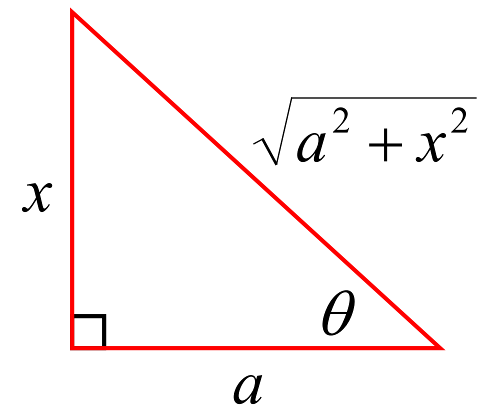
Trig Substitutions

$\sqrt{a^2 + x^2}$	use $x = a\tan\theta$
$\sqrt{a^2 - x^2}$	use $x = a\sin\theta$ or $x = a\cos\theta$
$\sqrt{x^2 - a^2}$	use $x = a\sec\theta$

$$\text{e.g. } (i) \int \frac{1}{\sqrt{a^2 + x^2}}\, dx$$

$$x = a \tan\theta$$

$$dx = a\sec^2\theta\, d\theta$$



$$= \int \frac{a\sec^2\theta\, d\theta}{\sqrt{a^2 + a^2\tan^2\theta}}$$

$$= \int \frac{a\sec^2\theta\, d\theta}{\sqrt{a^2\sec^2\theta}}$$

$$= \int \sec\theta\, d\theta$$

$$= \log|\sec\theta + \tan\theta| + c$$

$$= \log\left| \frac{\sqrt{a^2 + x^2}}{a} + \frac{x}{a} \right| + c$$

$$= \log\left| \sqrt{a^2 + x^2} + x \right| - \log|a| + c$$

$$= \log\left| \sqrt{a^2 + x^2} + x \right| + c$$

$$(ii)\int \frac{x}{\sqrt{1-x^2}}\,dx$$

$$= \int \frac{\sin\theta\cos\theta d\theta}{\sqrt{1-\sin^2\theta}}$$

$$= \int \frac{\sin\theta\cos\theta d\theta}{\sqrt{\cos^2\theta}}$$

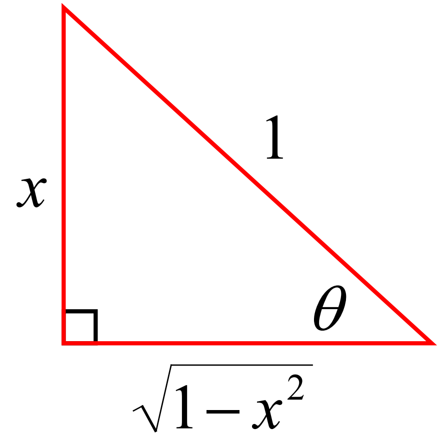
$$= \int \sin\theta d\theta$$

$$= -\cos\theta + c$$

$$= -\sqrt{1-x^2} + c$$

$$x = \sin\theta$$

$$dx = \cos\theta d\theta$$



$$(iii)\int\sqrt{x^2+3}\,dx$$

$$x=\sqrt{3}\tan\theta$$

$$dx=\sqrt{3}\sec^2\theta\,d\theta$$

$$=\int\left(\sqrt{3}\sec\theta\right)\left(\sqrt{3}\sec^2\theta\right)d\theta$$

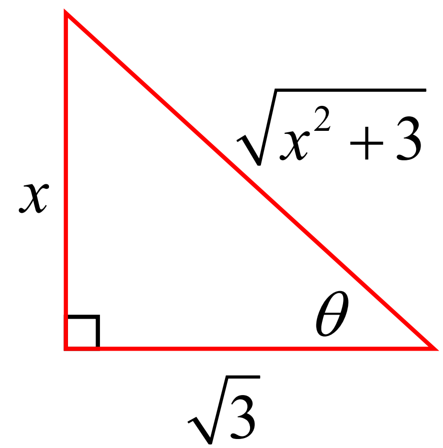
$$=3\int\sec^3\theta\,d\theta$$

$$u=\sec\theta \qquad v=\tan\theta$$

$$=3\int\sec\theta\sec^2\theta\,d\theta$$

$$du=\sec\theta\tan\theta\,d\theta \qquad dv=\sec^2\theta\,d\theta$$

$$=3\sec\theta\tan\theta-3\int\sec\theta\tan^2\theta\,d\theta$$



$$=3\sec\theta\tan\theta-3\int\sec^3\theta\,d\theta+3\int\sec\theta$$

$$=3\sec\theta\tan\theta-3\int\sec^3\theta\,d\theta+3\log|\sec\theta+\tan\theta|$$

$$=3\frac{\sqrt{x^2+3}}{\sqrt{3}}\times\frac{x}{\sqrt{3}}-\int\sqrt{x^2+3}\,dx+3\log\left|\frac{\sqrt{x^2+3}}{\sqrt{3}}+\frac{x}{\sqrt{3}}\right|$$

$$\therefore 2\int \sqrt{x^2+3}\,dx = x\sqrt{x^2+3} + 3\log\left|\frac{\sqrt{x^2+3}}{\sqrt{3}} + \frac{x}{\sqrt{3}}\right| + c$$

$$\int \sqrt{x^2+3}\,dx = \frac{x\sqrt{x^2+3}}{2} + \frac{3}{2}\log\left|\frac{\sqrt{x^2+3}}{\sqrt{3}} + \frac{x}{\sqrt{3}}\right| + c$$

$$\int \sqrt{x^2+3}\,dx = \frac{x\sqrt{x^2+3}}{2} + \frac{3}{2}\log\left|\sqrt{x^2+3} + x\right| + c$$

Patel **Exercise 2E; 1, 2, 3, 5, 6, 7, 9, 13, 17, 19, 20**