

# *Trig Substitutions*

$$\sqrt{a^2 + x^2} \quad \text{use } x = a \tan \theta$$

$$\sqrt{a^2 - x^2} \quad \text{use } x = a \sin \theta \text{ or } x = a \cos \theta$$

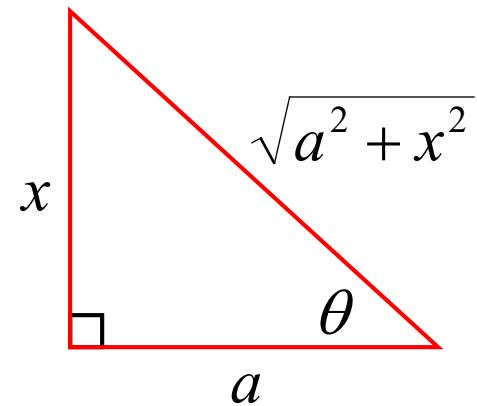
$$\sqrt{x^2 - a^2} \quad \text{use } x = a \sec \theta$$

$$\begin{aligned}
 & \text{e.g. (i)} \int \frac{1}{\sqrt{a^2 + x^2}} dx \\
 &= \int \frac{a \sec^2 \theta d\theta}{\sqrt{a^2 + a^2 \tan^2 \theta}} \\
 &= \int \frac{a \sec^2 \theta d\theta}{\sqrt{a^2 \sec^2 \theta}} \\
 &= \int \sec \theta d\theta \\
 &= \log|\sec \theta + \tan \theta| + c
 \end{aligned}$$

$$\begin{aligned}
 &= \log \left| \frac{\sqrt{a^2 + x^2}}{a} + \frac{x}{a} \right| + c \\
 &= \log \left| \sqrt{a^2 + x^2} + x \right| - \log|a| + c \\
 &= \log \left| \sqrt{a^2 + x^2} + x \right| + c
 \end{aligned}$$


---

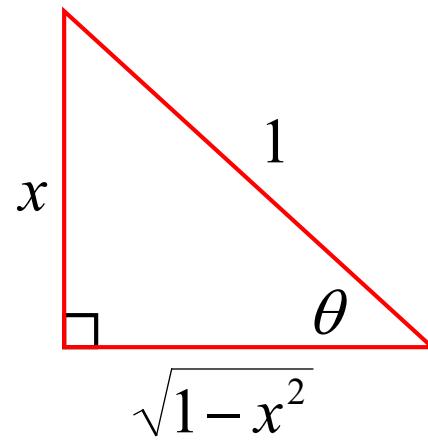
$$\begin{aligned}
 x &= a \tan \theta \\
 dx &= a \sec^2 \theta d\theta
 \end{aligned}$$



$$\begin{aligned}
 (ii) & \int \frac{x}{\sqrt{1-x^2}} dx \\
 &= \int \frac{\sin \theta \cos \theta d\theta}{\sqrt{1-\sin^2 \theta}} \\
 &= \int \frac{\sin \theta \cos \theta d\theta}{\sqrt{\cos^2 \theta}} \\
 &= \int \sin \theta d\theta \\
 &= -\cos \theta + c \\
 &= -\sqrt{1-x^2} + c
 \end{aligned}$$


---

$$\begin{aligned}
 x &= \sin \theta \\
 dx &= \cos \theta d\theta
 \end{aligned}$$



$$\begin{aligned}
 & (iii) \int \sqrt{x^2 + 3} dx \\
 &= \int (\sqrt{3} \sec \theta) (\sqrt{3} \sec^2 \theta) d\theta \\
 &= 3 \int \sec^3 \theta d\theta \\
 &= 3 \int \sec \theta \sec^2 \theta d\theta \\
 &= 3 \sec \theta \tan \theta - 3 \int \sec \theta \tan^2 \theta d\theta \\
 &= 3 \sec \theta \tan \theta - 3 \int \sec^3 \theta d\theta + 3 \int \sec \theta \\
 &= 3 \sec \theta \tan \theta - 3 \int \sec^3 \theta d\theta + 3 \log|\sec \theta + \tan \theta| \\
 &= 3 \frac{\sqrt{x^2 + 3}}{\sqrt{3}} \times \frac{x}{\sqrt{3}} - \int \sqrt{x^2 + 3} dx + 3 \log \left| \frac{\sqrt{x^2 + 3}}{\sqrt{3}} + \frac{x}{\sqrt{3}} \right|
 \end{aligned}$$

$$x = \sqrt{3} \tan \theta$$

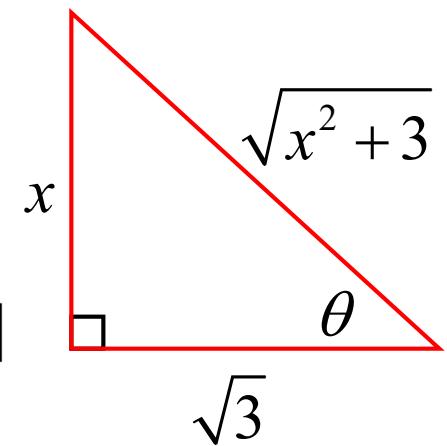
$$dx = \sqrt{3} \sec^2 \theta d\theta$$

$$u = \sec \theta$$

$$v = \tan \theta$$

$$du = \sec \theta \tan \theta d\theta$$

$$dv = \sec^2 \theta d\theta$$



$$\sqrt{3}$$

$$\therefore 2 \int \sqrt{x^2 + 3} dx = x\sqrt{x^2 + 3} + 3 \log \left| \frac{\sqrt{x^2 + 3}}{\sqrt{3}} + \frac{x}{\sqrt{3}} \right| + c$$

$$\int \sqrt{x^2 + 3} dx = \frac{x\sqrt{x^2 + 3}}{2} + \frac{3}{2} \log \left| \frac{\sqrt{x^2 + 3}}{\sqrt{3}} + \frac{x}{\sqrt{3}} \right| + c$$

$$\int \sqrt{x^2 + 3} dx = \frac{x\sqrt{x^2 + 3}}{2} + \frac{3}{2} \log \left| \sqrt{x^2 + 3} + x \right| + c$$

---

***Patel Exercise 2E; 1, 2, 3, 5, 6, 7, 9, 13, 17, 19, 20***