

Name:

Maths Class:

Year 12

Mathematics Extension 1

HSC Course

(Assessment 4)

TRIAL HSC

August, 2018

Time allowed: 120 minutes + 5 minutes reading time

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A NESA reference sheet is provided for use in this examination

Section 1 Multiple Choice 10 Marks

Questions 1 – 10

Allow approximately 15 minutes

Section II Questions 11 – 14 60 Marks

Allow 1 hour 45 minutes for this section

Section 1

10 marks

Attempt questions 1 - 10

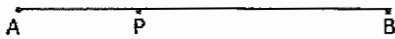
Allow about 15 minutes for this section.

Use the multiple choice answer sheet located in your answer booklet.

1. The acute angle between the two lines $3x + y - 1 = 0$ and $4x - 6y = 5$, is closest to;

- A. 15°
- B. 38°
- C. 52°
- D. 75°

2. The point P divides the interval AB in the ratio 3 : 5.

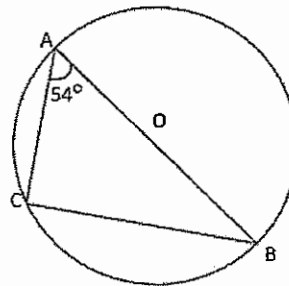


In what external ratio does A divide the interval PB?

- A. 3 : 8
- B. 8 : 3
- C. 5 : 8
- D. 8 : 5

3. In the figure below, AB is the diameter of the circle with radius 8 cm. What is the length of the minor arc BC?

- A. $\frac{12\pi}{5} \text{ cm}$
- B. $\frac{8\pi}{5} \text{ cm}$
- C. $\frac{24\pi}{5} \text{ cm}$
- D. $\frac{24}{5} \text{ cm}$



4. When $P(x)$ is divided by $x^2 + 5x - 6$, the remainder is the polynomial $R(x) = 2x - 5$.

What is the remainder when $P(x)$ is divided by $(x - 1)$?

- A. -7
- B. -6
- C. 7
- D. -3

5. A particle moving with acceleration $\ddot{x} = 12 \sin 3t \text{ m/s}^2$, starts at rest at $x = 4$. What is the maximum speed of the particle?

- A. 0 m/s
- B. 4 m/s
- C. 8 m/s
- D. 12 m/s

6. If $f(x) = x^2 - 2x$ then for $x \geq 1$, $y = f^{-1}(x)$ has the equation?

- A. $f^{-1}(x) = (x + 1)$
- B. $f^{-1}(x) = 1 - \sqrt{x + 1}$
- C. $f^{-1}(x) = 1 + \sqrt{x + 1}$
- D. $f^{-1}(x) = y^2 - 2y$

7. $\frac{d}{dx} \left[\cos^{-1} \left(\frac{1}{x} \right) \right]$ is;

- A. $\frac{-1}{\sqrt{x^2 - 1}}$
- B. $\frac{-1}{x\sqrt{x^2 - 1}}$
- C. $\frac{1}{\sqrt{x^2 - 1}}$
- D. $\frac{1}{x\sqrt{x^2 - 1}}$

8. $\int_0^1 5^{2x} dx$ is;

A. $12\ln 5$

B. $\frac{12}{\ln 5}$

C. $\frac{24}{\ln 5}$

D. $24\ln 5$

9. How many solutions does the equation $\sin x = \sin 2x$ have in the domain, $0 < x < 2\pi$?

A. 2

B. 3

C. 4

D. 5

10. A glass of water with a temperature of 4 degrees is placed in a room with a temperature of 18 degrees. After 5 minutes the temperature of the water has risen by 6 degrees.

The equation satisfying this situation is;

A. $T = 18 + 14e^{-\frac{t}{5} \ln \frac{4}{7}}$

B. $T = 18 - 4e^{\frac{t}{5} \ln 2}$

C. $T = 18 - 14e^{\frac{t}{5} \ln \frac{4}{7}}$

D. $T = 18 - 14e^{\frac{t}{5} \ln \frac{6}{7}}$

End of Section 1

Section 11

60 marks

Attempt questions 11 – 14

Allow about 1 hour and 45 minutes for this section.

Start each question at the TOP of a NEW page in your answer booklet

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 15 marks

a. Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$ 2

b. Solve $\frac{x-1}{x+2} \leq 4$ 3

c. Find,

i. $\frac{d}{dx}(4 \sin^{-1} 3x)$ 2

ii. $\int (\cos^2 3x) dx$ 2

d. Give the exact value of;

$$\int_0^{0.5} \frac{2}{1+4x^2} dx$$
 2

e. Show that $\int_0^{\frac{\pi}{4}} (\tan^2 x + \tan x + 1) dx = 1 + \ln \sqrt{2}$ 2

f. Write down the general solution to

$$\sin 2x = -\frac{1}{2}$$
 2

Question 12

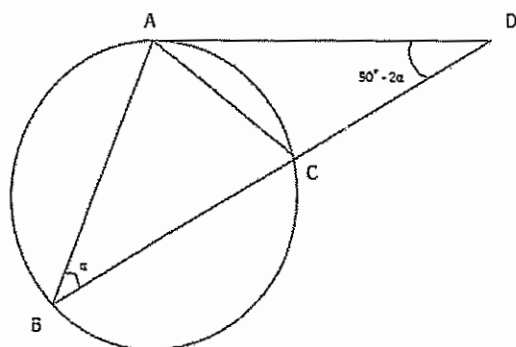
15 marks

- a. The equation $4x^3 - 8x + 1 = 0$ has roots α , β and γ
What is the value of

i. $\alpha\beta\gamma$ 1

ii. $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ 2

- b. Triangle ABC is inscribed in a circle. The tangent to the circle at A meets the secant BC at D $\angle ABC = \alpha$ and $\angle ADB = 90^\circ - 2\alpha$.



- i. Find an expression, in terms of α for $\angle ACB$, giving reasons 2

- ii. Show that BC is a diameter of the circle. 2

c. Use the substitution $u = x + 1$ to evaluate $\int_0^{15} \frac{x}{\sqrt{x+1}} dx$ 3

- d. A particle moving in Simple Harmonic motion obeys the rule $x = A \cos(2t + \alpha)$, where α is acute.
Initially, the particle is 6 metres to the left of the origin, travelling with a velocity of $12\sqrt{3}$ m/s.

- i. Find the period of the particle's motion. 1

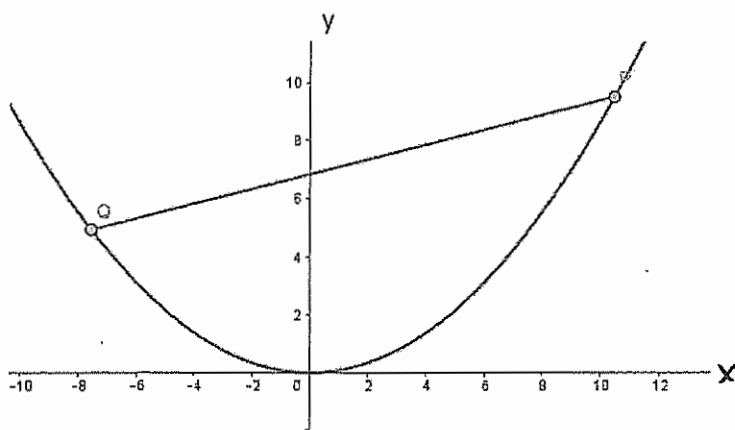
- ii. Find the values of A and α 2

- iii. When does the particle first reach the centre of motion? 2

Question 13

15 marks

a. The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola with equation $x^2 = 4ay$.



- i. Show that the chord PQ has equation $2y = (p + q)x - 2apq$ 2
- ii. The chord PQ crosses the x -axis at $(-2a, 0)$.
Show that $p + q = -pq$ 1
- iii. Hence, find the locus of M , the midpoint of PQ . 3

b. A particle is moving along the x -axis with velocity $v = \sqrt{8x - x^2}$ m/s.

- i. Find the acceleration of the particle at displacement 3 m. 2
- ii. By considering the velocity and acceleration of the particle, describe the motion of the particle at $x = 3$ 2

c. Consider the function $f(x) = e^x - e^{-x}$

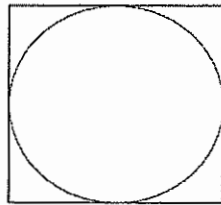
- i. Show that $y = f(x)$ is increasing for all values of x 1
- ii. Show that $f^{-1}(x) = \ln \left[\frac{\sqrt{x^2 + 4} + x}{2} \right]$ and state its domain. 3
- iii. Hence, or otherwise solve the equation $e^x - e^{-x} = 3$ 1

Question 14 15 marks

- a. Use Mathematical Induction to show that for all positive integers $n \geq 1$:

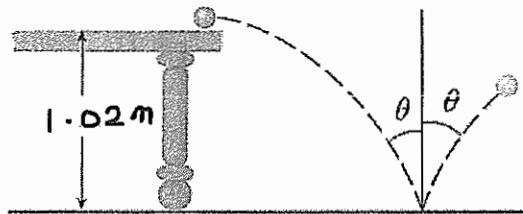
$$\frac{3}{1 \times 2 \times 2^1} + \frac{4}{2 \times 3 \times 2^2} + \dots + \frac{n+2}{n(n+1) \times 2^n} = 1 - \frac{1}{(n+1) \times 2^n} \quad 4$$

- b. A circle is inscribed in a square so that the sides of the square are tangents to the circle.
The circumference of the circle is increasing at a constant rate of 2 m/s, causing the sides of the square to increase, so that the sides remain touching as tangents.



Find the rate at which the area of the square is increasing when its' perimeter is 16cm. 3

- c. A ball rolls off a table with a speed of 0.6 m/s. The table is 1.02m high.



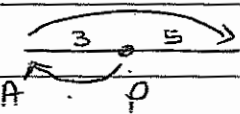
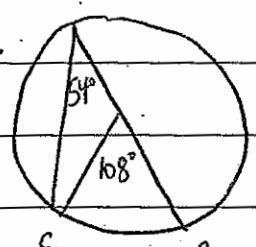
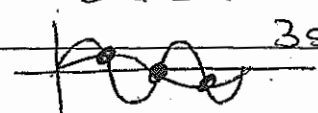
- i. Ignoring air resistance, setting the origin at ground level, gravity as 10 m/s^2 ,
Show that the trajectory of the ball obeys the rule, $y = 1.02 - \frac{125}{9}x^2$ 2
- ii. Determine the point at which the ball hits the floor and find its speed at the instant of impact. 2
- iii. Find the angle, θ , between the path of the ball and the vertical line drawn through the point of impact. 1
- iv. Suppose the ball rebounds from the floor at the same angle with which it hits the floor, but loses 20% of its speed due to energy absorbed by the ball on impact.

Where does the ball strike the floor for the second bounce, measured along the horizontal, from the edge of the table? 3

End of assessment

Year 12 - 2018 - Trial Extension 1
Suggested Solutions

Section 1	
1. D	6. C
2. A	7. D
3. C	8. B
4. D	9. B
5. C	10. C

Solutions to M/C.	
1. $3x + y - 1 = 0$ $m_1 = -3$	6. $f(x) = x^2 - 2x \quad x > 1$ $f^{-1}(x) \rightarrow 1 + \sqrt{x+1}$ [C]
$4x = 5 + 6y \quad m_2 = 2/3$	$x = y^2 - 2y \quad y = 1 \pm \sqrt{x+1}$
$\tan \theta = \left \frac{m_1 - m_2}{1 + m_1 m_2} \right $	7. $\frac{d}{dx} [\cos^{-1}(1/x)]$
$\theta \approx 75^\circ$ [D]	$= \frac{-1}{\sqrt{1^2 - (1/x)^2}} \times \frac{-1}{x^2}$
2.  PA : AB $= 3 : 8$ [A]	$= \frac{+1}{x\sqrt{x^2 - 1}}$ [D]
3.  $l = r\theta$ (θ in rad) $= 8 \times \frac{108\pi}{180}$ $= \frac{24\pi}{5}$ [C]	8. $\int_0^1 5^{2x} dx = \int_0^1 e^{\ln 5^{2x}} dx$ $= \left. \frac{5^{2x}}{2 \ln 5} \right _0^1$ $= \frac{5^2 - 5^0}{2 \ln 5} = \frac{24}{2 \ln 5}$ [B]
4. $P(x) = (x^2 + 5x - 6)Q(x) + 2x - 5$ $P(1) = 0 \times Q(1) + 2(1) - 5$ $= -3$ [D]	9. watch domain <u>does not</u> include 0 & 2π  3 sol ⁿ [B]
5. $\ddot{x} = 12 \sin 3t$ $\dot{x} = \int 12 \sin 3t dt \quad t=0$ $v = -4 \cos 3t + C_1 \quad v=0$ $0 = -4 \times \cos 0 + C_1 \quad C_1 = 4$	10. $T = B + A e^{kt}$ $4 = 18 + A e^0 \quad A = -14$ Now $10 = 18 - 14 e^{k(5)}$ $k = \frac{1}{5} \ln 4/7$ \therefore [C]
$v = 4 - 4 \cos 3t$ $= -4 \cos 3t + 4$ -4 to 4 $\therefore 0 \leq v \leq 8$ [C]	

Section II

Question 11

$$a) \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{1 - (1 - 2\sin^2 x)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x^2}$$

$$= 2$$

$$b) \frac{x-1}{x+2} \leq 4 \quad (x \neq -2)$$

$$(x-1)(x+2) \leq 4(x+2)^2$$

$$(x-1)(x+2) - 4(x+2)^2 \leq 0$$

$$(x+2)[x-1-4x-8] \leq 0$$

$$(x+2)(-3x-9) \leq 0$$

$$-3(x+2)(x+3) \leq 0$$



$$\therefore x \leq -3 \text{ or } x > -2$$

$$c) \frac{d}{dx} (4\sin^{-1} 3x)$$

$$= 4 \times \frac{1}{\sqrt{1-(3x)^2}} \times 3$$

$$= \frac{12}{\sqrt{1-9x^2}}$$

$$\sqrt{1-9x^2}$$

$$d) \int \frac{1}{2} (1 + \cos 6x) dx$$

$$= \frac{1}{2} \left[x + \frac{1}{6} \sin 6x \right] + C$$

$$d) \int_0^{0.5} \frac{2}{1+4x^2} dx$$

$$= 2 \int_0^{0.5} \frac{1}{4(\frac{1}{4} + x^2)} dx$$

$$= \frac{1}{2} \int_0^{0.5} \frac{1}{(\frac{1}{2})^2 + x^2} dx$$

$$= \frac{1}{2} \times \frac{1}{\frac{1}{2}} \tan^{-1}(2x) \Big|_0^{0.5}$$

$$= \left[\tan^{-1} 2x \right]_0^{0.5}$$

$$= \tan^{-1} 1 - \tan^{-1} 0$$

$$= \pi/4 - 0$$

$$= \pi/4$$

$$e) \int_0^{\pi/4} (\tan^2 x + \tan x + 1)$$

$$= \int_0^{\pi/4} \sec^2 x + \frac{\sin x}{\cos x} dx$$

$$= \tan x + -\ln(\cos x) \Big|_0^{\pi/4}$$

$$= \tan \pi/4 - \ln(\cos \pi/4) - (\tan 0 - \ln \cos 0)$$

$$= 1 - \ln(1/\sqrt{2}) - (0 - \ln 1)$$

$$= 1 - \ln(1/\sqrt{2})$$

$$= 1 + \ln \sqrt{2}$$

$$f) 2\theta = n\pi + (-i)^n \sin^{-1}(-1/2)$$

$$\theta = \frac{n\pi}{2} + \frac{(-1)^n (-\pi/6)}{2} \leftarrow$$

either

$$\theta = \frac{n\pi}{2} - (-1)^n \left(\frac{\pi}{12} \right) \leftarrow$$

Question 12

a) $a=4$ $b=0$ $c=-8$ $d=1$

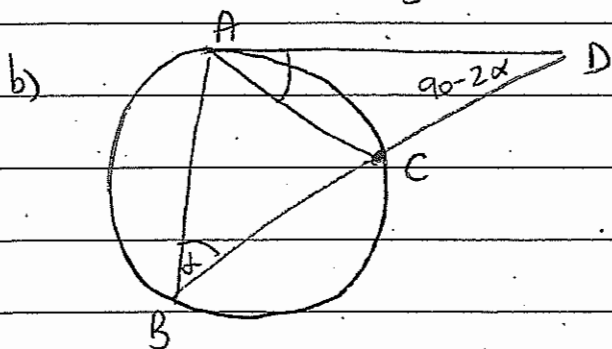
$$1. \alpha\beta\gamma = -d/a$$

$$= -1/4$$

$$11. \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \alpha\gamma + \beta\gamma}{\alpha\beta\gamma}$$

$$= -8/4 \div -1/4$$

$$= 8$$



$\angle DAC = \angle ABC$ (angle between a tangent and a chord equals angle in the alternate segment)

$= \alpha$

Now

$$\angle ACB = \angle CAD + \angle ADC$$

$$= \alpha + (90 - 2\alpha)$$

$$= 90 - \alpha$$

(exterior angle of $\triangle ACD$).

11. Now $\triangle ABC$

$$90 - \alpha + \alpha + \angle BAC = 180^\circ$$

(angle sum)

$$\therefore \angle BAC = 90^\circ$$

and BC is the diameter of

circle as angle in the semi-circle is 90° .

c) $u=x+1$ $x=15$ $u=16$

$x=0$ $u=1$

$$= \int_1^{16} \frac{u-1}{\sqrt{u}} du \quad du=dx$$

$$= \int_1^{16} \sqrt{u} - u^{-1/2} du$$

$$= \left[\frac{2}{3} u^{3/2} - 2 u^{1/2} \right]_1^{16}$$

$$= \frac{2}{3} (16)^{3/2} - 2\sqrt{16} - \left(\frac{2}{3} - 2 \right)$$

$$= 36$$

d) $x = A \cos(2t + \alpha)$

$t=0$

$x = -6$ 0

$v = 12\sqrt{3}$

1. Period = $2\pi/n = \pi$

11. $t=0$ $x = -6$

$$-6 = A \cos \alpha \quad \text{--- (1)}$$

$$v = -2A \sin(2t + \alpha)$$

$t=0$ $v = 12\sqrt{3}$

$$12\sqrt{3} = -2A \sin \alpha$$

$$-6\sqrt{3} = A \sin \alpha \quad \text{--- (2)}$$

(2)/(1) $\tan \alpha = \frac{6\sqrt{3}}{6}$

$$\alpha = \pi/3$$

Now $-6 = A \cos \pi/3$

$$A = -12$$

111) centre $\ddot{x} = 0$ @ $x = 0$

$$0 = -12 \left(\cos(2t + \pi/3) \right)$$

$$2t + \pi/3 = \pi/2 \quad (\text{1st time})$$

$$t = \pi/12 \quad (t > 0)$$

Question 13.

$$\begin{aligned} \text{a) } M_{pq} &= \frac{ap^2 - aq^2}{2ap - 2aq} \\ &= \frac{a(p+q)(p-q)}{2a(p-q)} \end{aligned}$$

$$M_{pq} = \frac{p+q}{2}$$

$$\circ \circ y - ap^2 = \frac{p+q}{2}(x - 2ap)$$

$$2y - 2ap^2 = (p+q)x - 2ap^2 - 2apq$$

$$2y = (p+q)x - 2apq$$

* OED

$$\text{ii) } 0 = -2a(p+q) - 2apq$$

$$2a(p+q) = -2apq$$

$$p+q = -pq$$

iii)

$$\text{Midpt } M = \left[\frac{2ap + 2aq}{2}, \frac{ap^2 + aq^2}{2} \right]$$

$$x = a(p+q) \quad y = \frac{a}{2}(p^2 + q^2)$$

Now

$$\frac{x}{a} = p+q \quad \text{and} \quad pq = -\frac{x}{a}$$

from (ii)

$$\frac{2y}{a} = (p+q)^2 - 2pq$$

$$\frac{2y}{a} = \left(\frac{x}{a}\right)^2 - 2\left(-\frac{x}{a}\right)$$

$$2ay = x^2 + 2ax \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{either}$$

$$\text{OR } y = \frac{x^2}{2a} + x$$

$$\text{b) } v = \sqrt{8x - x^2} \quad \text{m/s.}$$

$$\begin{aligned} \text{i. } \ddot{x} &= \frac{d}{dx} \left(\frac{1}{2}v^2 \right) \\ &= \frac{d}{dx} \left[\frac{1}{2}(8x - x^2) \right] \\ &= \dots \\ &= 4 - x \end{aligned}$$

$$\circ \circ x = 3 \quad \ddot{x} = 1 \text{ m/s}^2$$

$$\text{ii. at } x = 3 \quad v = \sqrt{15} \text{ m/s}$$
$$\ddot{x} = 1 \text{ m/s}^2$$

$\circ \circ \ddot{x} > 0$ & $v > 0$ means

the particle is moving to the right (+ve direction) speeding up.

$$\text{c) } f(x) = e^x - e^{-x}$$

$$\begin{aligned} \text{i. } f'(x) &= e^x - (-e^{-x}) \\ &= e^x + e^{-x} \end{aligned}$$

as $e^x > 0$ & $e^{-x} > 0$
for all x

$f'(x) > 0$ for all x
and $f(x)$ is increasing
for all x .

cont
next pg

Finding Inverse

$$x = e^y - e^{-y}$$

$$x = e^y - \frac{1}{e^y}$$

$$e^y x = e^{2y} - 1$$

$$e^{2y} - e^y x - 1 = 0$$

Solve as a quadratic

in e^y

$$\therefore e^y = \frac{x \pm \sqrt{x^2 - 4 \times 1 \times (-1)}}{2}$$

$$e^y = \frac{x \pm \sqrt{x^2 + 4}}{2}$$

Making

$$y = f^{-1}(x) \quad x > 0$$

$$y \in \mathbb{R}$$

which means

$$e^y = \frac{x + \sqrt{x^2 + 4}}{2}$$

$$\text{OR } f^{-1}(x) = \ln \left[\frac{x + \sqrt{x^2 + 4}}{2} \right]$$

D: all real x

$$\text{III. } e^x - e^{-x} = 3$$

means (x, y)

↓

$(x, 3)$ into $f(x)$

OR $(3, y)$ into $f^{-1}(x)$

either

$$\begin{cases} \therefore x = \ln \left[\frac{\sqrt{9+4} + 3}{2} \right] \\ x = \ln \left[\frac{\sqrt{13} + 3}{2} \right] \end{cases}$$

Need x equals

alternatively

$$e^x - \frac{1}{e^x} = 3$$

$$e^{2x} - 3e^x - 1 = 0$$

$$\therefore e^x = \frac{3 \pm \sqrt{9 - 4(1)(-1)}}{2 \times 1}$$

$$e^x = \frac{3 \pm \sqrt{13}}{2}$$

as $e^x > 0 \quad \therefore e^x = \frac{3 + \sqrt{13}}{2}$

$$\text{OR } x = \ln \left[\frac{3 + \sqrt{13}}{2} \right]$$

must explain why taking $x > 0$

*

Question 14.

a) Test $n=1$

$$\begin{aligned} \text{LHS} &= \frac{3}{1 \times 2 \times 2} \\ &= \frac{3}{4} \end{aligned} \quad \begin{aligned} \text{RHS} &= 1 - \frac{1}{(1+1) \times 2^1} \\ &= 1 - \frac{1}{4} \\ &= \frac{3}{4} \end{aligned}$$

\therefore LHS = RHS and the statement is true for $n=1$

Assume true for $n=k$

$$\text{i.e. } \frac{3}{1 \times 2 \times 2^1} + \frac{4}{2 \times 3 \times 2^2} + \dots + \frac{k+2}{k(k+1) \cdot 2^k} = 1 - \frac{1}{(k+1) \cdot 2^k}$$

Prove true for $n=k+1$

Aim to Prove:

$$\frac{3}{1 \times 2 \times 2^1} + \frac{4}{2 \times 3 \times 2^2} + \dots + \frac{k+2}{k(k+1) \cdot 2^k} + \frac{k+1+2}{(k+1)(k+2) \cdot 2^{k+1}} = 1 - \frac{1}{(k+2) \cdot 2^{k+1}}$$

$$\text{LHS} = \frac{3}{1 \times 2 \times 2^1} + \dots + \frac{k+2}{k(k+1) \cdot 2^k} + \frac{k+3}{(k+1)(k+2) \cdot 2^{k+1}}$$

$$= 1 - \frac{1}{(k+1) \cdot 2^k} + \frac{k+3}{(k+1)(k+2) \cdot 2^{k+1}} \quad (\text{using the assumption})$$

$$= 1 - \left[\frac{1}{(k+1) \cdot 2^k} - \frac{k+3}{(k+1)(k+2) \cdot 2^{k+1}} \right]$$

$$= 1 - \left[\frac{2(k+2) - (k+3)}{(k+1)(k+2) \cdot 2^{k+1}} \right]$$

$$= 1 - \left[\frac{2k+4-k-3}{(k+1)(k+2) \cdot 2^{k+1}} \right]$$

$$= 1 - \left[\frac{k+1}{(k+1)(k+2) \cdot 2^{k+1}} \right]$$

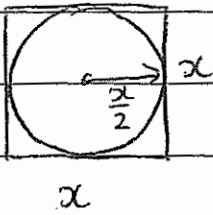
$$= 1 - \frac{1}{(k+2) \cdot 2^{k+1}}$$

= RHS

\therefore If the statement is true for $n=k$ it is also true for $n=k+1$. As it is true for $n=1$ it is also true for $n=2, 3, \dots$ hence true all positive integer n .

Question 14 (cont)

b)



$$\frac{dc}{dt} = 2 \text{ m/s}$$

$$\frac{dc}{dt} = 200 \text{ cm/s}$$

$$A = x^2$$

$$p = 16 \text{ cm}$$

$$\therefore x = 4 \text{ cm}$$

$$\text{Now } C = 2\pi r$$

$$= 2\pi \cdot x/2$$

$$= \pi x$$

$$\frac{dc}{dx} = \pi$$

$$\frac{dx}{dc} = \frac{1}{\pi}$$

convert units

$$\frac{dA}{dt} = \frac{dA}{dx} \cdot \frac{dx}{dt}$$

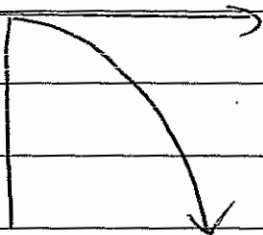
$$= 2x \cdot \left[\frac{dx}{dc} \cdot \frac{dc}{dt} \right]$$

$$= 2 \times x \times \frac{1}{\pi} \times 200$$

$$= 2 \times 4 \times \frac{1}{\pi} \times 200$$

$$= \frac{1600}{\pi} \text{ cm}^2/\text{s} \quad \text{or} \quad \frac{0.16}{\pi} \text{ m}^2/\text{s}$$

c)



$$x = 0$$

$$\dot{x} = 0$$

$$\dot{y} = -10$$

$$y = 1.02$$

$$\dot{x} = 0.6 \cos 0$$

$$\dot{y} = -10t$$

$$x = 0$$

$$= 0.6$$

$$y = -5t^2$$

$$v = 0.6$$

$$x = 0.6t$$

$$+ 1.02$$

$$\text{Now } x = \frac{0.6t}{0.6} \quad \therefore y = -5t^2 + 1.02$$

$$y = -5 \left(\frac{x}{0.6} \right)^2 + 1.02$$

$$y = 1.02 - \frac{125x^2}{9}$$

$$\text{ii) } y = 0 \quad \frac{125x^2}{9} = 1.02 \quad x > 0$$

$$x = 0.270998 \dots \text{ m.}$$

$\hat{=}$ 27 cm to the right of the table

When $x = 0.270998 \text{ m}$ (27 cm)

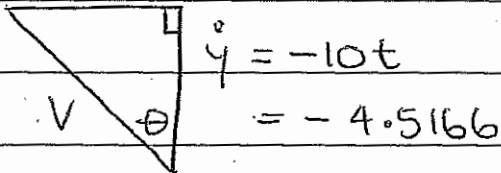
then

$$0.6t = 0.270998$$

$$t = 0.45166$$

speed @ impact

$$\dot{x} = 0.6$$



Pythagoras

$$V = \sqrt{0.6^2 + (-4.5166 \dots)^2} \\ = 4.6 \text{ m/s.}$$

iii) Use above diagram to calculate θ

$$\tan \theta = \frac{0.6}{4.5166 \dots}$$

$$\theta \doteq 7.5^\circ \text{ (1dp).}$$

Partiv con't

$$\ddot{x} = 0 \quad \ddot{y} = -10$$

$$\dot{x} = 3.68 \cos 82.5^\circ \quad \dot{y} = -10t + 3.68 \sin 82.5^\circ$$

$$x = 3.68 \cos 82.5^\circ t + 0.27 \quad y = -5t^2 + 3.68 \sin 82.5^\circ t$$

$$y = -5t^2 + 3.68 \sin 82.5^\circ t$$

Now set $y = 0$

$$0 = t [3.68 \sin 82.5^\circ - 5t]$$

$$t = 0 \quad \therefore t = \frac{3.68 \sin 82.5^\circ}{5}$$

(start)

$$t = 0.7297 \dots$$

$$x = 3.68 \cos 82.5^\circ \times 0.7297 + 0.27$$

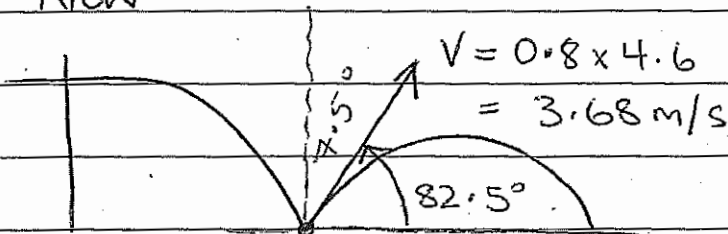
$$= 0.6205 \dots$$

$$= 0.62 \text{ m}$$

(62 cm to the right of the foot of

the table)

iv. Now



$$x = 0.27$$

$$t = 0$$

$$y = 0$$

Question 11

- a) needed to write in terms of $\frac{\sin x}{x}$ for the limit
- b) draw a diagram to help
- c) i. incorrect use of chain rule
- c) ii. Need to use cos double angle formula to integrate
- d) 4 is 2^2 and $\frac{1}{4}$ is $\left(\frac{1}{2}\right)^2$; learn the standard integral formats of known inverse trig functions
- e) Needed to be shown clearly
- f) Use the rule that is on the reference sheet and understand that the general solution formula is used as soon as the inverse sin is applied (see solution)

Question 12

- b(i) A of boys did not use correct terminology for "Alternate segment theorem" reason.
 - (ii) Reason for "Angle sum of a triangle" had to be stated, otherwise mark deducted.
- c) Errors occurred through transcription or carelessness. Ours is a careful subject so BE CAREFUL!
- d(ii) When solving for A and alpha, half of the cohort didn't realise that x=6 to the left meant x=-6. They also assumed A is amplitude and had to be positive. Learn from this one!!
 - (iii) No marks deducted for carry through error.

Question 13

- (b) (i) a lot of students did not use the chain rule correctly. Some of them did not use $\ddot{x} = \frac{1}{2}v^2$
- (b) (ii) the question says "by considering the velocity and acceleration of the particle" but some students did not consider the velocity and acceleration when describing the motion of the particle.
- (c) (ii) some students did not state the domain

Question 14

- a) Poor algebraic skills led to many students not obtaining full mark.
- b) Students needed to convert units in the questions. They also needed to show that some form of calculus (chain rule) to obtain marks.
- c) The question was a show that, so students needed to derive the formulas.
- ciii) The ball hits the floor when $y=0$. Students were able to find the x value yet finding the speed caused many problems for the majority.
- ciii) Many candidates had problems finding the angle.
- civ) POORLY completed question. Many variations for the angle. Students could not find when the ball hit the floor the 2nd time.