(E) Multiplication of Functions

The graph of $y = f(x).\ g(x)$ can be graphed by first graphing $y = f(x)$ and $y = g(x)$ separately and then examining the sign of the product. Special note needs to be made of points where $f(x) = 0$ or 1, or $g(x) = 0$ or 1.

NOTE: The regions on the number plane through which the graph must pass should be shaded in as the first step.

Things to keep in mind:

Discontinuities: any exclusions in the domain of the original function(s) remain in the new function

***x*-intercept:** If $f(x) = 0$ or $g(x) = 0$, then $y = f(x) \times g(x) = 0$

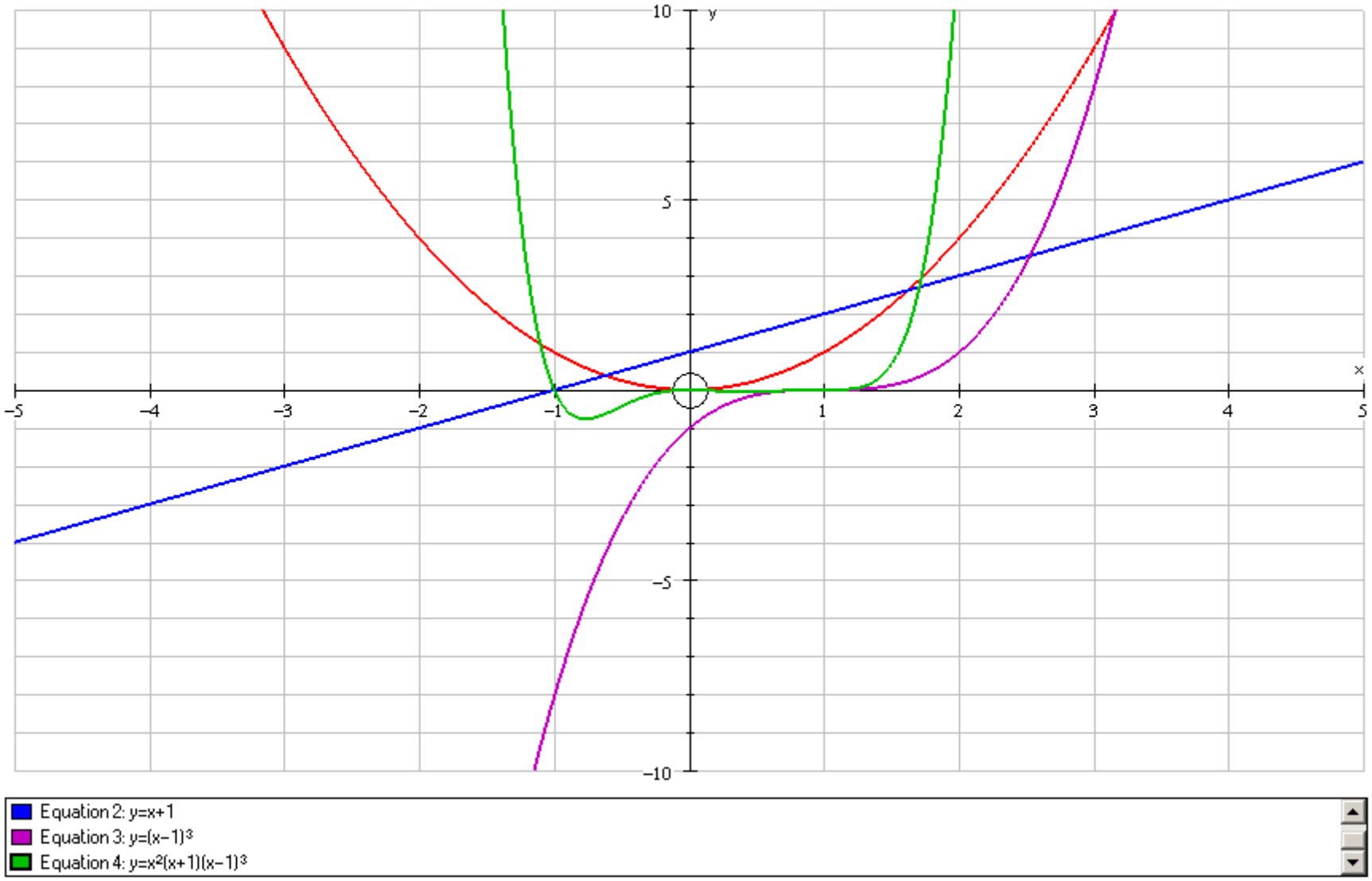
symmetry: symmetric graphs will retain some form of symmetry

odd function × odd function = even function

odd function × even function = oddfunction

even function × even function = even function

e.g. $y = x^2(x+1)(x-1)^3$


10
y
5
x
−5
−4
−3
−2
−1
1
2
3
4
5
−5
−10
Equation 2: y=x+1
Equation 3: y=(x−1)³
Equation 4: y=x²(x+1)(x−1)³

(F) Division of Functions

The graph of $y = \dfrac{f(x)}{g(x)}$ can be graphed by;

Step 1: First graph $y = f(x)$ and $y = g(x)$ separately.

Step 2: Mark in vertical asymptotes

Step 3: Shade in regions in which the curve must be (same as multiplication.

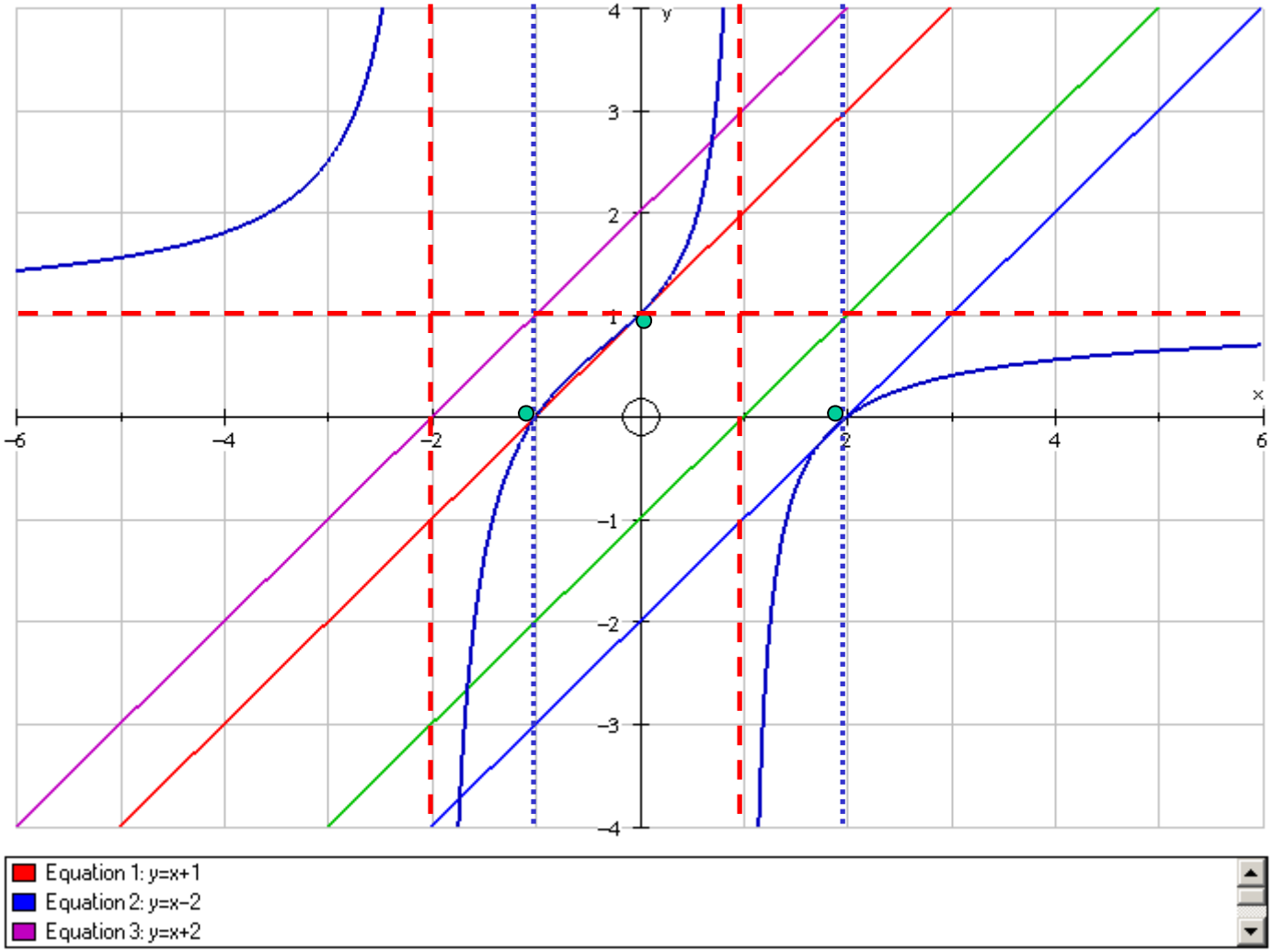
Step 4: Investigate the behaviour of the function for large values of x (find horizontal/oblique asymptotes , look at dominance)

$$y = \frac{(x+1)(x-2)}{(x+2)(x-1)}$$

$$= \frac{x^2 - x - 2}{x^2 + x - 2}$$

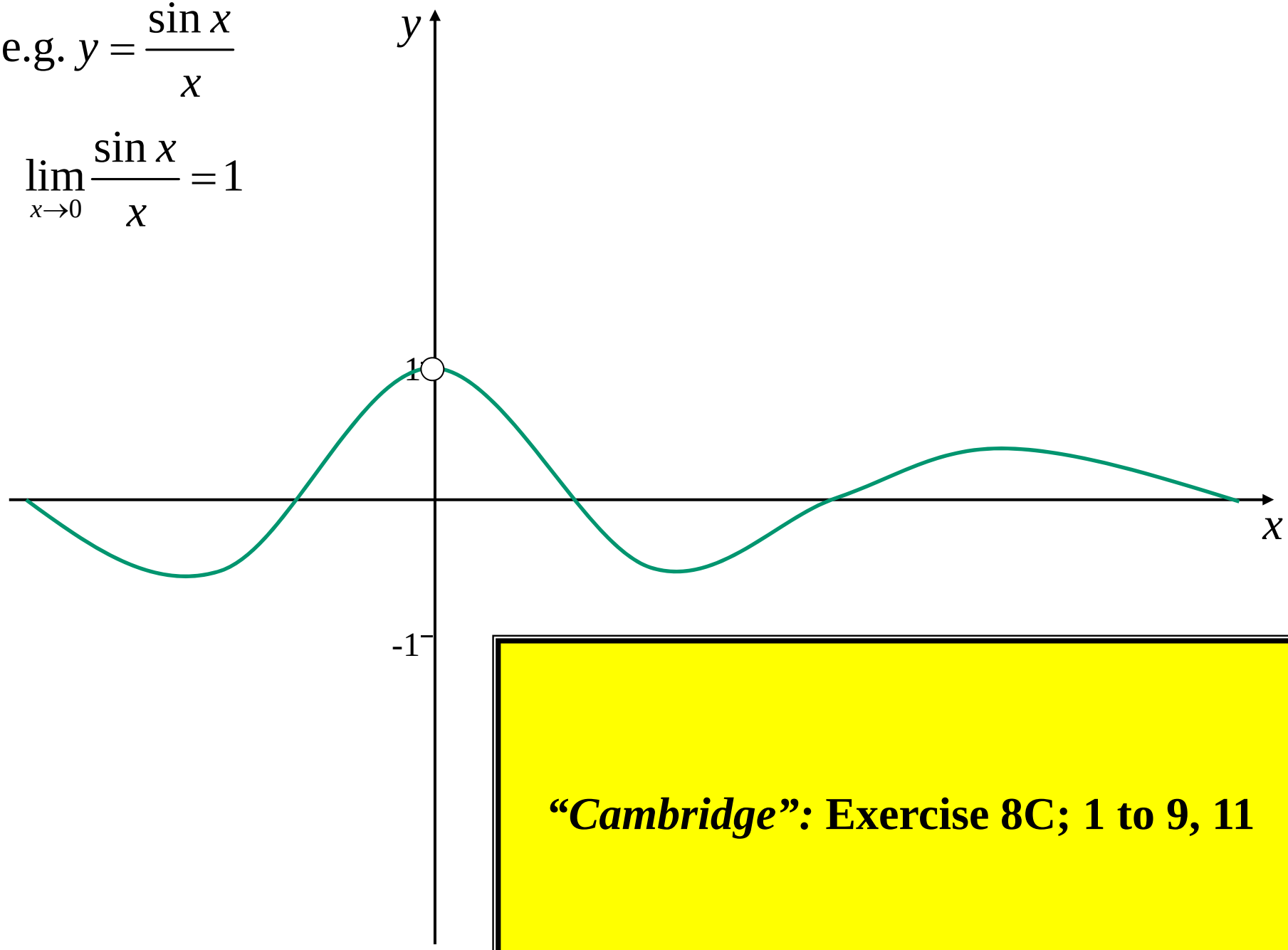
$$= 1 - \frac{2x}{x^2 + x - 2} \qquad \therefore \text{horizontal asymptote}: y = 1$$

e.g. $y = \dfrac{(x+1)(x-2)}{(x+2)(x-1)}$



e.g. $y = \dfrac{\sin x}{x}$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$



“*Cambridge*”: Exercise 8C; 1 to 9, 11