Factorial Notation

$$\begin{cases} 0! = 1 \\ n! = n \times (n-1)! , \text{ for } n \ge 1 \end{cases}$$

From this definition we can conclude;

$$1! = 1 \times 0! = 1 \times 1 = 1$$

$$2! = 2 \times 1! = 2 \times 1 = 2$$

$$3! = 3 \times 2! = 3 \times 2 = 6$$

$$4! = 4 \times 3! = 4 \times 6 = 24$$

$$\vdots$$

$$n! = n \times (n-1)! = n(n-1) \times (n-2)!$$

$$n! = n(n-1)(n-2)(n-3) \cdots 3 \times 2 \times 1$$

e.g.
$$(i)\frac{12!}{9!} = \frac{12 \times 11!}{9!}$$
 $(ii)\frac{16!}{10! \times 7!} = \frac{16 \times 15 \times 14 \times 13 \times 12 \times 11}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$
 $= \frac{12 \times 11 \times 10!}{9!}$ $= \frac{12 \times 11 \times 10 \times 9!}{9!}$ $= \frac{12 \times 11 \times 10 \times 9!}{9!}$ $= \frac{16 \times 13 \times 12 \times 11}{6 \times 5 \times 4 \times 3}$
 $= \frac{12 \times 11 \times 10}{9!}$ $= \frac{12 \times 11 \times 10}{9!}$ $= \frac{4 \times 13 \times 2 \times 11}{6 \times 4}$ $= 4 \times 13 \times 2 \times 11$ $= 4 \times 26 \times 11$ $= 104 \times 11$ $= 104 \times 11$ $= 1144$
 $(iii)\frac{1}{k!} + \frac{1}{(k-2)!} = \frac{k(k-1)+1}{k(k-1)(k-2)!}$ $k^2 - k + 1$

= -

k!

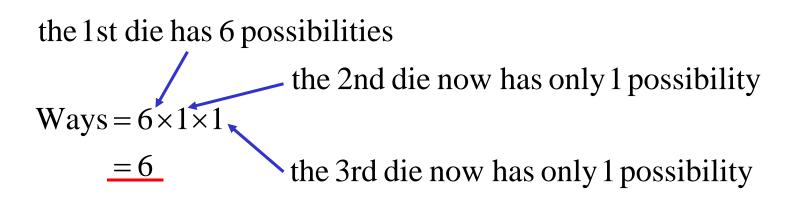
The Basic Counting Principle

If one event can happen in m different ways and after this another event can happen in n different ways, then the two events can occur in mn different ways.

e.g. 3 dice are rolled (i) How many ways can the three dice fall? the 1st die has 6 possibilities the 2nd die has 6 possibilities Ways = $6 \times 6 \times 6$ the 3rd die has 6 possibilities = 216

The number of ways of arranging *n* distinct objects, with replacement, in *k* different ways is n^k

(*ii*) How many ways can all three dice show the same number?



(*iii*) What is the probability that all three dice show the same number?

$$P(\text{all 3 the same}) = \frac{6}{216}$$
$$= \frac{1}{36}$$

1996 Extension 1 HSC Q5c)

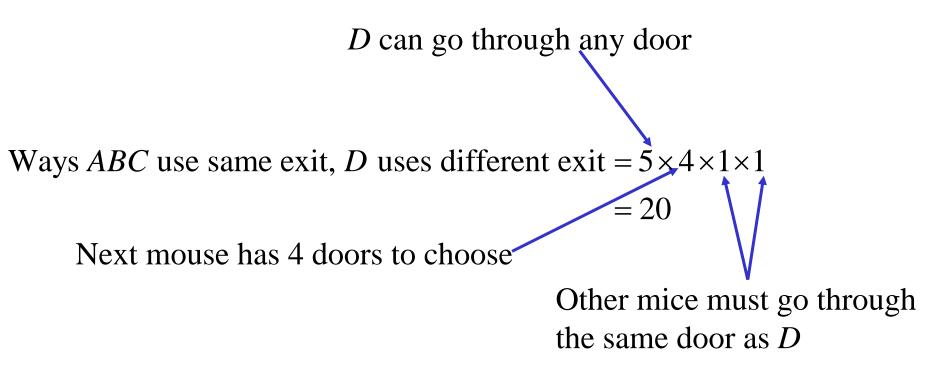
Mice are placed in the centre of a maze which has five exits.

Each mouse is equally likely to leave the maze through any of the five exits. Thus, the probability of any given mouse leaving by a particular exit is $\frac{1}{2}$

Four mice, A, B, C and D are put into the maze and behave independently.

(*i*) What is the probability that *A*, *B*, *C* and *D* all come out the same exit?

Total possibilities = 5^4 = 625 First mouse can go through any door Ways go through same door = $5 \times 1 \times 1 \times 1$ = 5 $P(all use same exit) = <math>\frac{5}{625}$ = $\frac{1}{125}$ must go through same = 5door (*ii*) What is the probability that *A*, *B* and *C* come out the same exit and *D* comes out a different exit?



$$P(ABC \text{ use same exit, } D \text{ uses different exit}) = \frac{20}{625}$$

= $\frac{4}{125}$

(*iii*) What is the probability that *any* three of the four mice come out the same exit and the other comes out a different exit?

$$P(D \text{ uses different exit}) = \frac{4}{125}$$

$$\therefore P(A \text{ uses different exit}) = \frac{4}{125}$$

$$P(B \text{ uses different exit}) = \frac{4}{125}$$

$$P(C \text{ uses different exit}) = \frac{4}{125}$$

$$\therefore P(\text{any mouse uses different exit}) = 4 \times \frac{4}{125}$$

$$= \frac{16}{125}$$

(*iv*) What is the probability that no more than two mice come out the same exit?

P(no more than 2 use same exit) = 1 - P(all same) - P(3 use same)

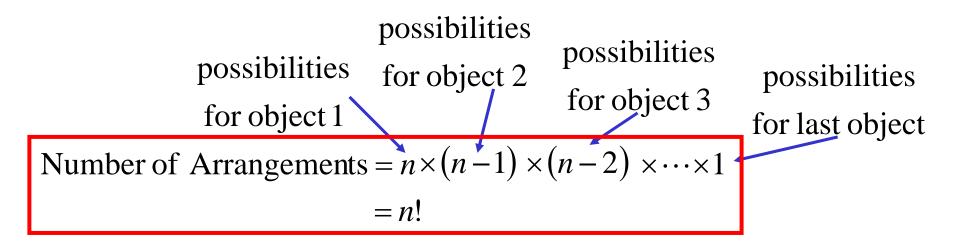
$$=1 - \frac{1}{125} - \frac{16}{125}$$
$$= \frac{108}{125}$$

Permutations

A permutation is an **ordered** set of objects i.e. an **arrangement**

Case 1: Ordered Sets of *n* Different Objects, from a Set of *n* Such Objects (*i.e.* use all of the objects)

If we arrange *n* different objects in a line, the number of ways we could arrange them are;



e.g. In how many ways can 5 boys and 4 girls be arranged in a line if;

(*i*) there are no restrictions?

Arrangements = 9!With no restrictions, arrange 9 people= 362880gender does not matter

(*ii*) boys and girls alternate? (*ALWAYS look after any restrictions first*) first person MUST be a boy Arrangements = $1 \times 5! \times 4!$ = 2880 (*ii*) boys and girls alternate? number of ways of arranging the boys arranging the girls (*iii*) What is the probability of the boys and girls alternating?

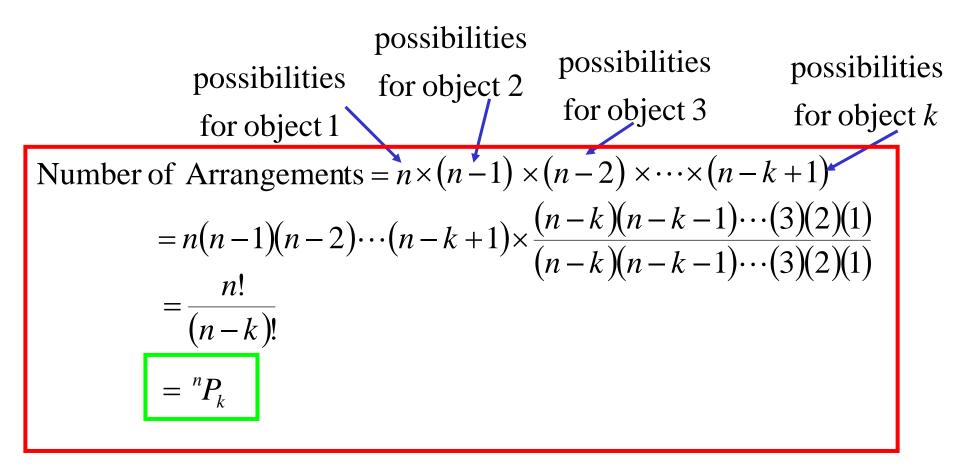
$$P(\text{boys \& girls alternate}) = \frac{2880}{362880}$$
$$= \frac{1}{126}$$

(*iv*) Two girls wish to be together?

the number of ways the girls can be arranged Arrangements = $2! \times 8!$ number of ways of arranging 8 objects (2 girls) + 7 others = 80640

Case 2: Ordered Sets of k Different Objects, from a Set of n Such Objects (k < n) (i.e. use some of the objects)

If we have *n* different objects in a line, but only want to arrange *k* of them, the number of ways we could arrange them are;

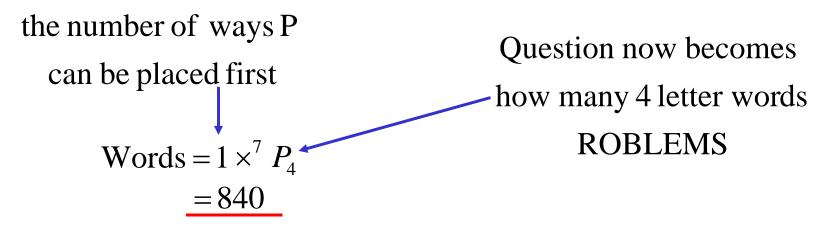


e.g. (*i*) From the letters of the word **PROBLEMS** how many 5 letter words are possible if;

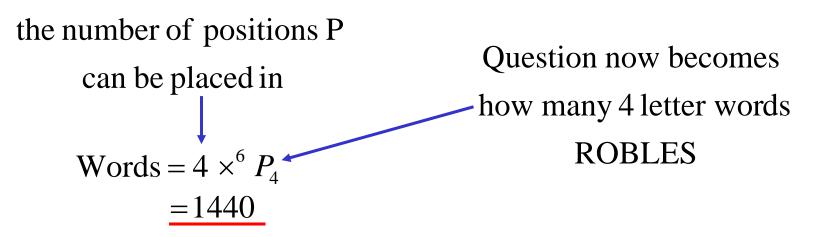
a) there are no restrictions?

Words $= {}^{8}P_{5}$ = 6720

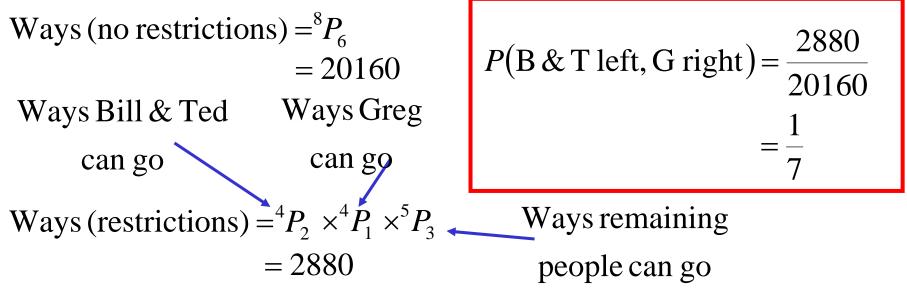
b) they must begin with **P**?



c) **P** is included, but not at the beginning, and **M** is excluded?



(*ii*) Six people are in a boat with eight seats, for on each side.What is the probability that Bill and Ted are on the left side and Greg is on the right?



2006 Extension 1 HSC Q3c)

Sophia has five coloured blocks: one red, one blue, one green, one yellow and one white.

She stacks two, three, four or five blocks on top of one another to form a vertical tower.

(*i*) How many different towers are there that she could form that are three blocks high?

$$Fowers = {}^{5}P_{3}$$
$$= 60$$

(*ii*) How many different towers can she form in total?

2 block Towers $={}^{5}P_{2} = 20$ 3 block Towers $={}^{5}P_{3} = 60$ 4 block Towers $={}^{5}P_{4} = 120$ 5 block Towers $={}^{5}P_{5} = 120$ Total number of Towers = 320 **Exercise 14A; 2bdfg, 3adfh, 5aceg, 6bdf, 7, 9, 12, 13 Exercise 14B; 2, 4, 6, 8ac, 10, 12, 14, 16, 18, 20, 23, 25 Exercise 14C; 2, 4, 5, 7, 9, 10, 12, 14, 16, 18, 19, 21, 22**