Logarithms

The **logarithm** of any number to a given **base** is the index of the power to which the base must be raised in order to equal the given number.

$$\text{If } N = a^x,\ a > 0,\ a \neq 1$$
$$\text{then } x = \log_a N$$

e.g. (i) $x = \log_{10} 1000$

$$10^x = 1000$$
$$\underline{x = 3}$$

(ii) $x = \log_5 \frac{1}{25}$

$$5^x = \frac{1}{25}$$
$$\underline{x = -2}$$

(iii) $\log_x 64 = 2$

$$x^2 = 64$$
$$\underline{x = 8}$$

(iv) $\log_6 x = 2$

$$x = 6^2$$
$$\underline{x = 36}$$

Log Laws

1) $\log_a a^x = x$

2) $a^{\log_a x} = x \ , x > 0$

logs and exponentials are inverse functions

3) $\log_a 1 = 0$ $\quad \left(\text{as } a^0 = 1\right)$

4) $\log_a a = 1$ $\quad \left(\text{as } a^1 = a\right)$

5) $\log_a x + \log_a y = \log_a xy$ $\quad \left(\text{as } a^x \times a^y = a^{x+y}\right)$

6) $\log_a x - \log_a y = \log_a \dfrac{x}{y}$ $\quad \left(\text{as } a^x \div a^y = a^{x-y}\right)$

7) $\log_a x^p = p \log_a x$ $\quad \left(\text{as } a^{px} = \left(a^x\right)^p\right)$

e.g. (i) $\log_{10} 20 + \log_{10} 5$

$$= \log_{10} 100$$

$$= 2$$

(ii) $\log_2 18 - \log_2 9$

$$= \log_2 2$$

$$= 1$$

(iii) $\log_4 9 + \log_4 8 - 2\log_4 6$

$$= \log_4 \frac{9 \times 8}{6^2}$$

$$= \log_4 2$$

$$= \frac{1}{2}$$

(iv) If $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$ evaluate;

a) $\log_{10} 27$

$$= \log_{10} 3^3$$
$$= 3\log_{10} 3$$
$$= 3(0.4771)$$
$$= 1.4313$$

b) $\log_{10} \sqrt{2}$

$$= \frac{1}{2}\log_{10} 2$$
$$= \frac{1}{2}(0.3010)$$
$$= 0.1505$$

c) $\log_{10} 12$

$$= \log_{10}\left(3 \times 2^2\right)$$
$$= \log_{10} 3 + 2\log_{10} 2$$
$$= 0.4771 + 2(0.3010)$$
$$= 1.0791$$

Exercise 8C; 3bceln, 4afk, 5afio, 8acef, 11afkpsv, 12

Exercise 8D; 1afk, 2cf, 5abdg, 6b, 7c, 8d, 9h, 12bcgh, 14a, 15bc, 17