Differentiating Non-Cardinal Powers

(1) $y = \frac{1}{x}$

$$f(x) = \frac{1}{x}$$

$$f(x+h) = \frac{1}{x+h}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{x - x - h}{hx(x+h)}$$

$$= \lim_{h \to 0} \frac{-h}{hx(x+h)}$$

$$= \lim_{h \to 0} \frac{-1}{x(x+h)}$$

$$= \frac{-1}{x^2}$$

Note:

$$f(x) = x^{-1}$$

$$f'(x) = -x^{-2}$$

$$= \frac{-1}{x^2}$$

(2) $y=\sqrt{x}$

$$f(x)=\sqrt{x}$$

$$f(x+h)=\sqrt{x+h}$$

$$f'(x)=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\times\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$=\lim_{h\to 0}\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=\lim_{h\to 0}\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=\lim_{h\to 0}\frac{1}{\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=\frac{1}{\sqrt{x}+\sqrt{x}}$$

$$=\frac{1}{2\sqrt{x}}$$

Note:

$$f(x)=x^{\frac{1}{2}}$$

$$f'(x)=\frac{1}{2}x^{-\frac{1}{2}}$$

$$=\frac{1}{2\sqrt{x}}$$

e.g. (i) $y = 3x + \frac{1}{x^2}$

$$= 3x + x^{-2}$$

$$\frac{dy}{dx} = 3 - 2x^{-3}$$

$$= 3 - \frac{2}{x^3}$$

(ii) $y = x^2\sqrt{x}$

$$= x^{\frac{5}{2}}$$

$$\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}}$$

$$= \frac{5}{2}x\sqrt{x}$$

(iii) $y = \frac{2}{(5x+4)^2}$

$$= 2(5x+4)^{-2}$$

$$\frac{dy}{dx} = -4(5x+4)^{-3}(5)$$

$$= -20(5x+4)^{-3}$$

$$= \frac{-20}{(5x+4)^3}$$

(iv) $y = \sqrt{x^2 - 3}$

$$= \left(x^2 - 3\right)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(x^2 - 3\right)^{-\frac{1}{2}}(2x)$$

$$= x\left(x^2 - 3\right)^{-\frac{1}{2}}$$

$$= \frac{x}{\sqrt{x^2 - 3}}$$

Exercise 9F; 1ace, 2bd, 3ad, 5a, 6b, 8a, 9b, 11, 14, 15

Exercise 9G; 1bdf, 2c iii, 4bd, 5b, 6b, 8a, 9c, 10ad, 13, 15