Integration Involving Inverse Trig

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c \quad \textbf{OR} \quad -\cos^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c \quad \textbf{OR} \quad -\cos^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 + [f(x)]^2}}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

e.g.

$$(i)\int \frac{dx}{\sqrt{4-x^2}}$$

$$= \sin^{-1}\left(\frac{x}{2}\right)+c$$

$$(ii)\int \frac{dx}{9+x^2}$$

$$= \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+c$$

$$(iii)\int_0^1 \frac{dx}{\sqrt{2-x^2}} = \left[\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]_0^1$$

$$= \sin^{-1}\frac{1}{\sqrt{2}} - \sin^{-1} 0$$

$$= \frac{\pi}{4} - 0$$

$$= \frac{\pi}{4}$$

(iv) Find $\frac{d}{dx}\{\sin^{-1}e^x\}$ and hence evaluate $\int_{-\log 2}^{0}\frac{e^x}{\sqrt{1-e^{2x}}}dx$

$$\frac{d}{dx}\{\sin^{-1}e^x\}=\frac{e^x}{\sqrt{1-e^{2x}}}$$

$$\therefore \int_{-\log 2}^{0}\frac{e^x}{\sqrt{1-e^{2x}}}dx=\left[\sin^{-1}e^x\right]_{-\log 2}^{0}$$

$$=\sin^{-1}e^0-\sin^{-1}e^{-\log 2}$$

$$=\sin^{-1}1-\sin^{-1}\frac{1}{2}$$

$$=\frac{\pi}{2}-\frac{\pi}{6}$$

$$=\frac{\pi}{3}$$

$(v)\int\frac{dx}{\sqrt{4-9x^2}}$

$$=\frac{1}{3}\int\frac{3\,dx}{\sqrt{4-9x^2}}$$

$$=\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)+c$$

Exercise 12B; 2, 3, 4a, 5b, 6bdf, 7bd, 8, 11, 13, 14, 15, 18, 19, 20, 22, 23