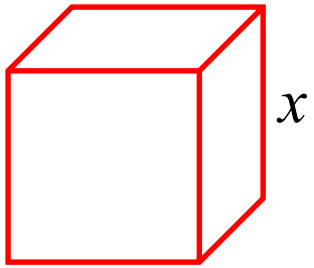
# *Rates of Change*

In some cases two, or more, rates must be found to get the equation in terms of the given variable.

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

e.g. (*i*) A block of ice in the form of a cube has one edge 10 cm long. It is melting so that its dimensions decrease at the rate of 1 mm/s.

At what rate is the volume decreasing when the edge is 5cm long?




$\frac{dV}{dt} = ?$ $\quad \frac{dx}{dt} = -\frac{1}{10}$

$V = x^3$

$\frac{dV}{dx} = 3x^2$

$$\frac{dV}{dt} = \frac{dx}{dt} \times \frac{dV}{dx}$$

$$= 3x^2 \times -\frac{1}{10}$$

$$= -\frac{3x^2}{10}$$

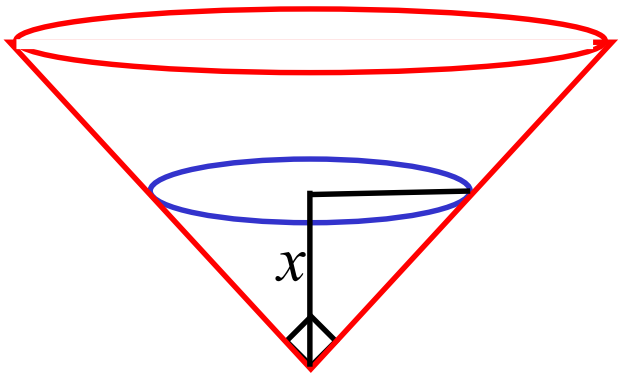
when $x = 5$, $\frac{dV}{dt} = -\frac{3(5)^2}{10}$

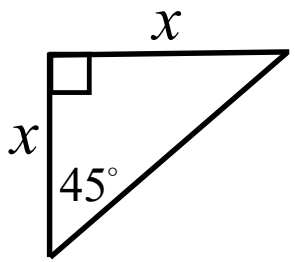
$= -7.5$

$\therefore$ volume is decreasing at 7.5 $cm^3/s$

(*ii*) A vessel is in the form of an inverted cone with a vertical angle of $90^\circ$
If the depth of the water in the vessel is $x$ cm;

a) find the volume of water.




$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi x^2 x$$
$$= \frac{1}{3}\pi x^3$$

b) If water is poured in at a rate of 0.2 $\text{cm}^3$/min, find the rate the depth is increasing when the water depth is 4 cm.

$\frac{dx}{dt} = ?$

$\frac{dV}{dt} = \frac{1}{5}$

$V = \frac{1}{3}\pi x^3$

$\frac{dV}{dx} = \pi x^2$

$\frac{dx}{dt} = \frac{dV}{dt} \times \frac{dx}{dV}$

$= \frac{1}{5} \times \frac{1}{\pi x^2}$

$= \frac{1}{5\pi x^2}$

when $x = 4$, $\frac{dx}{dt} = \frac{1}{5\pi(4)^2}$

$= \frac{1}{80\pi}$

$\therefore$ depth is increasing

at $\frac{1}{80\pi}$ cm/min

(*iii*) A spherical bubble is expanding so that its volume increases at a constant rate of $70\text{mm}^3/\text{s}$

What is the rate of increase of its surface area when the radius is 10 mm?

$\frac{dS}{dt} = ?$ $\quad \frac{dV}{dt} = 70$ $\quad V = \frac{4}{3}\pi r^3$ $\quad S = 4\pi r^2$

$\frac{dV}{dr} = 4\pi r^2$ $\quad \frac{dS}{dr} = 8\pi r$

$$\frac{dS}{dt} = \frac{dV}{dt} \cdot \frac{dS}{dr} \cdot \frac{dr}{dV}$$

$$= (70)(8\pi r)\left(\frac{1}{4\pi r^2}\right)$$

$$= \frac{140}{r}$$

when $r = 10, \frac{dV}{dt} = \frac{140}{10}$

$= 14$

$\therefore$ when radius is 10mm the surface area is increasing at a rate of $14\text{mm}^2/\text{s}$

**Exercise 16A; 1a, 2a, 4, 6, 7, 8, 9, 10, 13, 15, 16, 18**