

The Second Derivative

$$y'', f''(x), \frac{d^2}{dx^2} \{f(x)\}, \frac{d^2 y}{dx^2}$$

e.g. (i) $y = 3x^2 + x + 2$

$$\frac{dy}{dx} = 6x + 1$$

$$\frac{d^2 y}{dx^2} = 6$$

(ii) $y = \sqrt{x^2 + 3}$

$$= (x^2 + 3)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2} (x^2 + 3)^{-\frac{1}{2}} (2x)$$

$$= x (x^2 + 3)^{-\frac{1}{2}}$$

$$\frac{d^2 y}{dx^2} = (x) \left[-\frac{1}{2} (x^2 + 3)^{-\frac{3}{2}} (2x) \right] + (x^2 + 3)^{-\frac{1}{2}} (1)$$

$$= -x^2 (x^2 + 3)^{-\frac{3}{2}} + (x^2 + 3)^{-\frac{1}{2}}$$

$$= (x^2 + 3)^{-\frac{3}{2}} [-x^2 + x^2 + 3]$$

$$= \frac{3}{(x^2 + 3)\sqrt{x^2 + 3}}$$

Exercise 4D; 1 to 7 bdf etc in all, 8, 9b, 10b, 11, 13, 14ac, 15