Solving LHS > RHS

An efficient way of solving LHS > RHS is to rewrite the inequation as

LHS – RHS > 0

This is because finding when functions are positive (or negative) can be discovered by investigating the critical points of their domain.

A function can only change sign at;

- an x-intercept *OR*
- a discontinuity in the domain

Bracket Interval Notation

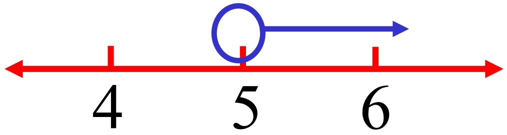
[: interval endpoint is included

(: interval endpoint is not inculded

[*a*,*b*]: closed **-** all endpoints are included

(*a*,*b*) or [*a*,*b*) or (*a*,*b*] : open - an endpoint is not included

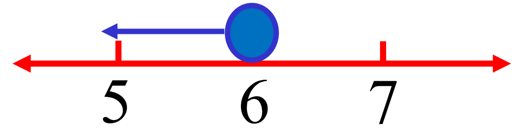
unbounded: if an interval extends to infinity in either direction e.g. $(i)\ x > 5$



open unbounded interval

$(5,\infty) = \{x : x > 5\}$

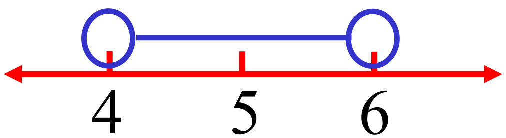
$(ii)\ x \le 6$



closed unbounded interval

$(-\infty, 6] = \{x : x \le 6\}$

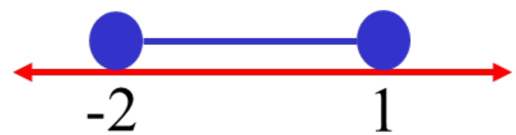
$(iii)\ 4 < x < 6$



open bounded interval

$(4,6) = \{x : 4 < x < 6\}$

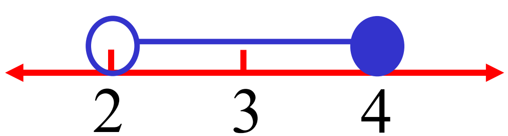
$(iv)\ -2 \le x \le 1$



closed bounded interval

$[-2,1] = \{x : -2 \le x \le 1\}$

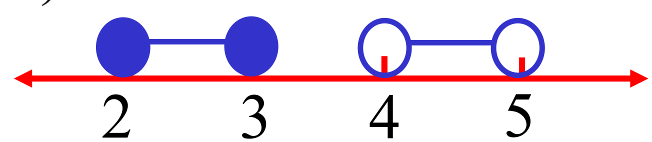
$(v)\ 2 < x \le 4$



bounded interval

$(2,4] = \{x : 2 < x \le 4\}$

$(vi)\ 2 \le x \le 3$ or $4 < x < 5$



union of intervals

$[2,3] \cup (4,5) = \{x : 2 \le x \le 3\} \cup \{x : 4 < x < 5\}$

Composite Functions

When two or more functions combine to create a new function.

$$f(g(x)) = f \circ g(x) \qquad \text{(substitute } g(x) \text{ into } f(x))$$

e.g. $f(x) = \dfrac{2x}{4-x}$ and $g(x) = \dfrac{1}{x^2}$

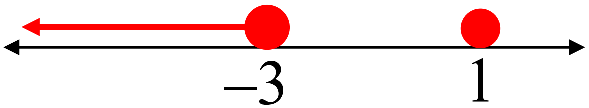
$$f \circ g(x) = \frac{2\left(\frac{1}{x^2}\right)}{4 - \frac{1}{x^2}}$$

$$= \frac{2}{4x^2 - 1}$$

$$g \circ f(x) = \frac{1}{\left(\frac{2x}{4-x}\right)^2}$$

$$= \frac{(4-x)^2}{4x^2}$$

e.g. $(i)\ (x-1)^2(x+3) \le 0$



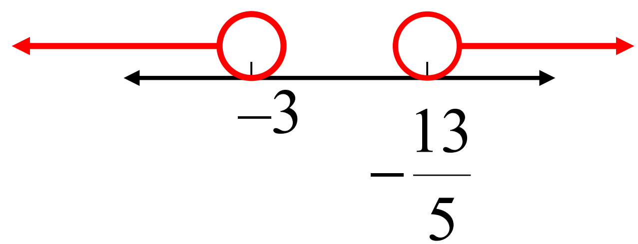
$x \le -3 \quad \text{or} \quad x = 1$

$(ii)\ \dfrac{2}{x+3} < 5$

$$\frac{2}{x+3} - 5 < 0$$

$$\frac{2-5(x+3)}{x+3} < 0$$

$$\frac{-13-5x}{x+3} < 0$$



$$\therefore x < -3 \quad \text{or} \quad x > -\frac{13}{5}$$

Exercise 3A; 4, 5bc, 6ace, 7acef, 8b, 9a, 10b, 12ac, 13bd, 14, 17, 19