Definite Integral

Fundamental Theorem of Calculus

$$F'(x) = \frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

Properties of the Definite Integral

$$(1)\ \int_a^b x^n\,dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b$$

$$(2)\ \int_a^b kf(x)dx = k\int_a^b f(x)dx$$

(can only factorise constants)

$$(3)\ \int_a^b \{f(x) \pm g(x)\}dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$(4)\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

$$(5)\int_a^b f(x)\,dx > 0 \qquad ,\text{if } f(x) > 0 \text{ for } a < x < b$$

$$< 0 \qquad ,\text{if } f(x) < 0 \text{ for } a < x < b$$

$$(6)\int_a^b f(x)dx < \int_a^b g(x)dx \qquad ,\text{if } f(x) < g(x) \text{ for } a < x < b$$

$$(7)\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$(8)\int_{-a}^{a} f(x) = 0, \qquad \text{if } f(x) \text{ is odd}$$

$$(9)\int_{-a}^{a} f(x) = 2\int_0^a f(x), \qquad \text{if } f(x) \text{ is even}$$

$NOTE:$

$odd \times odd = even$

$odd \times even = odd$

$even \times even = even$

e.g. (i) $\int_1^2 6x^2 dx = 6\left[\frac{1}{3}x^3\right]_1^2$

$= 2\left(2^3 - 1^3\right)$

$= 14$

(ii) $\int_0^5 \sqrt[3]{x} dx = \int_0^5 x^{\frac{1}{3}} dx$

$= \frac{3}{4}\left[x^{\frac{4}{3}}\right]_0^5$

$= \frac{3}{4}\left[x\sqrt[3]{x}\right]_0^5$

$= \frac{3}{4}\left(5\sqrt[3]{5} - 0\right)$

$= \frac{15\sqrt[3]{5}}{4}$

$$(iii)\int_{-2}^{2}\sin^5 x\,dx \;\underline{=0} \qquad (\text{odd function})^5 = \text{odd function}$$

$$(iv)\int_{-1}^{1}\left(x^3+2x^2+x+1\right)dx = 2\int_{0}^{1}\left(2x^2+1\right)dx$$

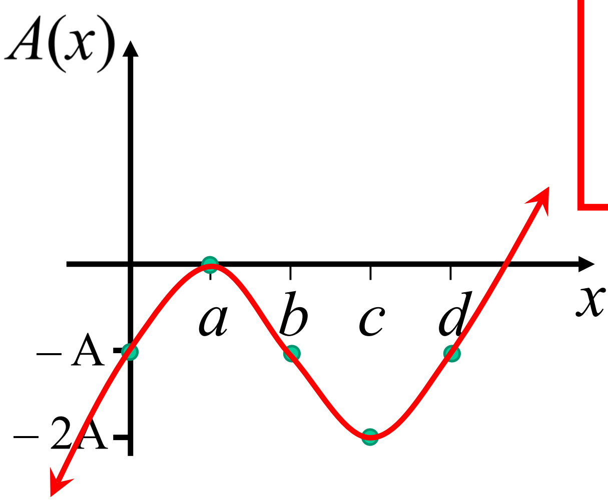
$$= 2\left[\frac{2}{3}x^3+x\right]_0^1$$

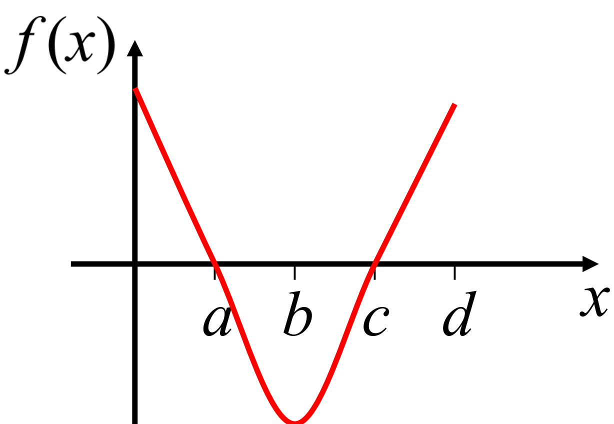
$$= 2\left\{\frac{2}{3}(1)^3+1\right\}-0$$

$$\underline{= \frac{10}{3}}$$

Sketching the Signed Area Function (primitive function)

$$A(x) = \int_a^x f(t)\,dt$$
$$= F(x) - F(a)$$





$A(a) = 0$ *NOTE: a is a stationary point as* $A'(x) = 0$

$$A(0) = \int_a^0 f(t)\,\mathrm{dt} = -\int_0^a f(t)\,\mathrm{dt} = -\mathrm{A}$$

$A(b) = -\mathrm{A}$

NOTE: b is a possible inflection point as $A''(x) = 0$

$A(c) = -2\mathrm{A}$

NOTE: c is a stationary point as $A'(x) = 0$

$A(d) = -\mathrm{A}$

NOTE: in the interval $[c, \infty)$ $A(x)$ is increasing as $A'(x) > 0$

Area $0 \to a$ = Area $a \to b$ = Area $b \to c$ = Area $c \to d$

Exercise 5C; 5, 6a, 7, 10, 11, 12a, 16, 18

Exercise 5D; 3, 4b, 7bc, 9