

# *Trig Equations*

## **Compound Angle Formulae**

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

## **Double Angles**

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\Rightarrow \cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\Rightarrow \sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$$

## Half Angles – $t$ results

$$\text{If } t = \tan \frac{\theta}{2};$$

$$\tan \theta = \frac{2t}{1-t^2} \quad \sin \theta = \frac{2t}{1+t^2} \quad \cos \theta = \frac{1-t^2}{1+t^2}$$

## Products to Sums

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$\mathbf{OR} \quad -\cos(A+B) + \cos(A-B)$$

## Sums to Products\*

$$\sin A + \sin B = 2 \sin \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)$$

$$\cos A + \cos B = 2 \cos \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)$$

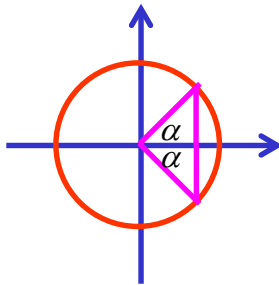
$$\cos A - \cos B = -2 \sin \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)$$

# Solving Trig Equations

- (1) Get all the angles the same
- (2) Match the required answer range to the angle
- (3) Get all the trig functions the same  
*if in doubt, change everything to sin and cos*
- (4) Check your answer(s) solve the original question,  
NOT just the transformed equation

e.g. (i)  $\cos 2\theta = \frac{1}{2}$        $0^\circ \leq \theta \leq 360^\circ$   
Q1, Q4       $0^\circ \leq 2\theta \leq 720^\circ$

$$\cos \alpha = \frac{1}{2}$$
$$\alpha = 60^\circ$$



$$2\theta = \alpha, 360 - \alpha$$

$$2\theta = 60^\circ, 360 - 60^\circ$$

$$2\theta = 60^\circ, 300^\circ, 420^\circ, 660^\circ$$

$$\underline{\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ}$$

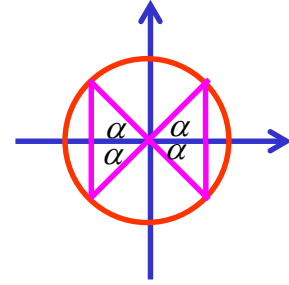
$$\mathbf{OR} \quad \cos 2\theta = \frac{1}{2} \quad 0^\circ \leq \theta \leq 360^\circ$$

$$2\cos^2 \theta - 1 = \frac{1}{2} \quad \text{Q1, Q2, Q3, Q4}$$

$$\cos^2 \theta = \frac{3}{4}$$

$$\cos \alpha = \frac{\sqrt{3}}{2}$$

$$\alpha = 30^\circ$$



$$\theta = \alpha, 180 - \alpha, 180 + \alpha, 360 - \alpha$$

$$\theta = 30^\circ, 180 - 30^\circ, 180 + 30^\circ, 360 - 30^\circ$$

$$\underline{\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ}$$

$$(ii) 4\sec^2 x = 3\tan x + 5$$

$$0^\circ \leq x \leq 360^\circ$$

$$4 + 4\tan^2 x = 3\tan x + 5$$

$$4\tan^2 x - 3\tan x - 1 = 0$$

$$(4\tan x + 1)(\tan x - 1) = 0$$

$$\tan x = -\frac{1}{4}$$

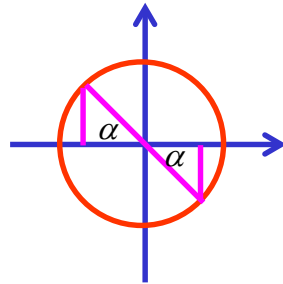
or

$$\tan x = 1$$

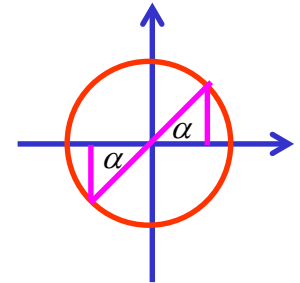
Q2, Q4

Q1, Q3

$$\tan \alpha = \frac{1}{4}$$



$$\tan \alpha = 1$$



$$\alpha = 14^\circ 2'$$

$$\alpha = 45^\circ$$

$$x = 180 - \alpha, 360 - \alpha$$

$$x = \alpha, 180 + \alpha$$

$$x = 180 - 14^\circ 2', 360 - 14^\circ 2'$$

$$x = 45^\circ, 180 + 45^\circ$$

$$x = 165^\circ 58', 345^\circ 58'$$

$$x = 45^\circ, 225^\circ$$

$$\underline{x = 45^\circ, 165^\circ 58', 225^\circ, 345^\circ 58'}$$

$$(iii) \cos 2\theta = 4\cos^2 \theta - 2\sin^2 \theta$$

$$0^\circ \leq \theta \leq 360^\circ$$

$$\cos^2 \theta - \sin^2 \theta = 4\cos^2 \theta - 2\sin^2 \theta$$

$$3\cos^2 \theta = \sin^2 \theta \quad \text{Q1, Q2, Q3, Q4}$$

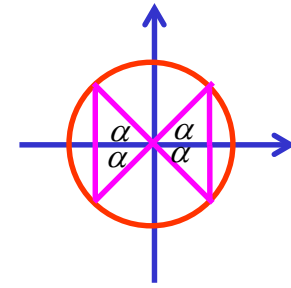
$$\tan^2 \theta = 3 \quad \tan \alpha = \sqrt{3}$$

$$\tan \theta = \pm\sqrt{3} \quad \alpha = 60^\circ$$

$$\theta = \alpha, 180 - \alpha, 180 + \alpha, 360 - \alpha$$

$$\theta = 60^\circ, 180 - 60^\circ, 180 + 60^\circ, 360 - 60^\circ$$

$$\underline{\theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ}$$



$$(iv) \cos 2\theta = \sin \theta,$$

$$0 \leq \theta \leq 360^\circ \quad \text{2000 Extension 1 HSC Q2c)}$$

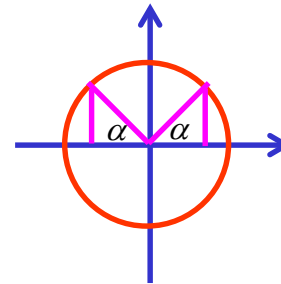
$$1 - 2\sin^2 \theta = \sin \theta$$

$$2\sin^2 \theta + \sin \theta - 1 = 0$$

$$(2\sin \theta - 1)(\sin \theta + 1) = 0$$

$$\sin \theta = \frac{1}{2} \quad \text{or} \quad \sin \theta = -1$$

$$\text{Q1, Q2} \quad \theta = 270^\circ$$



$$\sin \alpha = \frac{1}{2}$$

$$\alpha = 30^\circ$$

$$\theta = \alpha, 180 - \alpha$$

$$\theta = 30^\circ, 180 - 30^\circ$$

$$\theta = 30^\circ, 150^\circ$$

$$\underline{\theta = 30^\circ, 150^\circ, 270^\circ}$$

$$(v) \quad 2 \sin^2 \theta = \sin 2\theta, \quad 0 \leq \theta \leq 360^\circ \quad 1992 \text{ Extension 1 HSC Q2a)}$$

$$2 \sin^2 \theta = 2 \sin \theta \cos \theta$$

$$2 \sin^2 \theta - 2 \sin \theta \cos \theta = 0$$

$$2 \sin \theta (\sin \theta - \cos \theta) = 0$$

$$\sin \theta = 0 \quad \text{or} \quad \sin \theta = \cos \theta$$

$$\theta = 0^\circ, 180^\circ, 360^\circ \quad \tan \theta = 1$$

$$Q1, Q3$$

$$\tan \alpha = 1$$

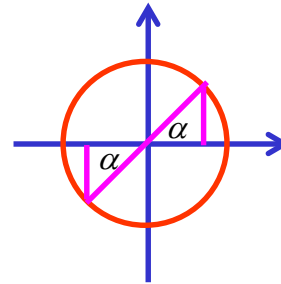
$$\alpha = 45^\circ$$

$$\theta = \alpha, 180 + \alpha$$

$$\theta = 45^\circ, 180 + 45^\circ$$

$$\theta = 45^\circ, 225^\circ$$

$$\underline{\theta = 0^\circ, 45^\circ, 180^\circ, 225^\circ, 360^\circ}$$



$$(vi) \quad \sin \theta \sin 3\theta = \frac{1}{2} \quad 0 \leq \theta \leq 2\pi$$

$$-\cos 4\theta + \cos 2\theta = 1$$

$$-2\cos^2 2\theta + 1 + \cos 2\theta = 1 \quad 0 \leq 2\theta \leq 4\pi$$

$$2\cos^2 2\theta - \cos 2\theta = 0$$

$$\cos 2\theta(2\cos 2\theta - 1) = 0$$

$$\cos 2\theta = 0 \quad \text{or} \quad \cos 2\theta = \frac{1}{2}$$

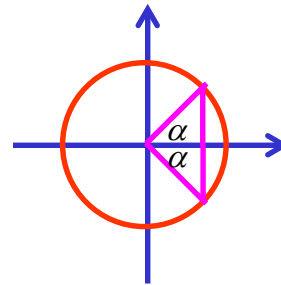
$$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

Q1, Q4

$$2\theta = \alpha, 2\pi - \alpha$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$\cos \alpha = \frac{1}{2}$$



$$= \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$$

$$\alpha = \frac{\pi}{3}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\theta = \frac{\pi}{6}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{6}$$


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**Exercise 11A; 1, 3, 4ad, 5be  
6dij, 9, 10, 11, 12cdf, 13b  
14, 15, 16**