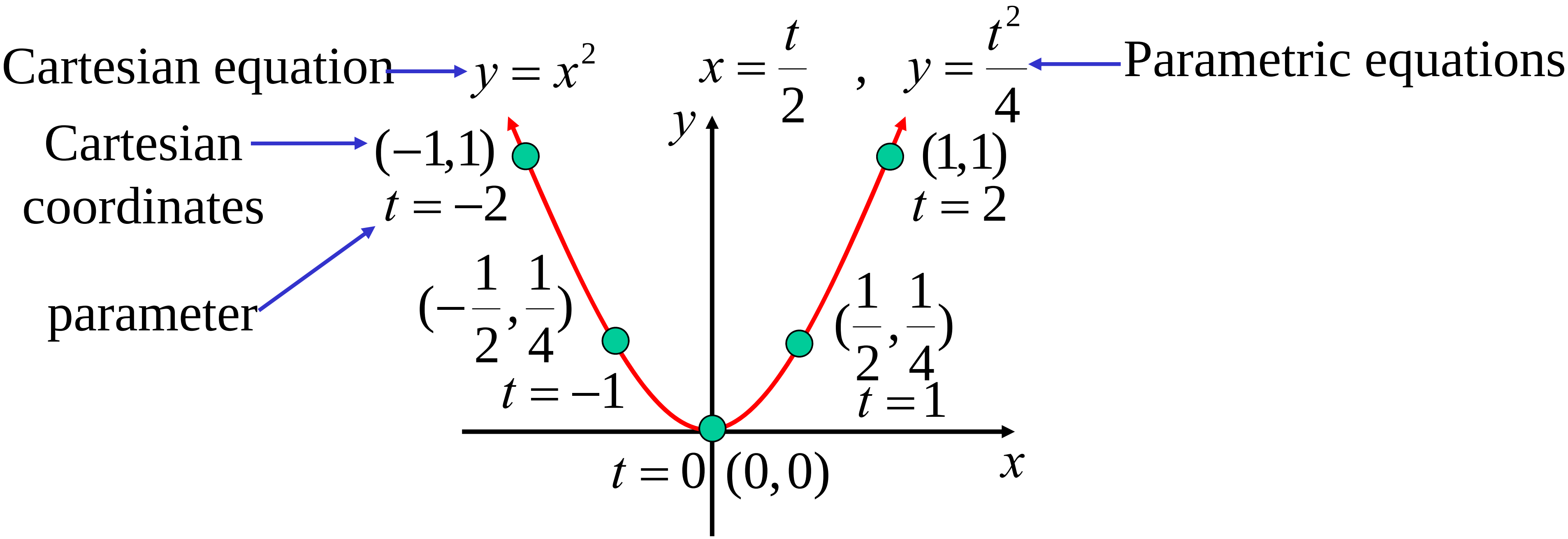
Parametric Coordinates

An alternative way of describing graphs

Cartesian Form: curve is described by one equation and points are described by two numbers.

Parametric Form: curve is described by two equations and points are described by one number ***(parameter)***.



Changing from parametric to Cartesian equations

If $x = f(t)$ and $y = g(t)$ are parametric equations of a curve C, and you eliminate the parameter (t) between the two equations, each point on the curve C lies on the curve represented by the resulting Cartesian equation.

e.g. (i) A curve is given parametrically by the equations $x = 2t + 1$, $y = 3t - 2$.

Show that the curve is a straight line.

$$x = 2t + 1 \qquad y = 3t - 2$$

$$t = \frac{1}{2}(x - 1) \qquad y = 3\left[\frac{1}{2}(x-1)\right] - 2$$

$$y = 3t - 2$$

$$y = \frac{3}{2}x - \frac{7}{2}$$ which is a straight line

(*ii*) Describe the curve represented by the parametric equations

$$x = 2 + \cos\theta \quad , \quad y = 1 + \sin\theta$$

$$\cos\theta = x - 2 \qquad \sin\theta = y - 1$$

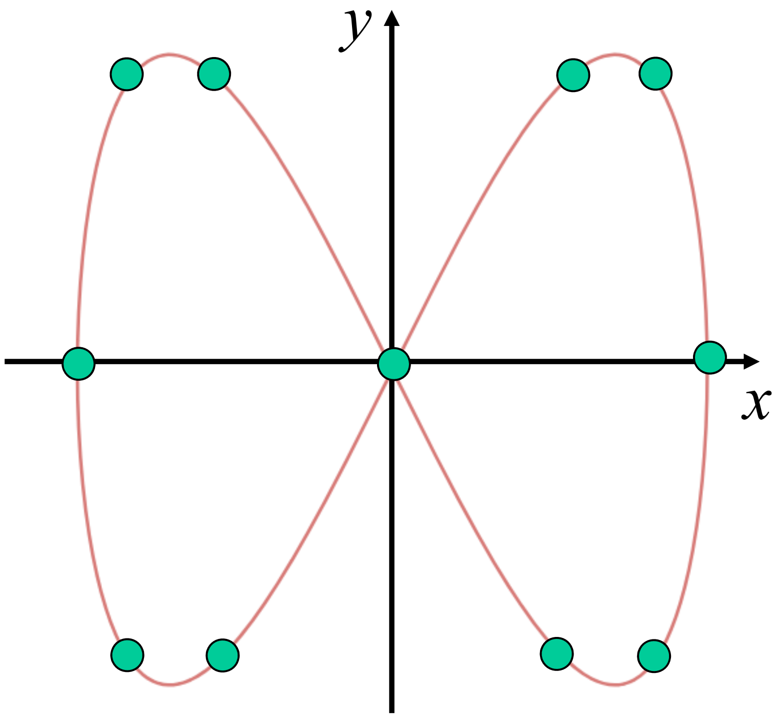
$$(x-2)^2 + (y-1)^2 = 1$$

In order to solve this problem, we need to use the trig identity $\sin^2\theta + \cos^2\theta = 1$

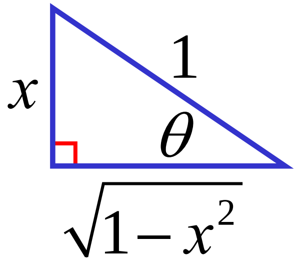
Curve is a circle; centre (2,1) and radius 1 unit

(*iii*) Complete the table of values for the curve $x = \sin\theta$, $y = \sin 2\theta$, taking the values 0°, 30°, 60°, 90°, 120°,…, 360°, and sketch the curve.

θ	0	30	60	90	120	150	180	210	240	270	300	330	360
x	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0
y	0	0.87	0.87	0	-0.87	-0.87	0	0.87	0.87	0	-0.87	-0.87	0



$$x = \sin\theta \qquad y = \sin 2\theta$$



$$y = 2\sin\theta\cos\theta$$

$$y = 2x\sqrt{1-x^2}$$

$$y^2 = 4x^2\left(1-x^2\right)$$

$$4x^4 - 4x^2 + y^2 = 0$$

Exercise 5H; 1, 3, 4, 6a, 7cd, 8a, 9, 15