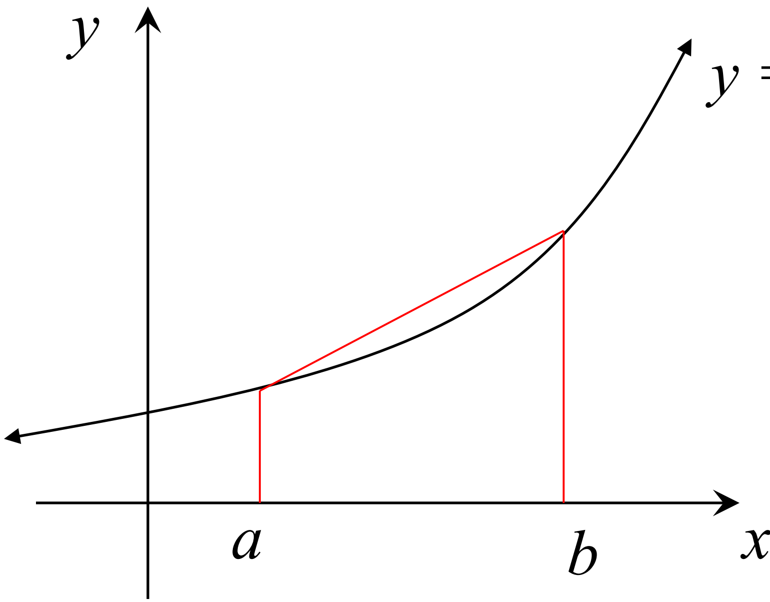
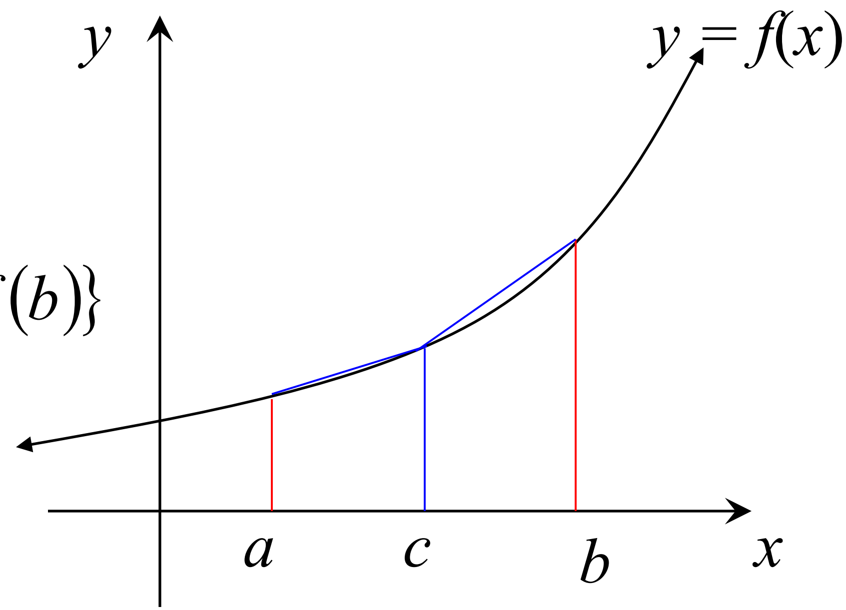
Trapezoidal Rule

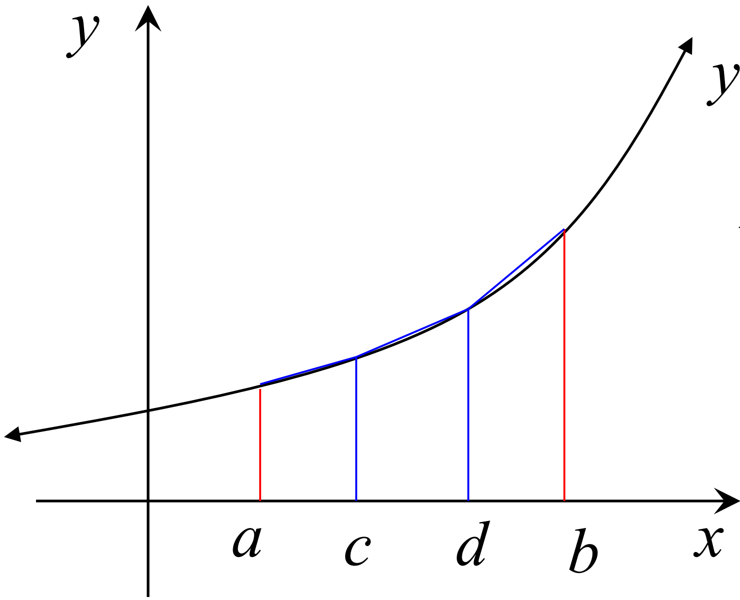


$$A \approx \frac{b-a}{2}\{f(a)+f(b)\}$$



$$A \approx \frac{c-a}{2}\{f(a)+f(c)\}+\frac{b-c}{2}\{f(c)+f(b)\}$$

$$= \frac{c-a}{2}\{f(a)+2f(c)+f(b)\}$$



$$A \approx \frac{c-a}{2}\{f(a)+f(c)\}+\frac{d-c}{2}\{f(c)+f(d)\}$$

$$+\frac{b-d}{2}\{f(d)+f(b)\}$$

$$=\frac{c-a}{2}\{f(a)+2f(c)+2f(d)+f(b)\}$$

<u>In general;</u>

$$\text{Area} = \int_a^b f(x)dx$$

$$\approx \frac{h}{2}\{y_0 + 2y_{others} + y_n\}$$

where $h = \dfrac{b-a}{n}$

$n =$ number of trapeziums

<u>NOTE:</u> there is always one more function value than interval e.g. Use the Trapezoidal Rule with 4 intervals to estimate the area under the curve $y = \left(4 - x^2\right)^{\frac{1}{2}}$, between $x = 0$ and $x = 2$ (correct to 3 decimal points)

$$h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$$

	$\boxed{1}$	$\boxed{2}$	$\boxed{2}$	$\boxed{2}$	$\boxed{1}$
x	0	0.5	1	1.5	2
y	2	1.9365	1.7321	1.3229	0

$$\text{Area} \approx \frac{h}{2}\left\{y_0 + 2y_{others} + y_n\right\}$$

$$= \frac{0.5}{2}\left\{2 + 2\left(1.9365 + 1.7321 + 1.3229\right) + 0\right\}$$

$$= 2.996 \text{ units}^2 \qquad (exact\ value\ =\ \pi)$$

$$\%\text{ error} = \frac{3.142 - 2.996}{3.142} \times 100$$

$$= 4.6\%$$

Alternative working out!!!

	$\boxed{1}$	$\boxed{2}$	$\boxed{2}$	$\boxed{2}$	$\boxed{1}$
x	0	0.5	1	1.5	2
y	2	1.9365	1.7321	1.3229	0

$$\text{Area} \approx \frac{2+2(1.9365+1.7321+1.3229)+0}{1+2+2+2+1}\times(2-0)$$

$$\underline{=2.996\ \text{units}^2}$$

Exercise 5H; 4, 5, 6, 7, 8ab, 10, 12, 14, 15