(A) Features You Should Notice About A Graph

(1) Basic Curves

The following *basic* curve shapes should be recognisable from the equation;

a) Straight lines: $y = x$ (both pronumerals are to the power of one)

b) Parabolas: $y = x^2$ (one pronumeral is to the power of one, the other the power of two)

NOTE: general parabola is $y = ax^2 + bx + c$

c) Cubics: $y = x^3$ (one pronumeral is to the power of one, the other the power of three)

NOTE: general cubic is $y = ax^3 + bx^2 + cx + d$

d) Polynomials in general

e) Hyperbolas: $y = \frac{1}{x}$ OR $xy = 1$

(one pronomeral is on the bottom of the fraction, the other is not *OR* pronumerals are multiplied together)

f) Exponentials: $y = a^x$ (one pronumeral is in the power)

g) Circles: $x^2 + y^2 = r^2$ (both pronumerals are to the power of two, coefficients are the same)

h) Ellipses: $ax^2 + by^2 = k$ (both pronumerals are to the power of two, coefficients are NOT the same)

(*NOTE: if signs are different then hyperbola*)

i) Logarithmics: $y = \log_a x$

j) Trigonometric: $y = \sin x, y = \cos x, y = \tan x$

k) Inverse Trigonmetric: $y = \sin^{-1} x, y = \cos^{-1} x, y = \tan^{-1} x$

(2) Odd & Even Functions

These curves have symmetry and are thus easier to sketch

a) ODD : $f(-x) = -f(x)$ (symmetric about the origin, i.e. 180 degree rotational symmetry)

b) EVEN : $f(-x) = f(x)$ (symmetric about the y axis)

(3) Symmetry in the line y = x

If x and y can be interchanged without changing the function, the curve is relected in the line $y = x$

e.g. $x^3 + y^3 = 1$ (in other words, the curve is its own inverse)

(4) Dominance

As x gets large, does a particular term dominate?

a) Polynomials: the leading term dominates

e.g. $y = x^4 + 3x^3 - 2x + 2$, x^4 dominates

b) Exponentials: e^x tends to dominate as it increases so rapidly

c) In General: look for the term that increases the most rapidly i.e. which is the steepest

NOTE: check by substituting large numbers e.g. 1000000

(5) Asymptotes

a) Vertical Asymptotes: the bottom of a fraction cannot equal zero

b) Horizontal/Oblique Asymptotes: Top of a fraction is constant, the fraction cannot equal zero

NOTE: if order of numerator $\geq$ order of denominator, perform a polynomial division. (*curves can cross horizontal/oblique asymptotes, good idea to check)*

(6) The Special Limit

Remember the special limit seen in 2 Unit i.e. $\lim_{x \to 0} \frac{\sin x}{x} = 1$

, it could come in handy when solving harder graphs.

(B) Using Calculus

Calculus is still a tremendous tool that should not be disregarded when curve sketching. However, often it is used as a final tool to determine **critical points, stationary points, inflections.**

(1) Critical Points

When $\frac{dy}{dx}$ is undefined the curve has a vertical tangent, these points are called **critical points.**

(2) Stationary Points

When $\frac{dy}{dx} = 0$ the curve is said to be **stationary**, these points may be minimum turning points, maximum turning points or points of inflection.

(3) Minimum/Maximum Turning Points

a) When $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0,$ the point is called a **minimum turning point**

b) When $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0,$ the point is called a **maximum turning point**

NOTE: testing either side of $\frac{dy}{dx}$ for change can be quicker for harder functions

(4) Inflection Points

a) When $\frac{d^2y}{dx^2} = 0$ and $\frac{d^3y}{dx^3} \neq 0,$ the point is called an **inflection point**

NOTE: testing either side of $\frac{d^2y}{dx^2}$ for change can be quicker for harder functions

b) When $\frac{dy}{dx} = 0, \frac{d^2y}{dx^2} = 0$ and $\frac{d^3y}{dx^3} \neq 0,$ the point is called a **horizontal point of inflection**

(5) Increasing/Decreasing Curves

a) When $\frac{dy}{dx} > 0$, the curve has a positive sloped tangent and is thus **increasing**

b) When $\frac{dy}{dx} < 0$, the curve has a negative sloped tangent and is thus **decreasing**

(6) Implicit Differentiation

This technique allows you to differentiate complicated functions

e.g. Sketch $x^3 + y^3 = 1$

Note: • the curve has symmetry in $y = x$
• it passes through (1,0) and (0,1)
• it is asymptotic to the line $y = -x$

$$\because y^3 = 1 - x^3$$

$$\text{i.e. } y^3 \neq -x^3$$

$$y \neq -x$$

On differentiating implicitly;

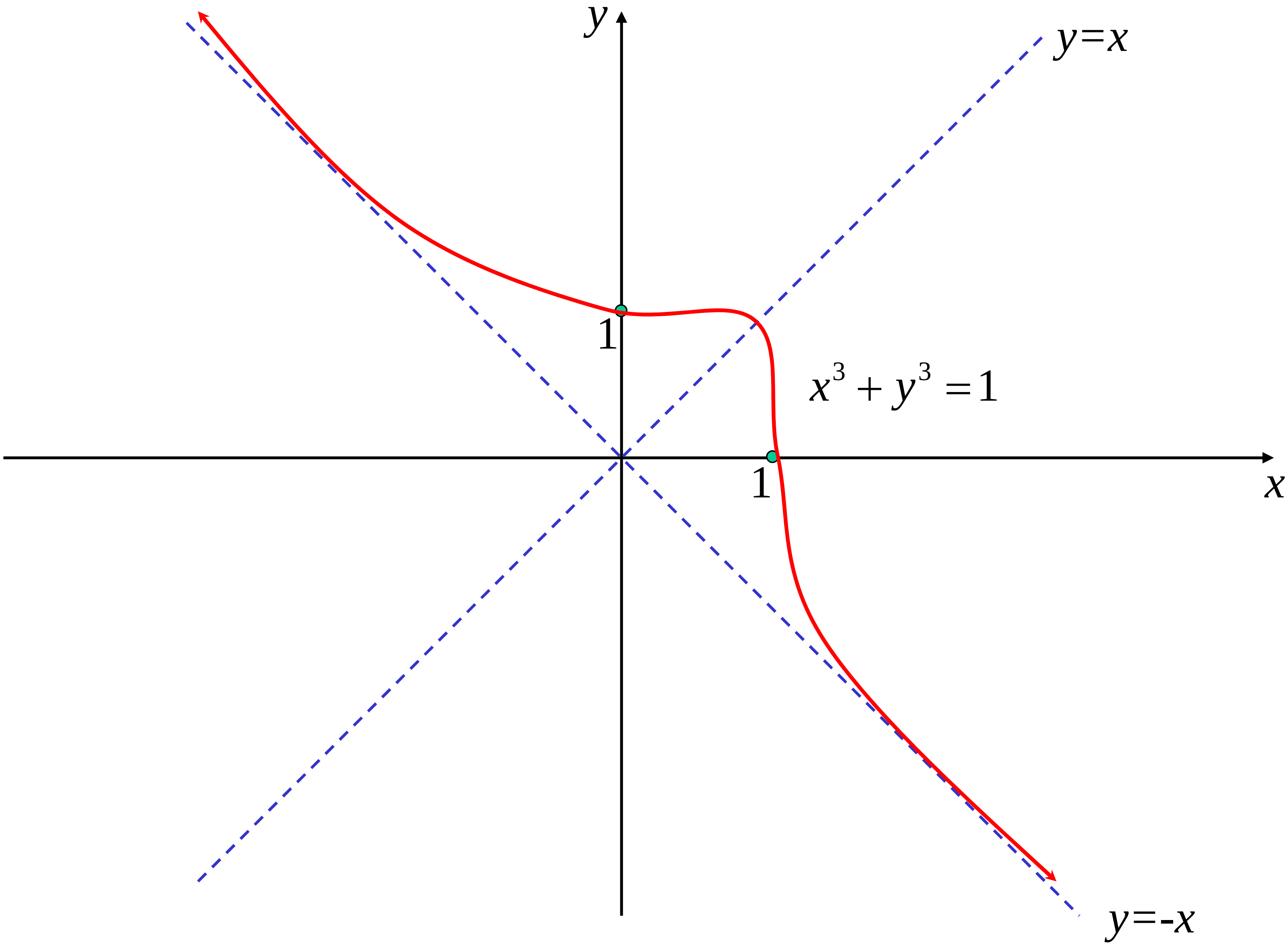
$$3x^2 + 3y^2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^2}{y^2}$$

This means that $\frac{dy}{dx} < 0$ for all x

Except at (1,0) : critical point &

(0,1): horizontal point of inflection


y
y=x
1
x³ + y³ = 1
1
x
y=-x