

Calculus Rules

3. Quotient Rule

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{vu' - uv'}{v^2}$$

“SQUARE the BOTTOM, write down the BOTTOM and DIFF the TOP, MINUS write down the TOP and DIFF the BOTTOM”

$$\text{e.g. } (i) \quad y = \frac{x}{1+2x}$$

$$\frac{dy}{dx} = \frac{(1+2x)(1) - (x)(2)}{(1+2x)^2}$$

$$= \frac{1+2x-2x}{(1+2x)^2}$$

$$= \frac{1}{(1+2x)^2}$$

$$(ii) \quad y = \frac{2x}{\sqrt{x^2 - 4}}$$

$$\frac{dy}{dx} = \frac{(x^2 - 4)^{\frac{1}{2}}(2) - (2x)\left\{ \frac{1}{2}(x^2 - 4)^{-\frac{1}{2}}(2x) \right\}}{(x^2 - 4)}$$

$$= \frac{2(x^2 - 4)^{\frac{1}{2}} - 2x^2(x^2 - 4)^{-\frac{1}{2}}}{(x^2 - 4)}$$

$$= \frac{2(x^2 - 4)^{-\frac{1}{2}} \{ (x^2 - 4) - x^2 \}}{-8(x^2 - 4)}$$

$$= \frac{-8}{(x^2 - 4)\sqrt{x^2 - 4}}$$

4. Reciprocal Rule

$$\frac{d}{dx} \left(\frac{k}{v} \right) = \frac{-kv'}{v^2}$$

“MINUS the DERIVATIVE on the FUNCTION SQUARED”

e.g. (i) $y = \frac{1}{x^2}$

$$\frac{dy}{dx} = \frac{-2x}{x^4}$$

$$= \frac{-2}{x^3}$$

(ii) $y = \frac{6}{4x^2 + 3}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{-6(8x)}{(4x^2 + 3)^2} \\ &= \frac{-48x}{(4x^2 + 3)^2}\end{aligned}$$

Exercise 9I; 2aceg, 3, 4a, 6a, 8, 9a, 10b, 11a, 13a, 15bejlo