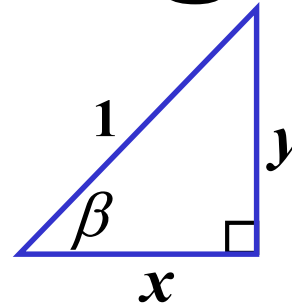
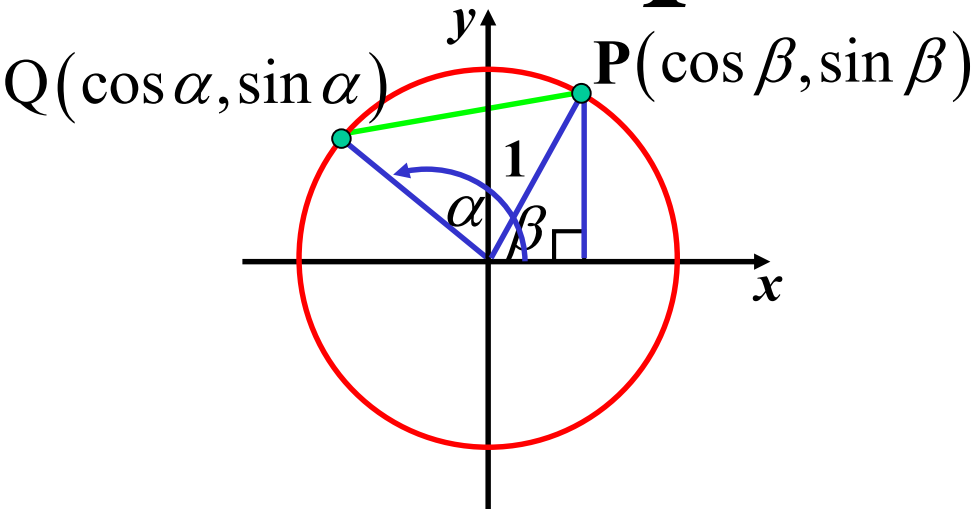


# Compound Angles



$$\frac{x}{1} = \cos \beta$$

$$x = \cos \beta$$

$$\frac{y}{1} = \sin \beta$$

$$y = \sin \beta$$

## By trigonometry

$$PQ^2 = 1^2 + 1^2 - 2 \times 1 \times 1 \times \cos(\alpha - \beta)$$

$$PQ^2 = 2 - 2 \cos(\alpha - \beta)$$

## By coordinate geometry

$$PQ^2 = (\cos \alpha - \cos \beta)^2 + (\sin \alpha - \sin \beta)^2$$

$$PQ^2 = \cos^2 \alpha - 2 \cos \alpha \cos \beta + \cos^2 \beta + \sin^2 \alpha - 2 \sin \alpha \sin \beta + \sin^2 \beta$$

$$PQ^2 = 2 - 2 \cos \alpha \cos \beta - 2 \sin \alpha \sin \beta$$

$$\therefore 2 - 2 \cos(\alpha - \beta) = 2 - 2 \cos \alpha \cos \beta - 2 \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

Replace  $\beta$  with  $-\beta$

$$\cos(\alpha + \beta) = \cos \alpha \cos(-\beta) + \sin \alpha \sin(-\beta)$$

$$\cos(-\beta) = \cos \beta \quad (\text{even function i.e. } f(-x) = f(x))$$

$$\sin(-\beta) = -\sin \beta \quad (\text{odd function i.e. } f(-x) = -f(x))$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

*cos, cos, sin, sin*



*If it's not the sine, it's not the sign*

Replace  $\alpha$  with  $90 - \alpha$

$$\cos(90 - \alpha - \beta) = \cos(90 - \alpha)\cos\beta + \sin(90 - \alpha)\sin\beta$$

$$\cos(90 - (\alpha + \beta)) = \cos(90 - \alpha)\cos\beta + \sin(90 - \alpha)\sin\beta$$


$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

Replace  $\beta$  with  $-\beta$

$$\sin(\alpha - \beta) = \sin\alpha\cos(-\beta) + \cos\alpha\sin(-\beta)$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

*sin, cos, cos, sin*

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$


*If it's the sine, it's the sign*

The diagram illustrates the mnemonic for the sine addition and subtraction formulas. It features the general formula  $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$  in the center. Above the formula, the words "sin, cos, cos, sin" are written in italics. Four blue arrows point from these words to the terms in the formula: "sin" points to  $\sin\alpha$ , "cos" points to  $\cos\beta$ , "cos" points to  $\cos\alpha$ , and "sin" points to  $\sin\beta$ . Below the formula, two thumbs-up emojis are positioned on either side. Two green arrows originate from the text "If it's the sine, it's the sign" at the bottom and point to the plus and minus signs in the formula.

$\tan(\alpha + \beta)$

$$\tan(\alpha + \beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)}$$

$$\tan(\alpha + \beta) = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta}$$

$$\tan(\alpha + \beta) = \frac{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}}$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

Replace  $\beta$  with  $-\beta$

$$\tan(\alpha - \beta) = \frac{\tan \alpha + \tan(-\beta)}{1 - \tan \alpha \tan(-\beta)}$$

$$\tan(-\beta) = -\tan \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

(odd function i.e.  $f(-x) = -f(x)$ )

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

*tan plus tan*

*on one minus tan, tan*

## Compound Angle Formulae

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

e.g. (i) Expand  $\cos(2\alpha - 3\beta)$

$$\underline{\cos(2\alpha - 3\beta) = \cos 2\alpha \cos 3\beta + \sin 2\alpha \sin 3\beta}$$

(ii) Simplify  $\frac{\tan 20^\circ + \tan 10^\circ}{1 - \tan 20^\circ \tan 10^\circ}$

$$\frac{\tan 20^\circ + \tan 10^\circ}{1 - \tan 20^\circ \tan 10^\circ} = \tan(20 + 10)$$

$$= \tan 30^\circ$$

$$= \underline{\frac{1}{\sqrt{3}}}$$

(iii) Find the exact value of  $\sin 15^\circ$

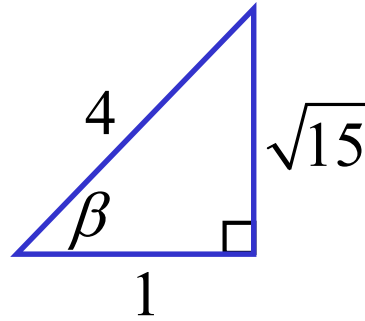
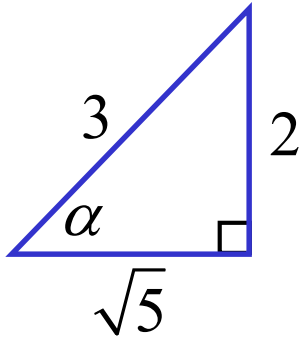
$$\sin 15^\circ = \sin(45 - 30)$$

$$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

(iv) If  $\sin \alpha = \frac{2}{3}$  and  $\cos \beta = \frac{1}{4}$ , find  $\sin(\alpha + \beta)$



$$\begin{aligned}\sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ &= \left(\frac{2}{3}\right)\left(\frac{1}{4}\right) + \left(\frac{\sqrt{5}}{3}\right)\left(\frac{\sqrt{15}}{4}\right) \\ &= \frac{2 + 5\sqrt{3}}{12}\end{aligned}$$

$$(vii) \tan\left(\sin^{-1}\frac{2}{3} + \cos^{-1}\frac{1}{4}\right)$$

$$= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$= \frac{\frac{2}{\sqrt{5}} + \sqrt{15}}{1 - \left(\frac{2}{\sqrt{5}}\right)(\sqrt{15})}$$

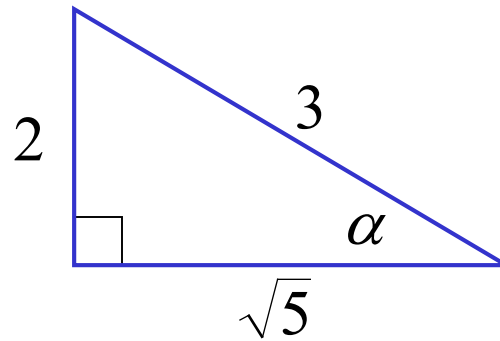
$$= \frac{2 + 5\sqrt{3}}{\sqrt{5} - 2\sqrt{15}}$$

$$= \frac{2 + 5\sqrt{3}}{\sqrt{5} - 2\sqrt{15}}$$

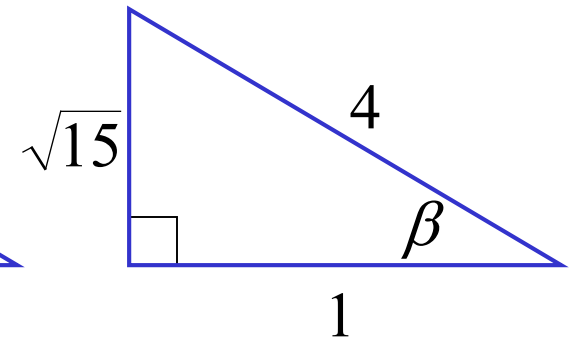
$$= \frac{2 + 5\sqrt{3}}{\sqrt{5} - 2\sqrt{15}}$$


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$$\text{let } \alpha = \sin^{-1}\frac{2}{3}$$



$$\text{let } \beta = \cos^{-1}\frac{1}{4}$$



**Exercise 17D; 1ade, 2bce, 4ac, 5,  
6ab, 7, 8, 9ac, 10ad, 11,  
12b, 13, 14a, 15ab, 17,  
18, 20**