

# *Translating Curves*

**Horizontal Shift** replace  $x$  with  $x - h$

$$\underline{y = f(x - h)}$$

moves  $f(x)$   $\xleftarrow{-}$  or  $\xrightarrow{+}$   $h$  units

*to find where the origin has been moved to; solve  $x - h = 0$*

**Vertical Shift** replace  $y$  with  $y - k$

$$(y - k) = f(x) \quad \text{OR} \quad \underline{y = f(x) + k}$$

moves  $f(x)$   $\uparrow\uparrow_{+}$  or  $\downarrow\downarrow_{-}$   $k$  units

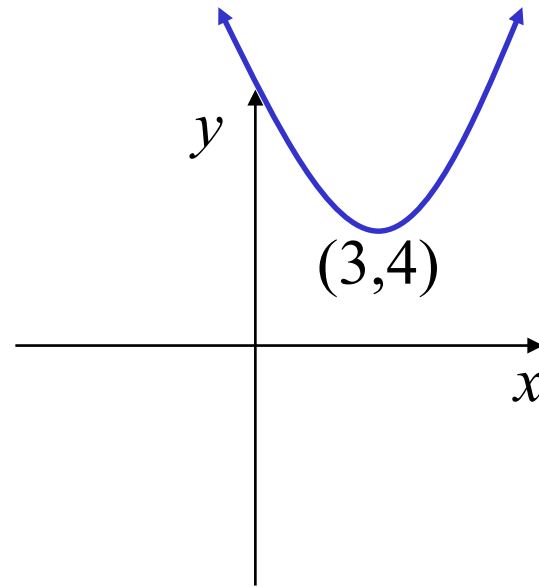
*to find where the origin has been moved to; solve  $y - k = 0$*

e.g. (i)  $y = (x - 3)^2 + 4$

1. *basic curve*:  $y = x^2$

2. *shift right 3 units*

3. *shift up 4 units*



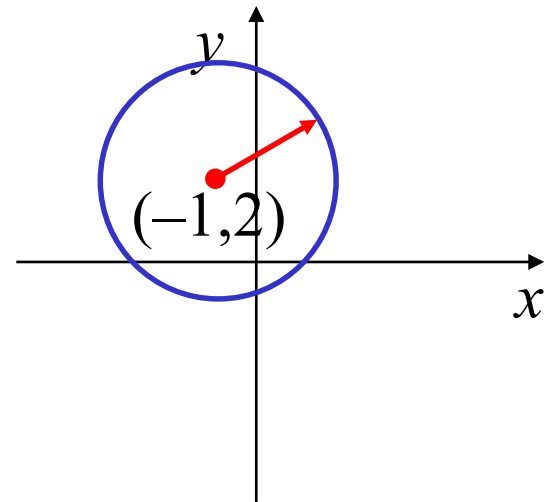
(ii)  $x^2 + 2x + y^2 - 4y - 4 = 0$

$$\begin{aligned}(x + 1)^2 + (y - 2)^2 &= 4 + 1 + 4 \\ &= 9\end{aligned}$$

1. *basic curve*:  $x^2 + y^2 = 9$

2. *shift left 1 unit*

3. *shift up 2 units*



$$(iii) y = 2x^2 + 8x + 5$$

$$\frac{y}{2} = x^2 + 4x + \frac{5}{2}$$

$$\frac{y}{2} = (x + 2)^2 + \frac{5}{2} - 4$$

$$= (x + 2)^2 - \frac{3}{2}$$

$$y = 2(x + 2)^2 - 3$$

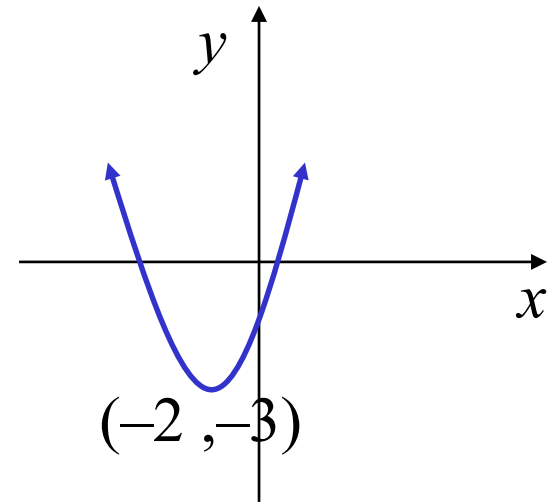
*It is easier to complete the square of  $x^2$  than  $2x^2$*

1. *basic curve:  $y = x^2$*

2. *curve gets steeper*

3. *shift left 2 units*

4. *shift down 3 units*



# Reflecting Curves

**Vertical Reflection** replace  $x$  with  $-x$

$$\underline{y = f(-x)}$$

Reflects in the  $y$ -axis

To reflect in the line  $x = a$ ; replace  $x$  with  $2a - x$

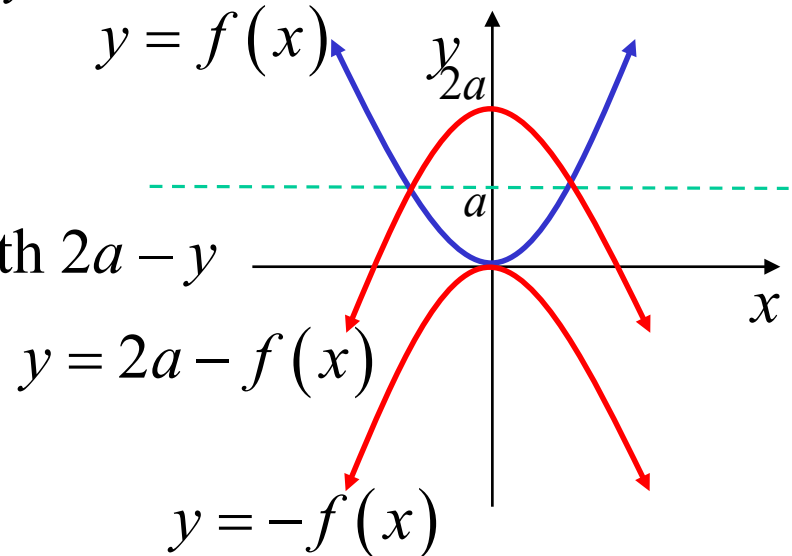
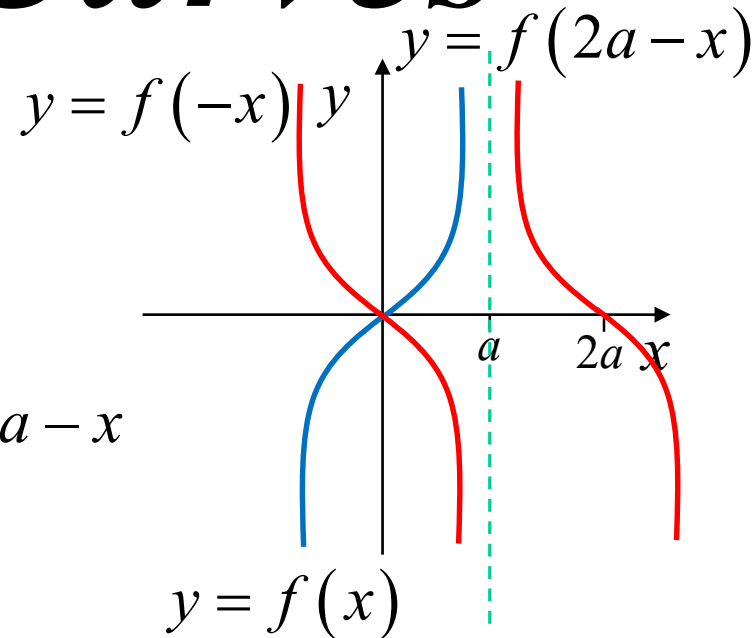
**Horizontal Reflection** replace  $y$  with  $-y$

$$-y = f(x) \quad \text{OR} \quad \underline{y = -f(x)}$$

Reflects in the  $x$ -axis

To reflect in the line  $y = a$ ; replace  $y$  with  $2a - y$

$$\text{i.e. } y = 2a - f(x)$$

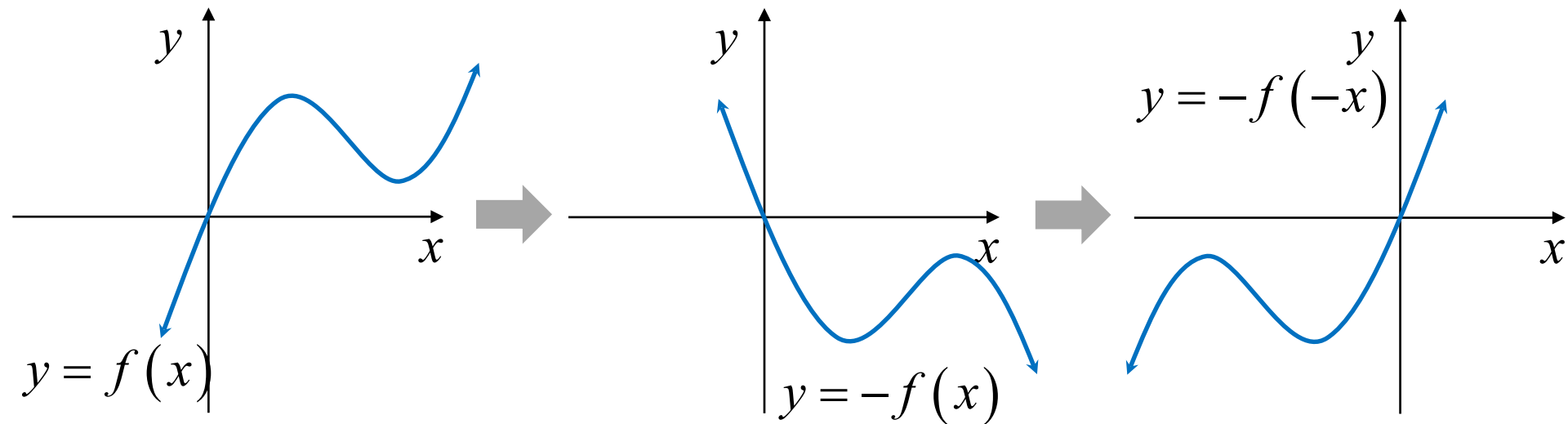


## Reflection in the origin

$$\underline{y = -f(-x)}$$

A reflection in the origin is the same as a rotation of  $180^\circ$

It is achieved by two other reflections, one in the  $x$ -axis, the other in the  $y$ -axis



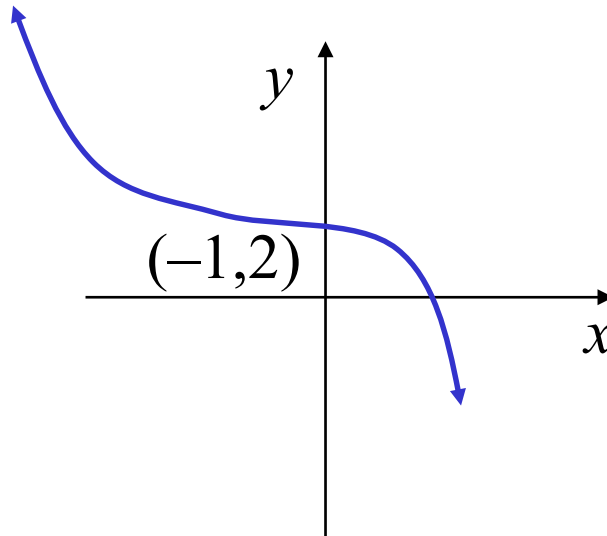
e.g. (i)  $y = -(x+1)^3 + 2$

1. *basic curve*:  $y = x^3$

2. *reflect in x axis*

3. *shift left 1 unit*

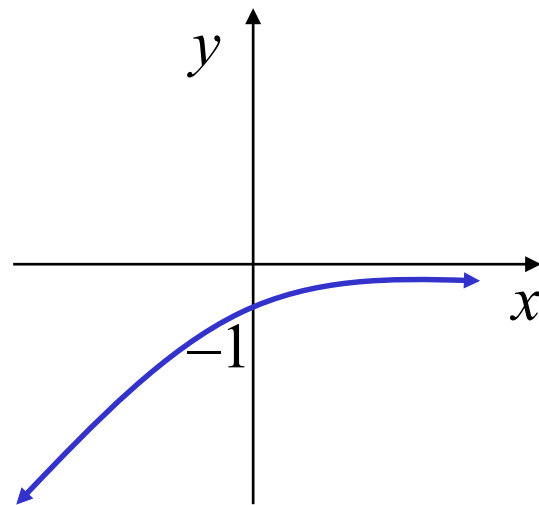
4. *shift up 2 units*



(ii)  $y = -2^{-x}$

1. *basic curve*:  $y = 2^x$

2. *reflect in origin*



**Exercise 3G; 1aceg, 2aceg, 4d, 5a, 6,  
7, 8ace, 10a, 12, 13**