The Gradient Function

The **function** $y = f(x)$ measures the distance of the graph from the x-axis

The **gradient function** $y = f'(x)$ measures how steep the graph is at the point (i.e. the gradient, or the rate that distance is changing)

Note: the gradient function is also known as the derivative

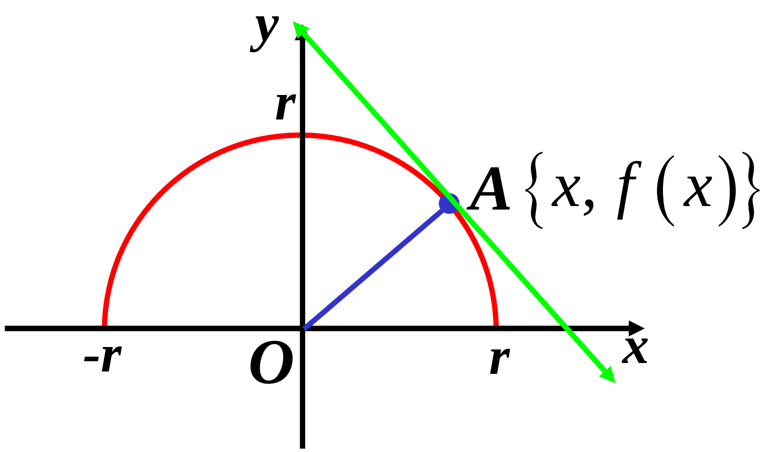
Geometry can be used to find the gradient function of lines and semi-circles.

Lines

horizontal line	oblique line
$f(x) = c$	$f(x) = mx + b$
$f'(x) = 0$	$f'(x) = m$

Semicircles

$$f(x)=\sqrt{r^2-x^2}$$



Unlike a line, a curve does not have a consistent slope.

The slope of a curve is defined as the slope of the tangent to the curve at any particular point.

With a semicircle, we know that the radius is perpendicular to the tangent

$$m_{OA}=\frac{f(x)}{x}$$

$$\therefore m_{\text{tangent}}=\frac{-x}{f(x)}$$

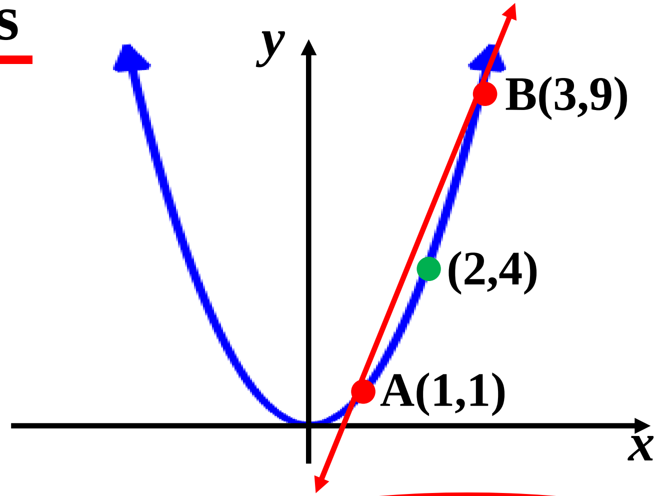
$$f(x)=\sqrt{r^2-x^2}$$

$$f'(x)=\frac{-x}{\sqrt{r^2-x^2}}$$

Any other graph needs to be estimated using secants around the point

Quadratics

$$f(x) = x^2$$



1. Choose two points either side of (2,4)

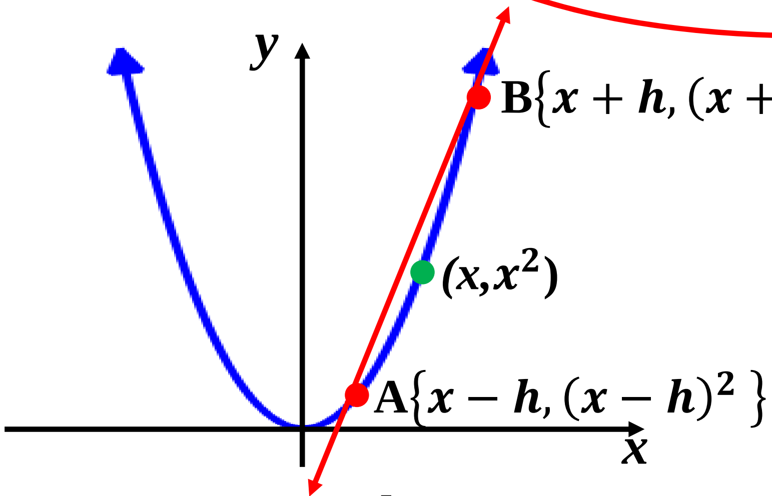
2. Calculate the slope of the secant joining the two points

$$m_{AB} = \frac{9-1}{3-1}$$

$$= 4$$

$$\therefore f'(2) \approx 4 \checkmark$$

Accuracy of the answer is totally dependent on the choice of the two points



$$m_{AB} = \frac{(x+h)^2 - (x-h)^2}{x+h-(x-h)}$$

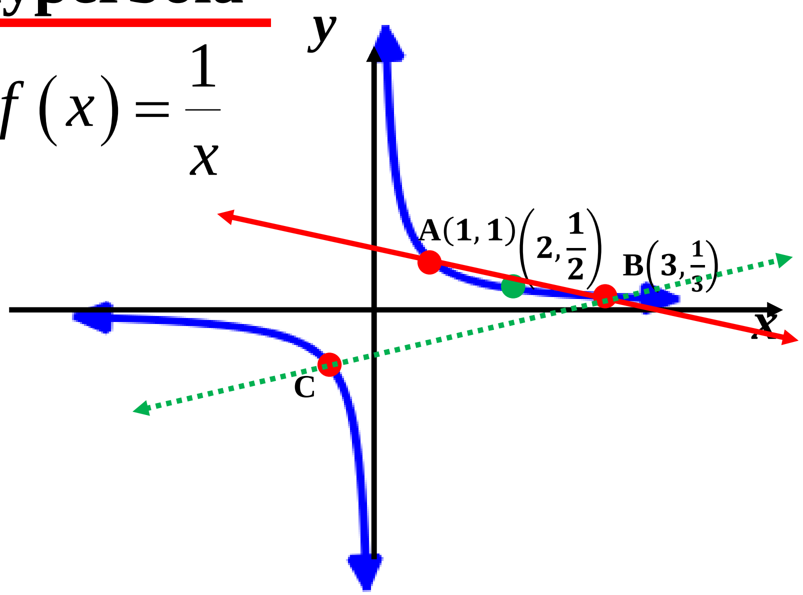
$$= \frac{4xh}{2h}$$

$$= 2x \quad \Rightarrow f'(x) = 2x \checkmark$$

For a quadratic, symmetrically chosen points will always give an exact answer

Hyperbola

$f(x) = \frac{1}{x}$



$$m_{AB} = \frac{\frac{1}{3} - 1}{3 - 1}$$

$$= -\frac{1}{3} \quad \Rightarrow f'(2) \approx -\frac{1}{3}$$ ×

$$\left(f'(2) = -\frac{1}{4} \right)$$

Note: It is vital that the graph is **continuous** between the two chosen points

If BC was used, clearly the estimate would be poor

If you have an accurate graph, it is usually better to draw the tangent and calculate its slope using $\frac{\text{rise}}{\text{run}}$

Exercise 9A; 1 or 2, 3 or 4, 5adg, 9 (*don't worry about sketch*), 10