# *Expected Value*

Let $X$ be a **continuous random variable**, then the expected value of $X$ is;

$$\boxed{E(X) = \int_{-\infty}^{\infty} x f(x)\,dx}$$

***Note:*** $\mathrm{E}(X) = \mu \quad$ (arithmetic mean)

e.g. It is proposed to model the quarterly salary, $X$, measured in thousands of dollars, paid to salespeople in a large company by the probability density function

$$f(x) = \begin{cases} 2560x^{-\frac{7}{2}} & x \geq 16 \\ 0 & \text{elsewhere} \end{cases}$$

By looking at the median and mean, describe the distribution of the salaries

$$2560\int_{16}^{M} x^{-\frac{7}{2}}dx = \frac{1}{2}$$

$$-1024\left[x^{-\frac{5}{2}}\right]_{16}^{M} = \frac{1}{2}$$

$$M^{-\frac{5}{2}} - \frac{1}{1024} = -\frac{1}{2048}$$

$$M = \left(\frac{1}{2048}\right)^{-\frac{2}{5}}$$

$$= 21.112$$

median quarterly salary is $21 112

$$\lim_{x \to \infty} x^{-\frac{3}{2}} = 0$$

$$\mu = 2560\int_{16}^{\infty} x \times x^{-\frac{7}{2}}dx$$

$$= 2560\int_{16}^{\infty} x^{-\frac{5}{2}}dx$$

$$= -\frac{5120}{3}\left[x^{-\frac{3}{2}}\right]_{16}^{\infty}$$

$$= -\frac{5120}{3} \times -\frac{1}{64}$$

$$= 26.667$$

mean quarterly salary is $26 667

as median < mean, the distribution is positively skewed

# *Variance*

Let $X$ be a **continuous random variable**, then the variance of $X$ is;

$$\boxed{\mathrm{Var}(X) = \mathrm{E}\left(X^2\right) - \mu^2}$$

where $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx$

e.g. Find the standard deviation of the quarterly salaries.

$$\mathrm{Var}(X) = 2560\int_{16}^{\infty} x^2 \times x^{-\frac{7}{2}} dx - \left(\frac{80}{3}\right)^2$$

$$= 2560\int_{16}^{\infty} x^{-\frac{3}{2}} dx - \left(\frac{6400}{9}\right)$$

$$\text{Var}(X) = -5120\left[x^{-\frac{1}{2}}\right]_{16}^{\infty} - \frac{6400}{9}$$

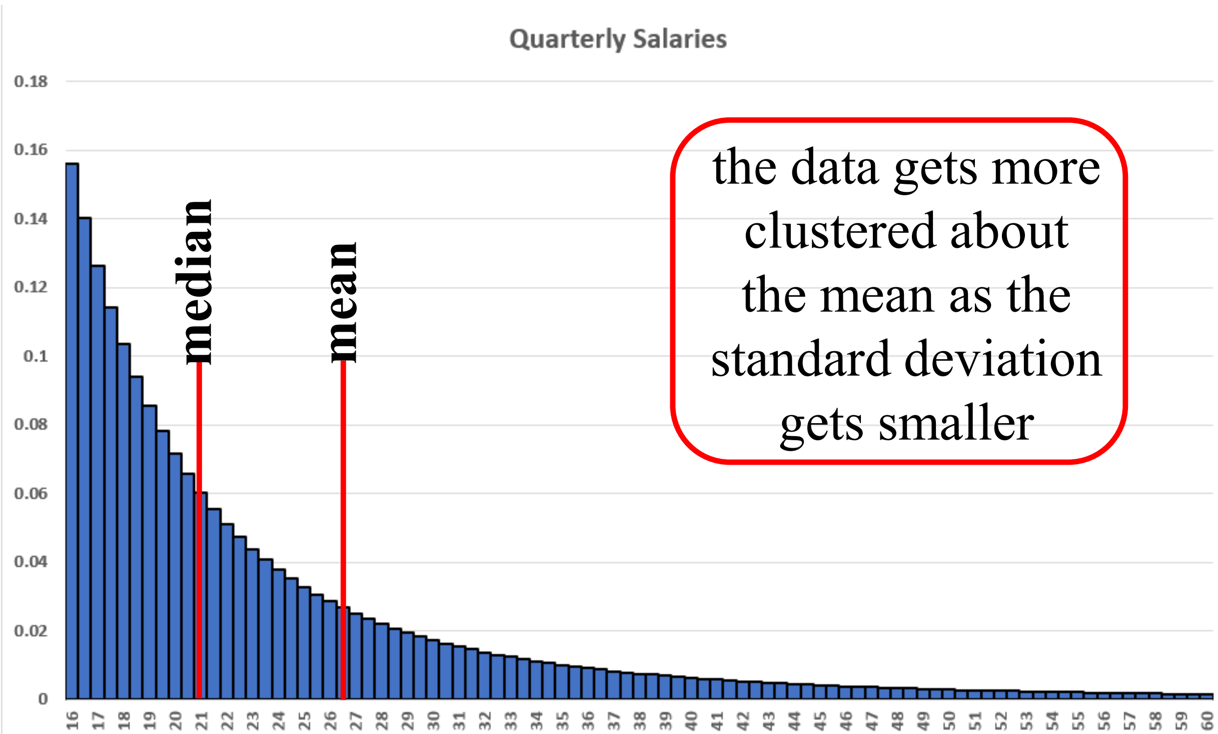
$$= -5120 \times -\frac{1}{4} - \frac{6400}{9}$$

$$= 568.889$$

$$\sigma = \sqrt{568.889}$$

$$= 23.851$$

standard deviation of the quarterly salary is \$23 851




**Exercise 16C; 1abce, 2, 3, 5, 6, 9, 10, 11, 12**