

# *Implicit Differentiation*

$$\frac{df}{dx} = \frac{df}{dy} \times \frac{dy}{dx}$$

e.g. (i)  $y = x^2$

$$\frac{d}{dx}(y) = \frac{d}{dx}(x^2)$$

$$\frac{d}{dy}(y) \times \frac{dy}{dx} = \frac{d}{dx}(x^2)$$

$$1 \times \frac{dy}{dx} = 2x$$

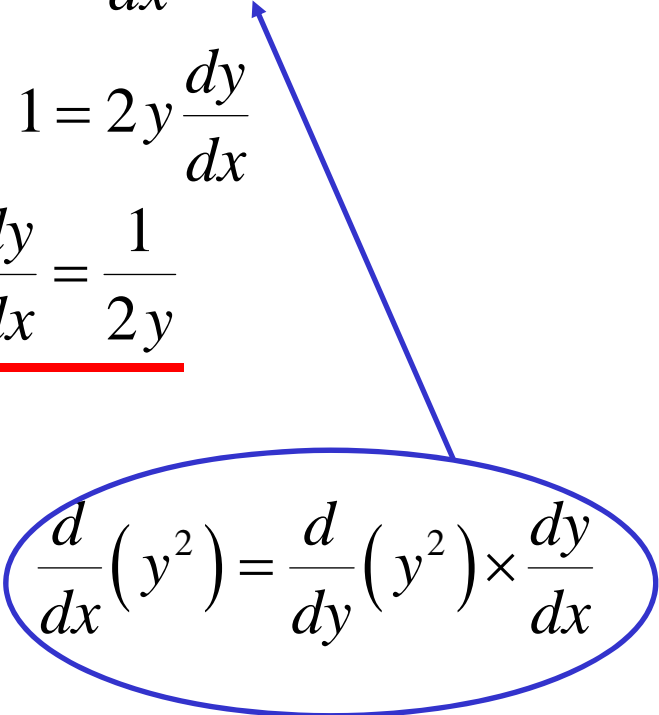
$$\underline{\frac{dy}{dx} = 2x}$$

(ii)  $x = y^2$

$$\frac{d}{dx}(x) = \frac{d}{dx}(y^2)$$

$$1 = 2y \frac{dy}{dx}$$

$$\underline{\frac{dy}{dx} = \frac{1}{2y}}$$


$$\frac{d}{dx}(y^2) = \frac{d}{dy}(y^2) \times \frac{dy}{dx}$$

$$\begin{aligned} \text{(iii)} \quad & \frac{d}{dx}(x^2 y^3) \\ &= (x^2) \left( 3y^2 \frac{dy}{dx} \right) + (y^3)(2x) \\ &= 3x^2 y^2 \frac{dy}{dx} + 2xy^3 \end{aligned}$$

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(iv) Find the equation of the tangent to  $x^2 + y^2 = 9$  at the point  $(1, 2\sqrt{2})$

$$x^2 + y^2 = 9$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\text{at } (1, 2\sqrt{2}), \frac{dy}{dx} = -\frac{1}{2\sqrt{2}}$$

$$\therefore \text{required slope} = -\frac{1}{2\sqrt{2}}$$

$$y - 2\sqrt{2} = -\frac{1}{2\sqrt{2}}(x - 1)$$

$$2\sqrt{2}y - 8 = -x + 1$$

$$\underline{x + 2\sqrt{2}y - 9 = 0}$$

**“Old Cambridge” Exercise 7K;  
1acegi, 2bdfh, 3a,  
4a, 7, 8**