A Curve Sketching Menu

1. Rewrite the function as one complete fraction

2. Find y-intercept i.e. substitute $x = 0$

3. Find x-intercepts i.e. numerator $= 0$

4. Perform a polynomial division

solve $R(x) = 0$ to find where (if anywhere) the curve cuts the horizontal/oblique asymptote

$$\boxed{y = \frac{P(x)}{A(x)} = Q(x) + \frac{R(x)}{A(x)}}$$

$y = Q(x)$ is the horizontal/oblique asymptote

solve $A(x) = 0$ to find vertical asymptotes

5. Take a note of any symmetry i.e. odd & even functions, symmetry in $y = x$

A note about asymptotes

1. **Vertical asymptotes:** created from exclusions in the domain
   Curves **do not** touch/cut vertical asymptotes

2. **Horizontal asymptotes:** created when a function **converges** to a specific value
   Curves **can** cut/touch horizontal asymptotes

Three limits that affect the value of the horizontal asymptote

$$\lim_{x \to \pm\infty} \frac{1}{x} = 0 \qquad \lim_{x \to \infty} e^{-x} = 0 \qquad \lim_{x \to -\infty} e^{x} = 0$$

e.g. (*i*)
$$\lim_{x \to \infty} \frac{x^2 + x}{1 - x^2} = \lim_{x \to \infty} \frac{\frac{x^2}{x^2} + \frac{x}{x^2}}{\frac{1}{x^2} - \frac{x^2}{x^2}} = \frac{1 + 0}{0 - 1} = \underline{-1}$$

(*ii*)
$$\lim_{x \to \infty} \frac{e^x + e^{-x}}{2e^x} = \lim_{x \to \infty} \frac{1 + e^{-2x}}{2} = \underline{\frac{1}{2}}$$

(*iii*)
$$\lim_{x \to -\infty} \frac{e^x + e^{-x}}{2e^x} = \lim_{x \to -\infty} \frac{e^{2x} + 1}{2e^{2x}} = \frac{1}{0} = \underline{\infty}$$

Example: Sketch the graph of $y = \dfrac{x^3 - 9x}{x^2 - 4}$, clearly indicating any asymptotes and any points where the graph meets the axes.

• y-intercepts: $x = 0$

• x-intercepts:
$$x^3 - 9x = 0$$
$$x(x^2 - 9) = 0$$
$$x = 0 \quad \text{and } x = \pm 3$$

$$y = \frac{x^3 - 9x}{x^2 - 4}$$
$$= x - \frac{5x}{x^2 - 4}$$

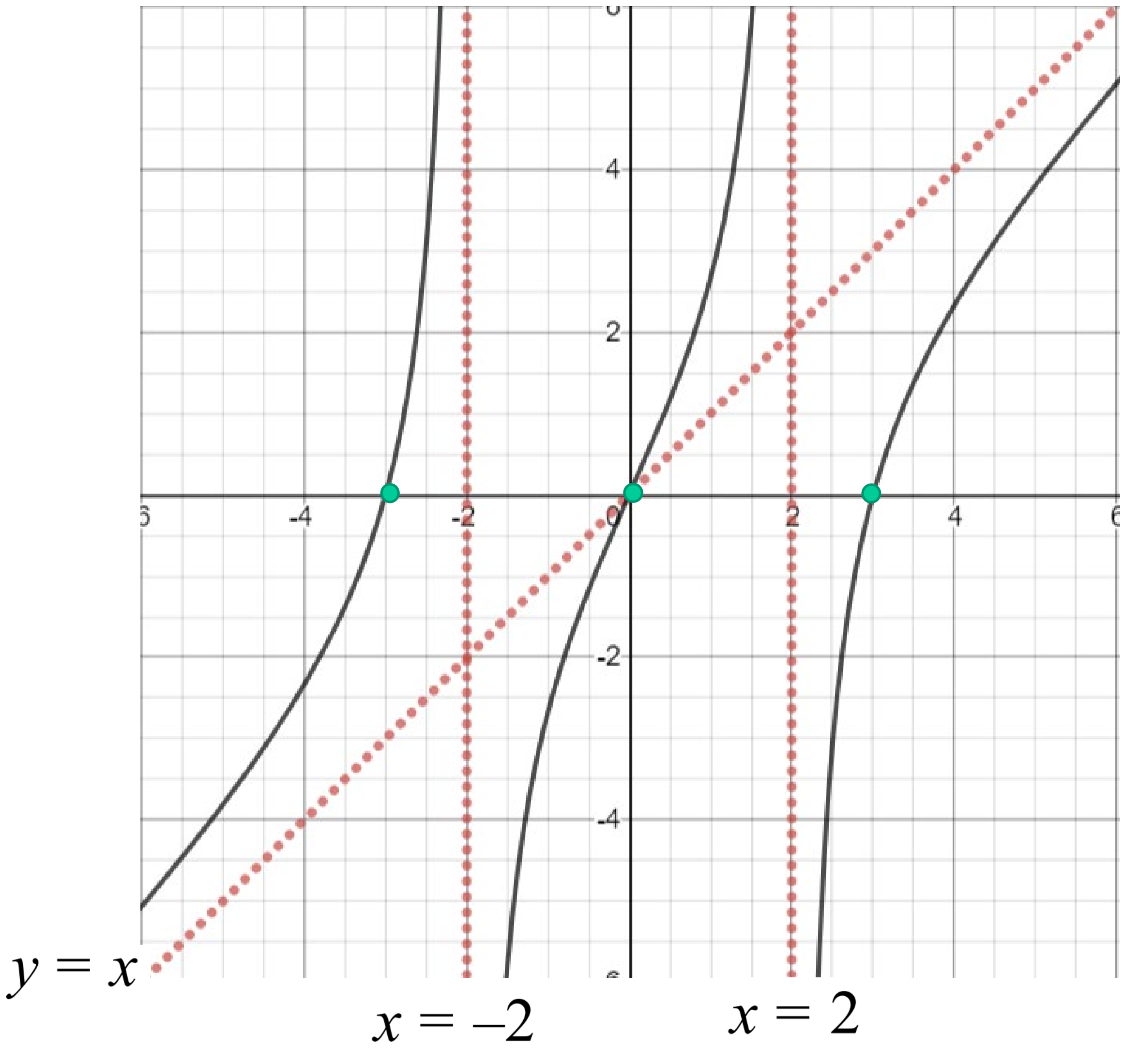
• vertical asymptotes:
$$x^2 - 4 = 0$$
$$x = \pm 2$$

• oblique asymptote: $y = x$

• curve meets asymptote:
$$5x = 0$$
$$x = 0$$

• odd function: $\Rightarrow$ rotational symmetry

$$y = \frac{x^3 - 9x}{x^2 - 4}$$



Exercise 3B; 1ace, 3, 8, 9, 10ac, 12a, 14, 15b, 16

Exercise 3C; 2, 5, 8, 11ce, 12, 13