Rates of Change & Integration

Integration allows the original quantity to be found from the rate of change.

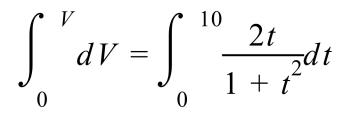
e.g. (*i*) 2017 Mathematics HSC Q13 d) The rate at which water flows into a tank is given by

$$\frac{dV}{dt} = \frac{2t}{1+t^2}$$

where *V* is the volume of water in the tank in litres and *t* is the time in seconds. Initially the tank is empty.

Find the exact amount of water in the tank after 10 seconds.

$$\frac{dV}{dt} = \frac{2t}{1+t^2}$$



Solving as a definite integral 1. separate the variables 2. match the limits of the variables eliminates the need to find the constant of integration

$$\int_{0}^{V} dV = \int_{0}^{10} \frac{2t}{1+t^{2}} dt$$
$$\left[V\right]_{0}^{V} = \left[\ln(1+t^{2})\right]_{0}^{10}$$
$$V = \ln\left(\frac{101}{1}\right)$$

$= \ln(101)$

: there is $\ln(101)$ litres of water left in the tank

(*ii*) 2010 Mathematics HSC Q7 a)(ii)The acceleration of a particle is given by;

 $\ddot{x} = 4\cos 2t$

where *x* is displacement in metres and *t* is time in seconds. Initially the particle is at the origin with a velocity of 1 ms^{-1}

Find when the particle first comes to rest.

 $\frac{dv}{dt} = 4\cos 2t$ $\sin 2t = -\frac{1}{2}$ $\int_{-1}^{0} dv = 4 \int_{0}^{t} \cos 2t dt$ $2t = \frac{7\pi}{6}$ $\left[v\right]_{1}^{0} = 2\left[\sin 2t\right]_{0}^{t}$ $t = \frac{7\pi}{12}$ $-1 = 2\sin 2t$ \therefore the particle first comes to rest after $\frac{7\pi}{12}$ seconds (*iii*) 2013 Extension 1 HSC Q13 a)

A spherical raindrop of radius r metres loses water through evaporation at a rate that depends upon its surface area. The rate of change of the volume V of the raindrop is given by

$$\frac{dV}{dt} = -10^{-4} A$$
where t is in seconds and A is the surface area of the rain drop.
a) Show that $\frac{dr}{dt}$ is a constant.

$$\frac{dr}{dt} = ? \quad \frac{dV}{dt} = -10^{-4} A \qquad V = \frac{4}{3} \pi r^{3}$$

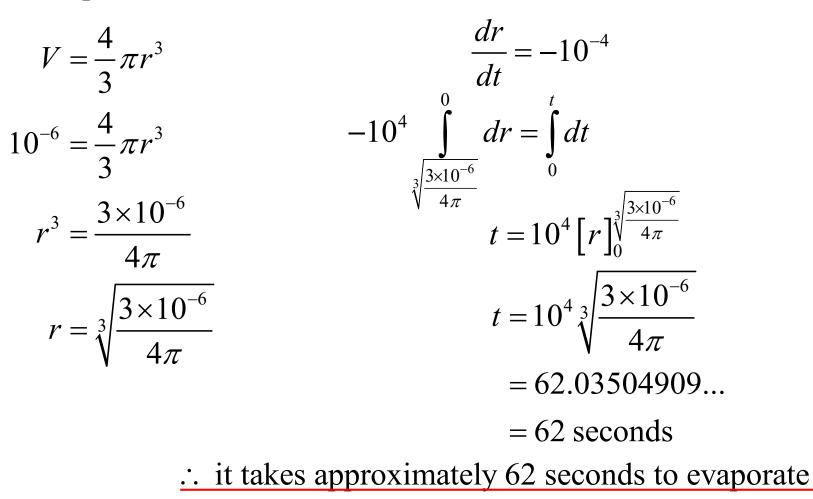
$$\frac{dV}{dt} = 4\pi r^{2} \qquad \frac{dr}{dt} = -10^{-4} A \cdot \frac{1}{A}$$

$$\frac{dV}{dr} = 4\pi r^{2} \qquad \frac{dr}{dt} = -10^{-4} A \cdot \frac{1}{A}$$

$$\therefore \frac{dV}{dr} = A$$

 \therefore radius decreases at a constant rate of 10^{-4} m / s

b) How long does it take for a raindrop of volume 10^{-6} m³ to completely evaporate?



Exercise 9F; 2, 3, 6, 8, 9, 11, 12, 13