Geometric Series

An geometric series is a sequence of numbers in which each term after the first is found by **multiplying** a constant amount to the previous term.

The constant amount is called the **common ratio**, symbolised, ***r***.

$$r = \frac{T_2}{a}$$

$$= \frac{T_3}{T_2}$$

$$\boxed{r = \frac{T_n}{T_{n-1}}}$$

$$\boxed{\begin{array}{c} T_n = rT_{n-1} \\ \textbf{\textit{(recursive formula)}} \end{array}}$$

$$T_1 = a$$

$$T_2 = ar$$

$$T_3 = ar^2$$

$$\boxed{T_n = ar^{n-1}}$$

When plotted on a number plane, the graph of a geometric sequence is an exponential function

$e.g.(i)$ Find r and the general term of $2, 8, 32, \dots$

$$a = 2, r = 4$$

$$\begin{aligned} T_n &= ar^{n-1} \\ &= 2(4)^{n-1} \\ &= 2\left(2^2\right)^{n-1} \\ &= 2(2)^{2n-2} \end{aligned}$$

$$\therefore T_n = 2^{2n-1}$$

(***ii***) If $T_2 = 7$ and $T_4 = 49$, find the general term

$$ar = 7$$
$$ar^3 = 49$$

$$r^2 = 7$$
$$r = \pm\sqrt{7} \quad \therefore a = \pm\sqrt{7}$$
$$T_n = (\sqrt{7})(\sqrt{7})^{n-1}$$
$$= (\sqrt{7})^n$$

OR

$$T_n = (-\sqrt{7})(-\sqrt{7})^{n-1}$$
$$= (-\sqrt{7})^n$$
$$= (-1)^n(\sqrt{7})^n$$

(*iii*) find the first term of 1, 4, 16, … to be greater than 500.

$$a = 1, r = 4 \qquad T_n = 1(4)^{n-1}$$
$$T_n > 500$$
$$4^{n-1} > 500$$
$$\log 4^{n-1} > \log 500$$
$$(n-1)\log 4 > \log 500$$
$$n-1 > 4.48$$
$$n > 5.48$$
$$T_6 = 1024, \text{is the first term} > 500$$

Arithmetic & Geometric Means

Arithmetic Mean(average)

$$AM = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}$$

Geometric Mean

$$GM = \pm\sqrt[n]{a_1 a_2 a_3 \ldots a_n}$$

e.g. Find the AM and GM of 4 and 25

$$AM = \frac{25+4}{2} = \frac{29}{2}$$

$$GM = \pm\sqrt{25 \times 4} = \pm\sqrt{100} = \pm 10$$

(ii) Find x and y if 2, x, y, 128 form a GP

$$x = \sqrt{2y} \qquad\qquad y = \sqrt{128x}$$

$$\begin{aligned} y^2 &= 128x \\ &= 128\sqrt{2y} \\ y^4 &= 32768y \end{aligned}$$

$$y(y^3 - 32768) = 0$$

$$y = 0 \quad \text{or} \quad y = 32$$

$\therefore y = 32$ (0 cannot be a term in a GP)

$\underline{x = 8 \;,\; y = 32}$

Exercise 1C; 4be, 6, 8cf, 9ad, 10f, 13, 14, 16c, 19b

Exercise 1D; 1ae, 2af, 3ace etc, 4 (*use AM & GM*), 5b, 6b, 9, 10a, 11, 12, 13bd, 14, 16, 18ab, 19, 20