

# *Series – The Sum Of A Sequence*

$$T_1 + T_2 + T_3 + \dots + T_n = S_n = \sum_{k=1}^n T_k$$

$$\begin{aligned} \text{e.g. (i)} \sum_{n=1}^5 2n + 3 &= (2 + 3) + (4 + 3) + (6 + 3) + (8 + 3) + (10 + 3) \\ &= \underline{45} \end{aligned}$$

(ii) If  $S_{10} = 29$  and  $S_{11} = 37$ , find  $T_{11}$

$$\begin{aligned} T_{11} &= S_{11} - S_{10} \\ &= 37 - 29 \\ &= \underline{8} \end{aligned}$$

(iii) If  $S_n = 5n^2 - 2$ , find  $T_n$

$$\begin{aligned}T_n &= S_n - S_{n-1} \\&= 5n^2 - 2 - [5(n-1)^2 - 2] \\&= 5n^2 - 2 - 5n^2 + 10n - 5 + 2 \\&= \underline{10n - 5}\end{aligned}$$

# *Sum Of An Arithmetic Series*

$$S_n = a + a + d + a + 2d + \dots + l - 2d + l - d + l$$

$$S_n = \underline{l + l - d + l - 2d + \dots + a + 2d + a + d + a}$$

$$2S_n = a + l + a + l + a + l + \dots + a + l + a + l + a + l$$
$$= n(a + l)$$

$$S_n = \frac{n}{2}(a + l)$$

if we know  $l$

otherwise;

$$S_n = \frac{n}{2}\{a + a + (n - 1)d\}$$

$$S_n = \frac{n}{2}\{2a + (n - 1)d\}$$

e.g. (i) If  $a = 3$  and  $T_6 = 96$ , find  $S_6$

$$S_n = \frac{n}{2}(a + l)$$

$$\begin{aligned} S_6 &= \frac{6}{2}(3 + 96) \\ &= \underline{297} \end{aligned}$$

(ii) Find the sum of the first 100 even numbers

$$a = 2, d = 2 \text{ and } n = 100$$

$$S_n = \frac{n}{2}\{2a + (n - 1)d\}$$

$$\begin{aligned} S_{100} &= \frac{100}{2}\{4 + (99)(2)\} \\ &= 50 \times 202 \\ &= \underline{10100} \end{aligned}$$

(iii) The sum of the first 10 numbers is 100 and the first 5 numbers is 25.

Find  $a, d$  and the general term.

$$\begin{array}{l} S_{10} = 100 \\ \frac{10}{2} \{2a + 9d\} = 100 \\ 2a + 9d = 20 \end{array} \qquad \begin{array}{l} S_5 = 25 \\ \frac{5}{2} \{2a + 4d\} = 25 \\ a + 2d = 5 \end{array}$$

$$2a + 9d = 20$$

$$\underline{2a + 4d = 10}$$

$$5d = 10$$

$$d = 2 \quad \therefore a = 1$$

$$T_n = a + (n-1)d$$

$$= 1 + (n-1)2$$

$$= 2n - 1$$

$$\underline{\therefore a = 1, d = 2, T_n = 2n - 1}$$

$$(iv) \sum_{n=1}^{10} (3n - 6)$$

$$a = -3, l = 24, n = 10$$

$$S_n = \frac{n}{2}(a + l)$$

$$S_{10} = \frac{10}{2}(-3 + 24)$$
$$= \underline{105}$$

**Exercise 1E; 4ace etc, 8, 9b, 10bdf, 11bdf, 12, 13, 15**

**Exercise 1F; 4ace, 6bd, 7ce, 9c, 10ad, 11a, 13,  
14c, 16ac, 17a, 18**