CARINGBAH HIGH SCHOOL

Name: ______________________

2018 | HSC TRIAL

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided.
- In Questions 11–14, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 6–11)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Marker's Use Only

Question	1 – 10	11	12	13	14	Total	
Mark	/10	/15	/15	/15	/15	/70	%

This paper must not be removed from the examination room

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

---

1 Which expression is equal to $\sin x + \cos x$?

(A) $\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$

(B) $\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)$

(C) $2\sin\left(x+\frac{\pi}{4}\right)$

(D) $2\sin\left(x-\frac{\pi}{4}\right)$

2 What is the acute angle between the lines $3x + y = 4$ and $x - 2y = 5$?

(A) $\tan^{-1} 0.5$

(B) $\tan^{-1} 1$

(C) $\tan^{-1} 2$

(D) $\tan^{-1} 7$

3 Which of the polynomials are divisible by $x + 1$?

(I) $x^{2018}-1$ (II) $x^{2017}-1$ (III) $x^{2018}+1$ (IV) $x^{2017}+1$

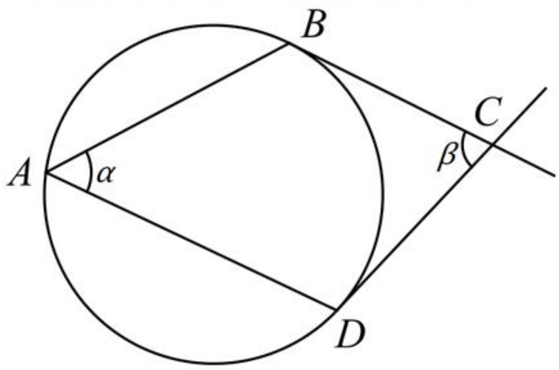
(A) (I) and (III) only

(B) (II) and (III) only

(C) (II) and (IV) only

(D) (I) and (IV) only

4 In the diagram below, if BC and DC are tangents then:



(A) $\alpha + \beta = 180°$

(B) $2\alpha + \beta = 180°$

(C) $\alpha + 2\beta = 180°$

(D) $2\alpha + 2\beta = 180°$

5 A particle is moving in simple harmonic motion which satisfies $\ddot{x} = -4x$.

The particle starts from $x = 0$ with initial velocity 4 m/s.

Which is a possible expression for the displacement, x, of the particle?

(A) $x = 2 \sin 2t$

(B) $x = 2 \sin 4t$

(C) $x = 4 \sin 2t$

(D) $x = 4 \sin 4t$

6 What is the derivative of $6\sin^{-1}\dfrac{x}{3}$?

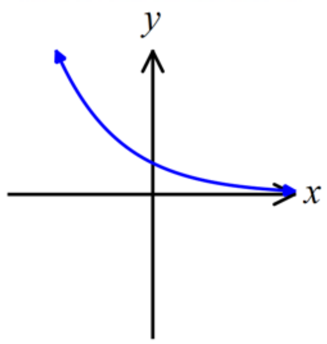
(A) $\dfrac{1}{\sqrt{9-x^2}}$

(B) $\dfrac{2}{\sqrt{9-x^2}}$

(C) $\dfrac{6}{\sqrt{9-x^2}}$

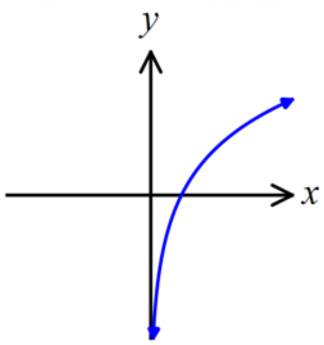
(D) $\dfrac{18}{\sqrt{9-x^2}}$

7 The diagram shows the graph of $y = f(x)$.

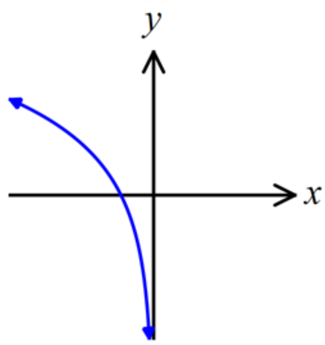


Which diagram shows the graph of $y = f^{-1}(x)$?

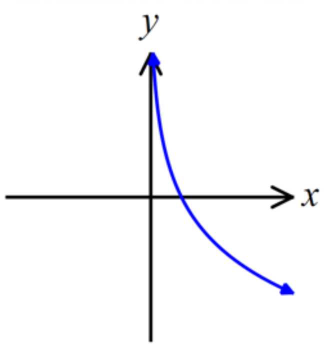
(A)



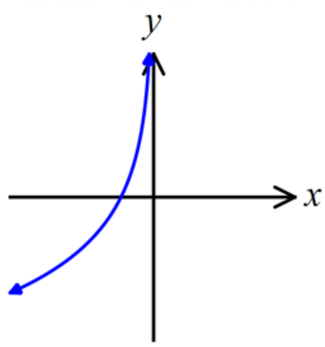
(B)



(C)



(D)



8 What are the asymptotes of $y = \dfrac{x+3}{(x-2)(x+1)}$?

(A) $y = 0, x = 2, x = -1$

(B) $y = 0, x = -2, x = 1$

(C) $y = 3, x = 2, x = -1$

(D) $y = 3, x = -2, x = 1$

9 Which of the following are the coordinates of the point of intersection of the normal $x + 2py = 8ap^3 + 4ap$ and the parabola $x^2 = 4ay$?

(A) $(2ap, ap^2)$

(B) $(4ap, 4ap^2)$

(C) $(-2ap, ap^2)$

(D) $(-4ap, 4ap^2)$

10 A particle moves in a straight line. Its position at any time t is given by

$$x = 3\cos 2t + 4\sin 2t.$$

The acceleration in terms of x is:

(A) $\ddot{x} = -2x$

(B) $\ddot{x} = -3x$

(C) $\ddot{x} = -4x$

(D) $\ddot{x} = -5x$

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question on SEPARATE writing paper. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Marks

Question 11 (15 marks) Use SEPARATE writing paper.

(a) A and B are the points (1, 4) and (5, 2) respectively. Find the coordinates of the point M which divides the interval AB externally in the ratio 2:3. **2**

(b) Solve $\frac{2}{x} \le \frac{x}{2}$. **3**

(c) Evaluate $\lim_{x \to 0} \frac{\sin\left(\frac{x}{3}\right)}{4x}$. **2**

(d) Water at a temperature of 24˚C is placed in a freezer that maintains a constant temperature at -12°C. After time t minutes the rate of change of temperature T of the water is given by the formula:

$$\frac{dT}{dt} = -k(T+12)$$

where t is the time in minutes and k is a positive constant.

(i) Show that $T = Ae^{-kt} - 12$ is a solution of this equation, where A is a constant. **1**

(ii) Show that the value of A is 36. **1**

(iii) After 15 minutes the temperature of the water falls to 9˚C. Show that the value of k is 0.0359 correct to 3 significant figures. **1**

(iv) Find to the nearest minute the time taken for the water to start freezing. (Freezing point of water is 0˚C). **2**

(e) (i) Factorise $e^{3x} + e^{3y}$. **1**

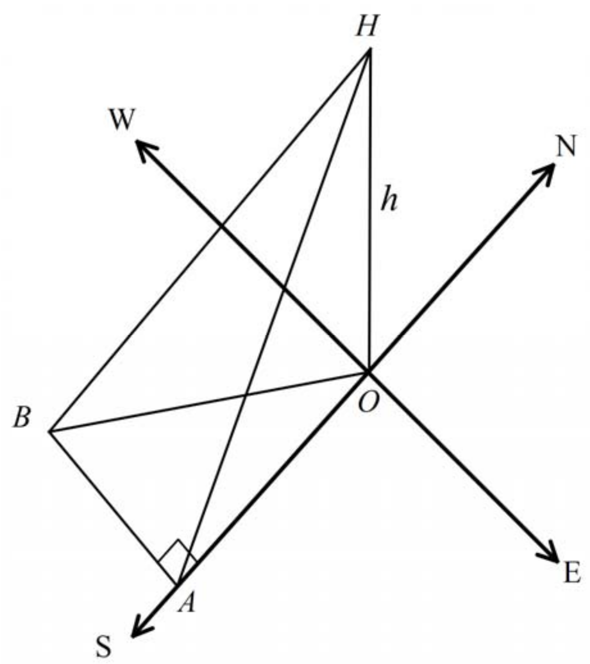
(ii) If $e^x + e^y = 3$ and $e^{3x} + e^{3y} = 10$, find the exact value of $x + y$. **2**

End of Question 11

Marks

Question 12 (15 marks) Use SEPARATE writing paper.

(a) Point A is due south of the base of a hill, the angle of elevation from A to the top of the hill, H, is 46º. Another point B is due west of A and the angle of elevation from B to the top of the hill is 35º. The distance AB is 220 m.



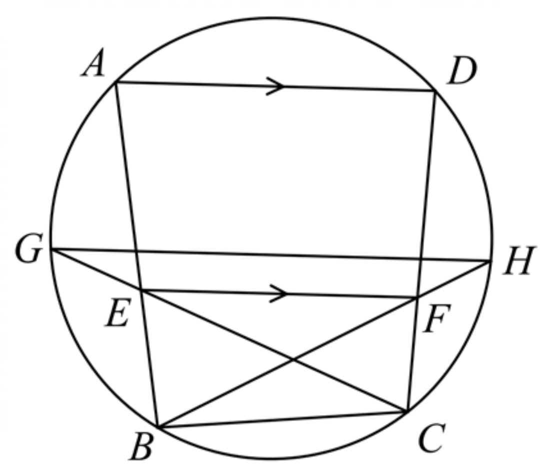
(i) Show that $OA = h \tan 44°$. **1**

(ii) Find the height of the hill, h correct to 1 decimal place. **2**

Question 12 continues on page 8

Question 12 (continued)

(b) In the diagram below $ABCD$ is a cyclic quadrilateral. E is a point on AB and F is a point on DC such that $EF \parallel AD$. BF produced and CE produced meet the circle through A, B, C, D at H and G respectively.



(i) Show that $EBCF$ is a cyclic quadrilateral. **2**

(ii) Show that $GH \parallel EF$, giving reasons. **2**

(c) Let $f(x) = 2\cos^{-1}\left(\frac{x}{3}\right)$.

(i) State the domain and range of the function $f(x)$. **2**

(ii) Sketch the graph of $y = f(x)$ using $\frac{1}{3}$ of a page. **1**

(iii) Determine the inverse function $y = f^{-1}(x)$, and write down the domain of this inverse function. **2**

(iv) Hence or otherwise find the exact value of the volume of the solid of revolution formed when the region bounded by the curve $y = f(x)$, the x–axis and the y–axis is rotated about the y-axis. **3**

End of Question 12

Marks

Question 13 (15 marks) Use SEPARATE writing paper.

(a) A spherical metal ball is heated so that its diameter is increasing at a constant rate of 0.005 m/s. Given the surface area of the sphere is $S = \pi D^2$ where D is the diameter of the sphere, at what rate is the surface area of the metallic ball increasing when its diameter is 6 metres? Correct your answer to 2 decimal places. **2**

(b) The speed v (cm/s) of a particle moving in a straight line is given by $v^2 = 12 + 8x - 4x^2$, where the magnitude of its displacement from a fixed point O is x (cm).

(i) Show that the motion is simple harmonic. **1**

(ii) Find the amplitude of the motion. **2**

(iii) Find the maximum speed of the particle and state where it occurs. **2**

(c) Let $f(x) = e^x + \ln x$.

(i) Show that $f(x)$ is a monotonically increasing function for $x > 0$. Hence explain why $f(x)$ has an inverse. **2**

(ii) The graphs of $y = f(x)$ and $y = f^{-1}(x)$ meet at exactly one point P. Let α be the x-coordinate of P. Explain why α is a root of the equation **1**

$$e^x + \ln x - x = 0.$$

(iii) Take 0.5 as a first approximation for α. Use one application of Newton's method to find a second approximation for α correct to 3 significant figures. **2**

(d) Prove by mathematical induction that, for $n \geq 1$, **3**

$$\frac{3}{1\times2\times2}+\frac{4}{2\times3\times2^2}+\frac{5}{3\times4\times2^3}+\cdots+\frac{n+2}{n(n+1)2^n}=1-\frac{1}{(n+1)2^n}$$

End of Question 13

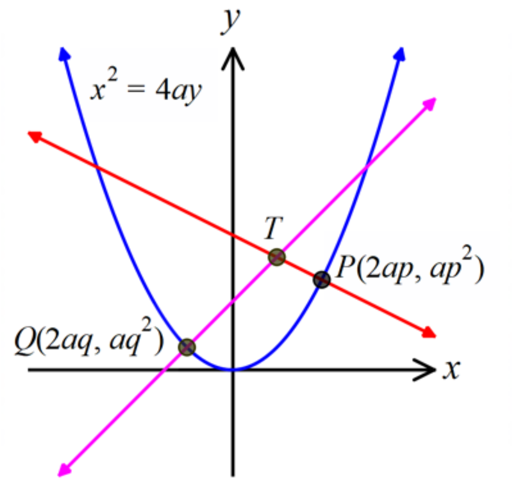
Marks

Question 14 (15 marks) Use SEPARATE writing paper.

(a) Use the substitution $u = 4 - x^2$ to evaluate **3**

$$\int_0^2 \frac{x}{\sqrt{4-x^2}}\,dx.$$

(b) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are points on the parabola $x^2 = 4ay$. The normal to the parabola at P and Q intersect at the point $T(-apq(p+q), a(2+p^2+pq+q^2))$. (DO NOT prove this)



(i) If $q = -3p$, show that the coordinates of T are $(-6ap^3, 2a + 7ap^2)$. **1**

(ii) There is a third normal to the parabola at the point $R\ (2ar, ar^2)$ that passes through the point T. Show that the parameter corresponding to the point R satisfies the equation **2**

$$r^3 - 7p^2r + 6p^3 = 0.$$

(iii) Hence find the coordinates of the point R in terms of p. **2**

Question 14 continues on page 11

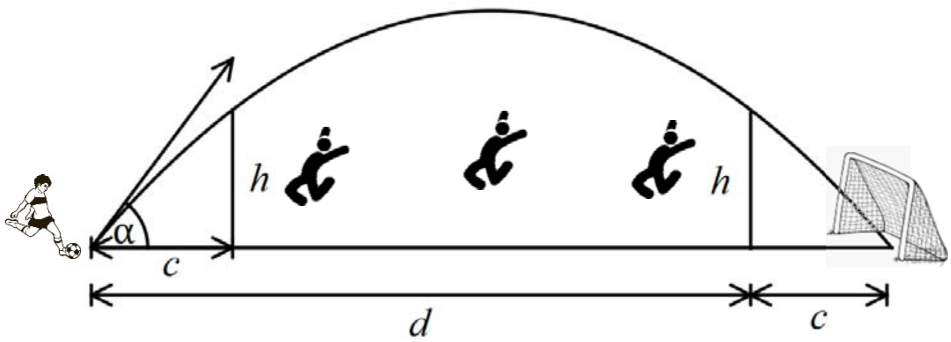
Question 14 (continued)

(c) A soccer ball is at ground level on a horizontal plane. The ball was kicked at an angle α and with velocity v metres per second. You may assume that if the origin is taken to be the point of projection, the path of the ball is given by the equation

$$y = x\tan\alpha - \frac{gx^2\sec^2\alpha}{2v^2}$$

where g ms^{-2} is the acceleration due to gravity. (DO NOT prove this)

Steve Beckham wants to calculate the "perfect" angle to kick the ball to reach the goal. The defenders are standing between c metres and at most d metres from Steve. The defenders cannot jump higher than h metres. The trajectory of the ball is shown in the diagram below.



(i) Explain why c and d are solutions to the equation **1**

$$\frac{g\sec^2\alpha}{2v^2}x^2 - x\tan\alpha + h = 0.$$

(ii) Show that $c + d = \dfrac{2v^2\tan\alpha}{g\sec^2\alpha}$. **1**

(iii) Hence show that $\tan\alpha = \dfrac{h(c+d)}{cd}$. **2**

(iv) The horizontal range of the soccer ball is given by $R = \dfrac{v^2\sin 2\alpha}{g}$. (DO NOT prove this) **3**

Use the result in part (iii) to show $\dfrac{v^2}{g} = \dfrac{c^2d^2 + h^2R^2}{2hcd}$.

End of Exam

Section I

Q1	Q2	Q3	Q4	Q5
(A)	(D)	(D)	(B)	(A)
Q6	**Q7**	**Q8**	**Q9**	**Q10**
(C)	(C)	(A)	(B)	(C)

Question 1

$\sin x + \cos x = R\sin(x+\alpha)$ where

$R = \sqrt{1^2+1^2} = \sqrt{2}$ and $\alpha = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$

∴ (A)

Question 2

$3x + y = 4 \ \rightarrow \ m_1 = -3$

$x - 2y = 5 \ \rightarrow \ m_2 = \frac{1}{2}$

$$\therefore \tan\theta = \left|\frac{-3-\frac{1}{2}}{1+(-3)\times\frac{1}{2}}\right| = 7$$

$\theta = \tan^{-1}(7)$

∴ (D)

Question 3

factor $x + 1 \rightarrow x = -1$ is a root

(I) $(-1)^{2018} - 1 = 0$ (IV) $(-1)^{2017} + 1 = 0$

∴ (D)

Question 4

Let O be the centre of circle

$\angle OBC = 90°$ (radius ⊥ tangent)

$\angle ODC = 90°$ (radius ⊥ tangent)

$\angle BOD = 2\alpha$

(angle at centre twice angle at circumference)

$2\alpha + 90° + 90° + \beta = 360°$

(angle sum of quadrilateral $OBCD$)

$\therefore 2\alpha + \beta = 180°$

∴ (B)

Question 5

$\ddot{x} = -4x \rightarrow n = 2.$

$x = a\sin 2t \rightarrow v = 2a\cos 2t,$

$t = 0, v = 4 \rightarrow 4 = 2a\cos 0 \rightarrow a = 2$

$\therefore x = 2\sin 2t$

∴ (A)

Question 6

$$\frac{d}{dx}\left(6\sin^{-1}\frac{x}{3}\right) = 6\times\frac{1}{\sqrt{3^2-x^2}} = \frac{6}{\sqrt{9-x^2}}$$

∴ (C)

Question 7

Reflect $y = f(x)$ about $y = x$.

∴ (C)

Question 8

Vertical asymptote, denominator = 0

$(x-2)(x+1) = 0 \rightarrow x = 2, x = -1$

Horizontal asymptote,

$$y = \lim_{x\to\infty}\frac{x+3}{(x-2)(x+1)} = \lim_{x\to\infty}\frac{x+3}{x^2-x-2}$$

$$= \lim_{x\to\infty}\frac{\frac{x}{x^2}+\frac{3}{x^2}}{\frac{x^2}{x^2}-\frac{x}{x^2}-\frac{2}{x^2}} = \lim_{x\to\infty}\frac{\frac{1}{x}+\frac{3}{x^2}}{1-\frac{1}{x}-\frac{2}{x^2}}$$

$$= \frac{0+0}{1-0-0} = 0$$

∴ (A)

Question 9

Point $(4ap, 4ap^2)$ lies on the equation of the normal by substitution into the equation of normal.

LHS $= x + 2py$

$= 4ap + 2p \times 4ap^2$

$= 4ap + 8ap^3 =$ RHS

∴ (B)

Question 10

$x = 3\cos 2t + 4\sin 2t.$

$v = -6\sin 2t + 8\cos 2t$

$\ddot{x} = -12\cos 2t - 16\sin 2t$

$= -4(3\cos 2t + 4\sin 2t)$

$= -4x$

∴ (C)

Section II

Question 11

(a) $x = \frac{-3\times1+2\times5}{2-3} = -7$, $y = \frac{-3\times4+2\times2}{2-3} = 8$

$\therefore M = (-7, 8)$

(b) $\dfrac{2}{x} \le \dfrac{x}{2} \qquad x \ne 0$

$\dfrac{2}{x} \times x^2 \le \dfrac{x}{2} \times x^2$

$2x \le \dfrac{x^3}{2}$

$4x \le x^3$

$0 \le x^3 - 4x$

$0 \le x(x-2)(x+2)$

$\therefore -2 \le x < 0, x \ge 2$

(c) $\lim\limits_{x\to 0} \dfrac{\sin\left(\frac{x}{3}\right)}{4x} = \dfrac{1}{4}\lim\limits_{x\to 0}\dfrac{\sin\left(\frac{x}{3}\right)}{x}$

$= \dfrac{1}{4} \times \dfrac{1}{3}\lim\limits_{x\to 0}\dfrac{\sin\left(\frac{x}{3}\right)}{\frac{x}{3}}$

$= \dfrac{1}{4} \times \dfrac{1}{3} \times 1 = \dfrac{1}{12}$

(d) **(i)** $\dfrac{dT}{dt} = -kAe^{-kt}$

$= -k\left(Ae^{-kt} - 12 + 12\right)$

$= -k(T+12)$

(ii) When $t = 0$, $T = 24°$

$24 = Ae^{-k\times 0} - 12$

$24 = A \times 1 - 12$

$A = 24 + 12 = 36$

(iii) $9 = 36e^{-k\times 15} - 12$

$21 = 36e^{-15k}$

$\dfrac{21}{36} = e^{-15k}$

$k = -\dfrac{1}{15}\ln\left(\dfrac{21}{36}\right) \approx 0.0359$

(iv) $0 = 36e^{-kt} - 12$

$12 = 36e^{-kt}$

$\dfrac{12}{36} = e^{-kt}$

$t = -\dfrac{1}{k}\ln\left(\dfrac{12}{36}\right)$

$\approx -\dfrac{1}{0.0359}\ln\left(\dfrac{12}{36}\right)$

$= 30.6... \approx 31\text{min}$

(e) **(i)** $e^{3x} + e^{3y} = \left(e^x\right)^3 + \left(e^y\right)^3$

$= \left(e^x + e^y\right)\left(\left(e^x\right)^2 - e^x e^y + \left(e^y\right)^2\right)$

(ii) $e^{3x} + e^{3y} = \left(e^x + e^y\right)\left(\left(e^x\right)^2 - e^x e^y + \left(e^y\right)^2\right)$

$10 = 3\left(\left(e^x\right)^2 - e^x e^y + \left(e^y\right)^2\right)$

$10 = 3\left(\left(e^x\right)^2 + 2e^x e^y + \left(e^y\right)^2 - 3e^x e^y\right)$

$10 = 3\left(\left(e^x + e^y\right)^2 - 3e^x e^y\right)$

$10 = 3\left((3)^2 - 3e^{x+y}\right)$

$e^{x+y} = \dfrac{17}{9}$

$x + y = \ln\left(\dfrac{17}{9}\right)$

Question 12

(a) **(i)** $\dfrac{OA}{h} = \tan\left(90° - 46°\right)$

$OA = h\tan 44°$

(ii) $\left(h\tan 55°\right)^2 = 220^2 + \left(h\tan 44°\right)^2$

$h^2\left(\tan^2 55° - \tan^2 44°\right) = 220^2$

$h = \dfrac{220}{\sqrt{\tan^2 55° - \tan^2 44°}}$

$= 209.09...$

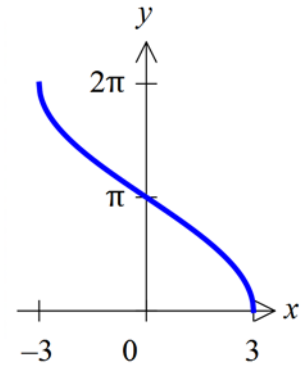
≈ 209.1

(b) **(i)** $\angle BEF = \angle EAD$ (corresponding angle on parallel lines, $AD \| EF$)
$\angle EAD + \angle FCB = 180$ (Opposite angles of cyclic quad ABCD)
$\therefore \angle BEF + \angle FCB = 180$
$\therefore$ *EBCF* is a cyclic quadrilateral (opposite angles are supplementary)

(ii) $\angle GHF = \angle GCB$ (angles in the same segment, cyclic quad *ABCD*)
$\angle EFB = \angle ECB$ (angles in the same segment, in cyclic quad *EBCF*)
$\angle GHF = \angle EFB$
$GH \| EF$
(corresponding angles are equal)

(c) **(i)** $-3 \le x \le 3,\ 0 \le y \le 2\pi$

(ii)



(iii)

$$x = 2\cos^{-1}\left(\frac{y}{3}\right)$$

$$\frac{x}{2} = \cos^{-1}\left(\frac{y}{3}\right)$$

$$\cos\left(\frac{x}{2}\right) = \frac{y}{3}$$

$$y = 3\cos\left(\frac{x}{2}\right)$$

$$\therefore f^{-1}(x) = 3\cos\left(\frac{x}{2}\right)$$

Domain: $0 \le x \le 2\pi$

(iv)

$$V = \pi\int_0^{\pi} x^2 dy$$

$$= \pi\int_0^{\pi}\left(3\cos\left(\frac{y}{2}\right)\right)^2 dy$$

$$= 9\pi\int_0^{\pi}\frac{1+\cos y}{2}dy$$

$$= \frac{9\pi}{2}\left[y + \sin y\right]_0^{\pi}$$

$$= \frac{9\pi^2}{2}$$

Question 13

(a)

$$\frac{dS}{dt} = \frac{dS}{dD}\times\frac{dD}{dt}$$

$$= 2\pi D\times\frac{dD}{dt}$$

$$= 2\pi\times 6\times 0.005$$

$$\approx 0.19$$

(b) **(i)**

$$\frac{1}{2}v^2 = 6 + 4x - 2x^2$$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 4 - 4x$$

$$\ddot{x} = -4(x-1)$$

i.e. $\ddot{x}$ is proportional to x.

(ii) $v = 0 \rightarrow 12 + 8x - 4x^2 = 0$

$-4(x^2 - 2x - 3) = 0$

$(x-3)(x+1) = 0 \rightarrow x = 3, -1$

Amplitude = 3 – 1 = 2

(iii) Max speed occur at the centre of motion, at $x = 1$,

$v^2 = 12 + 8(1) - 4(1)^2$

$v^2 = 16 \rightarrow v_{max} = 4$ m/s

(c) **(i)** Since $e^x > 0$ and $\frac{1}{x}$ for $x > 0$,

$$\therefore f'(x) = e^x + \frac{1}{x} > 0 \text{ for } x > 0.$$

A monotonic increasing function implies 1 to 1 function as there are no turning points.

(ii) The two graphs intersect on the line $y = x$. So α will satisfy $f(x) = x \rightarrow e^x + \ln x = x$.

Hence $e^x + \ln x - x = 0$

(iii)

$$x = 0.5 - \frac{e^{0.5} + \ln 0.5 - 0.5}{e^{0.5} + \frac{1}{0.5} - 1}$$

$$\approx 0.328$$

(d) Prove $n = 1$ is true

$$LHS = \frac{3}{1\times2\times2} = \frac{3}{4} \quad RHS = 1 - \frac{1}{2\times2} = \frac{3}{4}$$

Therefore n = 1 is true.

Assume $n = k$ is true

$$\frac{3}{1\times2\times2} + \frac{4}{2\times3\times2^2} + \cdots + \frac{k+2}{k(k+1)2^k} = 1 - \frac{1}{(k+1)2^k}$$

Prove $n = k + 1$ is true, i.e.

$$\frac{3}{1\times2\times2} + \cdots + \frac{k+2}{k(k+1)2^k} + \frac{k+3}{(k+1)(k+2)2^{k+1}}$$

$$= 1 - \frac{1}{(k+2)2^{k+1}}$$

$$LHS = \frac{3}{1\times2\times2}+\cdots+\frac{k+2}{k(k+1)2^k}+\frac{k+3}{(k+1)(k+2)2^{k+1}}$$
$$=1-\frac{1}{(k+1)2^k}+\frac{k+3}{(k+1)(k+2)2^{k+1}}$$
$$=1-\left[\frac{2(k+2)}{(k+1)(k+2)2^{k+1}}-\frac{k+3}{(k+1)(k+2)2^{k+1}}\right]$$
$$=1-\frac{2(k+2)-k-3}{(k+1)(k+2)2^{k+1}}$$
$$=1-\frac{\cancel{k+1}}{\cancel{(k+1)}(k+2)2^{k+1}}$$
$$=1-\frac{1}{(k+2)2^{k+1}}$$
$$= RHS$$

$\therefore$ $n = k + 1$ is true whenever $n = k$ is true.

$\therefore$ As it is true for $n = 1$, by induction, the result is true for $n \geq 1$.

Question 14

(a) $u = 4 - x^2 \rightarrow du = -2xdx$

$x = 2 \rightarrow u = 0, \quad x = 0 \rightarrow u = 4$

$$\int_0^2 \frac{x}{\sqrt{4-x^2}}dx = -\frac{1}{2}\int_4^0 \frac{1}{\sqrt{u}}du$$
$$=\frac{1}{2}\int_0^4 u^{-\frac{1}{2}}du = \left[\sqrt{u}\right]_0^4 = 2-0=2$$

(b) (i) Substitute $q = -3p$

$T(-ap(-3p)(p + -3p),$
$a(2 + p^2 + p(-3p) + (-3p)^2))$
$= (3ap^2(-2p), a(2 + p^2 - 3p^2 + 9p^2))$
$= (-6ap^3, 2a + 7ap^2)$

(ii) Equation of the normal at R is
$x + ry = ar^3 + 2ar$.
Since it passes through the point T
The coordinates of T satisfies the above equations.

$-6ap^3 + r(2a + 7ap^2) = ar^3 + 2ar$
$-6ap^3 + 2ar + 7ap^2r = ar^3 + 2ar$
$-6ap^3 + 7ap^2r = ar^3$
$ar^3 - 7ap^2r + 6ap^3 = 0$
$r^3 - 7p^2r + 6p^3 = 0$

(iii) The parameters of the three numbers satisfies the cubic equation in (ii).
Sum of roots = 0 → $p + -3p + r = 0$
$-2p + r = 0$ → $r = 2p$
Coordinates of R is $(2a(2p), a(2p)^2)$
Coordinates of R is $(4ap, 4ap^2)$

(c) (i) The projectile passes through the points (c, h) and (d, h)

$$h = x\tan\alpha - \frac{gx^2\sec^2\alpha}{2v^2}$$
$$\frac{gx^2\sec^2\alpha}{2v^2} - x\tan\alpha + h = 0$$

Therefore $x = c$ and $x = d$ are the roots of the quadratic equation.

(ii) Sum of the roots

$$c+d = -(-\tan\alpha)\div\frac{g\sec^2\alpha}{2v^2}$$
$$=\tan\alpha\times\frac{2v^2}{g\sec^2\alpha}=\frac{2v^2\tan\alpha}{g\sec^2\alpha}$$

(iii) Product of roots

$$cd = h\div\frac{g\sec^2\alpha}{2v^2}=\frac{2v^2h}{g\sec^2\alpha}$$

Therefore using the result from (ii)

$$\frac{c+d}{cd}=\frac{2v^2\tan\alpha}{g\sec^2\alpha}\div\frac{2v^2h}{g\sec^2\alpha}$$
$$=\frac{\cancel{2v^2}\tan\alpha}{\cancel{g\sec^2\alpha}}\times\frac{\cancel{g\sec^2\alpha}}{\cancel{2v^2}h}=\frac{\tan\alpha}{h}$$
$$\therefore \tan\alpha = \frac{h(c+d)}{cd}$$

(iv) Rearranging $R = \dfrac{v^2\sin2\alpha}{g}$

$$v^2 = \frac{Rg}{\sin2\alpha}$$
$$=\frac{Rg}{\frac{2t}{1+t^2}}\left(\text{where } t=\tan\alpha\right)$$
$$=\frac{Rg}{2t}\left(1+t^2\right)$$
$$=\frac{Rg}{2}\left(\frac{1}{t}+t\right)$$
$$=\frac{Rg}{2}\left(\frac{cd}{h(c+d)}+\frac{h(c+d)}{cd}\right)$$
$$=\frac{Rg}{2}\left(\frac{cd}{hR}+\frac{hR}{cd}\right)$$
$$=\frac{\cancel{R}g}{2}\left(\frac{c^2d^2+h^2R^2}{h\cancel{R}cd}\right)$$
$$\frac{v^2}{g}=\frac{c^2d^2+h^2R^2}{2hcd}$$