Applications of Calculus To The Physical World

Displacement (x)

Distance from a point, with direction.

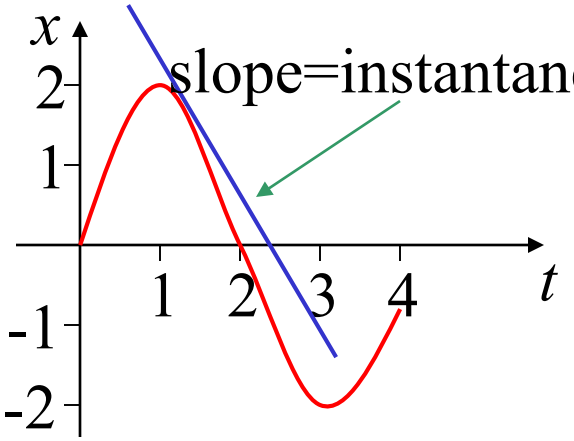
Velocity $\left(v, \frac{dx}{dt}, \dot{x} \right)$

The rate of change of displacement with respect to time i.e. speed with direction.

Acceleration $\left(a, \frac{dv}{dt}, \frac{d^2x}{dt^2}, \ddot{x}, \dot{v} \right)$

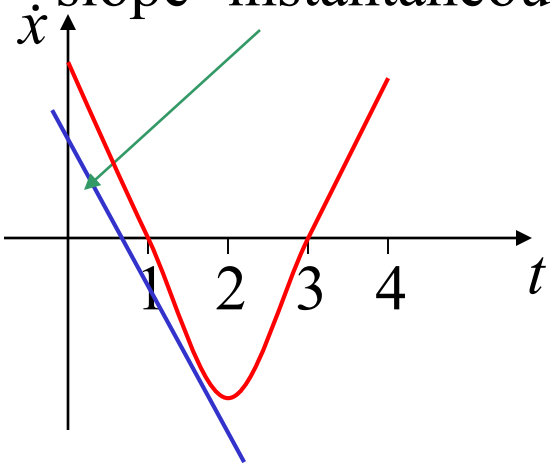
The rate of change of velocity with respect to time

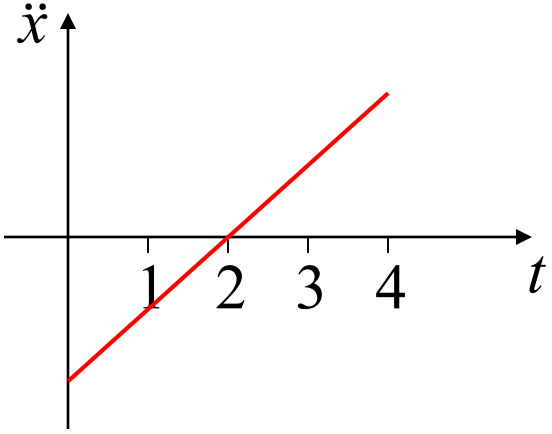
NOTE: "deceleration" or slowing down is when acceleration is in the opposite direction to velocity.



slope=instantaneous velocity

slope=instantaneous acceleration





e.g. (*i*) distance traveled $= 7\text{m}$

(*ii*) total displacement $= -1\text{m}$

(*iii*) average speed $= \frac{7}{4}\text{m/s}$

(*iv*) average velocity $= \frac{-1}{4}\text{m/s}$

e.g. (*i*) The displacement x from the origin at time t seconds, of a particle traveling in a straight line is given by the formula $x = t^3 - 21t^2$

a) Find the acceleration of the particle at time t.

$$x = t^3 - 21t^2$$

$$v = 3t^2 - 42t$$

$$a = 6t - 42$$

b) Find the times when the particle is stationary.

Particle is stationary when $v = 0$

$$\text{i.e. } 3t^2 - 42t = 0$$

$$3t(t - 14) = 0$$

$$t = 0 \text{ or } t = 14$$

Particle is stationary initially and again after 14 seconds

(*ii*) A particle is moving on the x axis. It started from rest at $t = 0$ from the point $x = 7$.
If its acceleration at time t is $2 + 6t$ find the position of the particle when $t = 3$.

$$a = 2 + 6t$$

$$v = 2t + 3t^2 + c$$

when $t = 0, v = 0$

i.e. $0 = 0 + 0 + c$

$$c = 0$$

$$\therefore v = 2t + 3t^2$$

$$x = t^2 + t^3 + c$$

when $t = 0, x = 7$

i.e. $7 = 0 + 0 + c$

$$c = 7$$

$$\therefore x = t^2 + t^3 + 7$$

when $t = 3, x = 3^2 + 3^3 + 7$

$$= 43$$

after 3 seconds the particle is 43 units to the right of O.

OR

$$\frac{dv}{dt} = 2 + 6t$$

$$\int_0^v dv = \int_0^t (2 + 6t)\,dt$$

$$v = \left[2t + 3t^2\right]_0^t$$

$$v = 2t + 3t^2$$

$$\frac{dx}{dt} = 2t + 3t^2$$

$$\int_7^x dx = \int_0^3 \left(2t + 3t^2\right)dt$$

$$[x]_7^x = \left[t^2 + t^3\right]_0^3$$

$$x - 7 = 3^2 + 3^3 - 0$$

$$x = 43$$

e.g. **2001 HSC Question 7c)**

A particle moves in a straight line so that its displacement, in metres, is given by $x = \dfrac{t-2}{t+2}$

where t is measured in seconds.

(i) What is the displacement when $t = 0$?

$$\text{when } t = 0, x = \frac{0-2}{0+2}$$

$$= -1$$

$\therefore$ the particle is 1 metre to the left of the origin

(ii) Show that $x = 1 - \dfrac{4}{t+2}$

Hence find expressions for the velocity and the acceleration in terms of t.

$$1 - \frac{4}{t+2} = \frac{t+2-4}{t+2}$$

$$= \frac{t-2}{t+2}$$

$$\therefore x = 1 - \frac{4}{t+2}$$

$$v = -\frac{4(-1)}{(t+2)^2}$$

$$v = \frac{4}{(t+2)^2}$$

$$a = \frac{4 \times -2(t+2)^1(1)}{(t+2)^4}$$

$$a = \frac{-8}{(t+2)^3}$$

(iii) Is the particle ever at rest? Give reasons for your answer.

$$v = \frac{4}{(t+2)^2} \neq 0$$

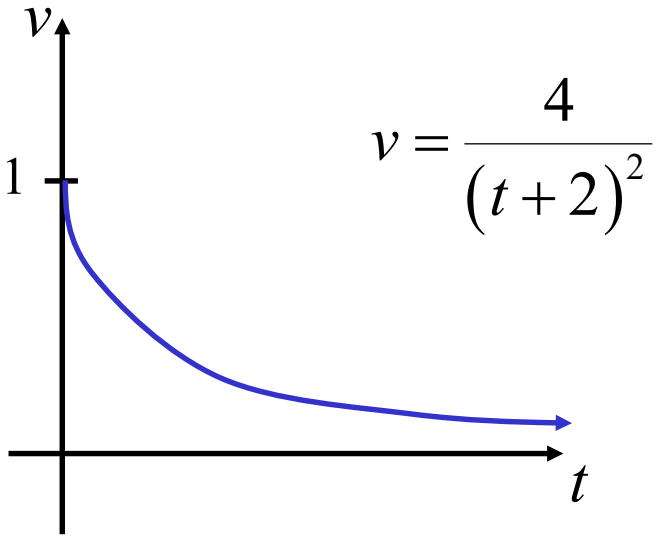
$\therefore$ the particle is never at rest

(iv) What is the limiting velocity of the particle as t increases indefinitely?

$$\lim_{t\to\infty} v = \lim_{t\to\infty} \frac{4}{(t+2)^2}$$

$$= 0$$

OR



$\therefore$ the limiting velocity of the particle is 0 m/s

(ii) **2002 HSC Question 8b)**

A particle moves in a straight line. At time t seconds, its distance x metres from a fixed point O in the line is given by $x = \sin 2t + 3$

(i) Sketch the graph of x as a function of t for $0 \le t \le 2\pi$

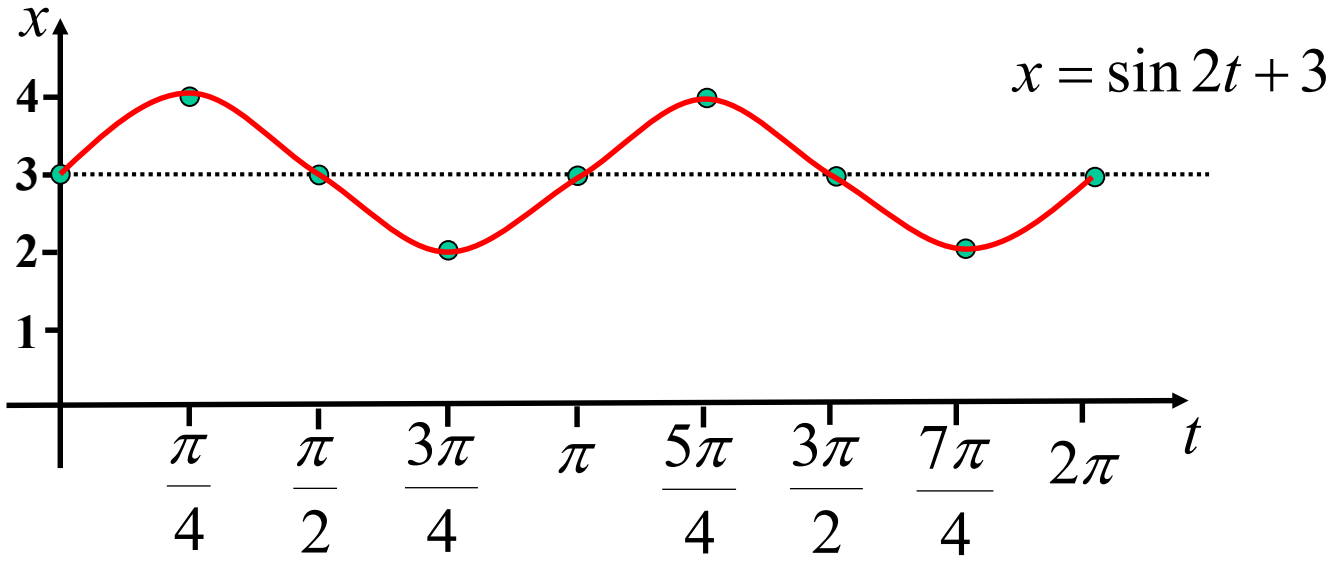
amplitude $= 1$ unit

period $= \dfrac{2\pi}{2}$

$= \pi$

divisions $= \dfrac{\pi}{4}$

shift $= \uparrow 3$ units



(ii) Using your graph, or otherwise, find the times when the particle is at rest, and the position of the particle at those times.

Particle is at rest when velocity = 0

$\frac{dx}{dt} = 0$ i.e. the stationary points

when $t = \frac{\pi}{4}$ seconds, $x = 4$ metres

$t = \frac{3\pi}{4}$ seconds, $x = 2$ metres

$t = \frac{5\pi}{4}$ seconds, $x = 4$ metres

$t = \frac{7\pi}{4}$ seconds, $x = 2$ metres

(iii) Describe the motion completely.

The particle oscillates between x=2 and x=4 with a period of π seconds

Exercise 9B; 3, 5, 7, 8, 10, 11, 13, 15