

Calculus Rules

2. Product Rule $\frac{d}{dx}(uv) = uv' + vu'$

“Write down the FIRST and DIFF the SECOND, PLUS write down the SECOND and DIFF the FIRST”

e.g. (i) $y = x^7(x^9 - 6)$

$$\begin{aligned}\frac{dy}{dx} &= (x^7)(9x^8) + (x^9 - 6)(7x^6) \\ &= 9x^{15} + 7x^{15} - 42x^6\end{aligned}$$

$$= \underline{\underline{16x^{15} - 42x^6}}$$

(ii) $y = (x-2)(2x-3)$

$$\begin{aligned}\frac{dy}{dx} &= (x-2)(2) + (2x-3)(1) \\ &= 2x - 4 + 2x - 3 \\ &= \underline{\underline{4x - 7}}\end{aligned}$$

(iii) $\frac{d}{dx}\{(x^7 - x^3)(3x^2 + 7)\}$

$$= (x^7 - x^3)(6x) + (3x^2 + 7)(7x^6 - 3x^2)$$

$$= 6x^8 - 6x^4 + 21x^8 - 9x^4 + 49x^6 - 21x^2$$

$$= \underline{\underline{27x^8 + 49x^6 - 15x^4 - 21x^2}}$$

$$(iv) \quad y = 3x(x^2 + 4)^5$$

$$\begin{aligned}\frac{dy}{dx} &= (3x) \left\{ 5(x^2 + 4)^4 (2x) \right\} + (x^2 + 4)^5 (3) \\&= 30x^2(x^2 + 4)^4 + 3(x^2 + 4)^5 \\&= (x^2 + 4)^4 \left\{ 30x^2 + 3(x^2 + 4) \right\} \\&= (x^2 + 4)^4 (33x^2 + 12) \\&= 3(x^2 + 4)^4 (11x^2 + 4)\end{aligned}$$

$$(v) \quad y = 2x\sqrt{2x-1}$$

$$= 2x(2x-1)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = (2x) \left\{ \frac{1}{2}(2x-1)^{-\frac{1}{2}} (2) \right\} + (2x-1)^{\frac{1}{2}} (2)$$

$$= (2x)(2x-1)^{-\frac{1}{2}} + 2(2x-1)^{\frac{1}{2}}$$

$$= 2(2x-1)^{-\frac{1}{2}} \{x + (2x-1)\}$$

$$= 2(2x-1)^{-\frac{1}{2}} (3x-1)$$

**Exercise 9H; 2ac, 3bdf,
4a, 5ac, 7, 9ac,
10, 12, 14**

$$\frac{dy}{dx} = \frac{2(3x-1)}{\sqrt{2x-1}}$$
